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"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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(Corrected to 1 January 1912.)

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- 1870 †Hardy, Ralph Price, F.F.A.,
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- 1893 †Harris, Arnold Stoughton, M.A.,
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- 1892 †Hart, James Robert,
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70 Lombard-street, E.C.
- 1879 Harvey, Chas. J.,
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- 1888 †Hemming, Arthur George, F.S.S.,
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- 1896 †Henderson, Robert, B.A., F.A.S.,
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New York, U.S.A.
- 1883 Hewat, Archibald, F.F.A., F.R.S.E.,
13 Eton-terrace, Edinburgh.
- 1908 †Hicks, Arthur Joseph,
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- 1874 †Higham, Charles Daniel, F.A.S.,
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81 King William-street, E.C.
- 1898 †Hodgson, William Horsford,
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- 1911 †Home, Noel Charles Minchin,
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E.C.
- 1888 †Hopkins, William Raynes,
Ibrox, 140 Highbury-new-park,
N.
- 1890 †Hovil, Lewis Frederick,
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National Provident Institution,
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(PAST-PRESIDENT, 1902-4),
62 Palace-road, Tulse-hill, S.W.

Date of
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a Fellow.

- 1906 †Humphreys, Henry Thompson,
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63 Threadneedle-street, E.C.
- 1894 †Hutcheson, William Anderson,
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Mutual Life Ins. Co. of New York,
Nassau-street, New York, U.S.A.
- 1893 †Hutton, William, F.F.A.,
Scottish Amicable Life Assur.
Society, 35 St. Vincent-place,
Glasgow.
- 1911 †Jamieson, Charles William Steele,
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Scottish Amicable Life Assur.
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- 1903 †Jarman, William Rees, B.A.,
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- 1910 †Jefferson, John Arthur,
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- 1869 †Justican, Edwin, F.S.S.,
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Limited. St. Mildred's-house,
Poultry, E.C.
- 1906 †Kelham, Cyril Stephen,
Prudential Assurance Company,
Holborn-bars, E.C.
- 1910 †Kelly, John Joseph,
Mutual Life & Citizens' Assur-
ance Co., Ltd., Sydney, Australia.
- 1902 †Kenchington, Charles William,
Prudential Assurance Company
Holborn-bars, E.C.
- 1897 †Kentish, Owen,
Alliance Assurance Co., Ltd.,
6 New Bridge-street, E.C.
- 1874 †King, George, F.F.A., F.A.S.,
15 Walbrook, E.C.
- 1907 †Laing, James Murray, F.F.A.,
Britannic Assurance Company,
Limited, Broad-street-corner,
Birmingham.
- 1882 Lancaster, Sir William John,
South Lynn, Putney-hill, S.W.
- 1909 †Langstaff, James Miles, F.A.S.,
C.A. (Ont.),
666 Bathurst-street, Toronto,
Canada.
- 1894 †Loughton, Alexander Millar, F.F.A.,
F.S.S.,
Victorian Government Statist,
Queen Street, Melbourne, Aus-
tralia.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1887	†Lemon, William Kent, Barrister-at-Law, 1 Vanbrugh-terrace, Blackheath, S.E.	1901	†Macphail, Donald, F.F.A., <i>Sauer's Buildings, Loveday-st., Johannesburg, South Africa.</i>
1910	†Levey, Ralph, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1910	†Makepeace, Francis Lucas, M.A., <i>National Standard Life Assurance Corporation, Ltd, 38 Church-street, Liverpool.</i>
1896	†Levine, Abraham, M.A., <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1908	†Malthy, Charles Hugh, <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>
1896	†Lewis, John Norman, F.F.A., <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>	1870	†Manly, Henry William, F.A.S., (PAST-PRESIDENT, 1898-1900). <i>Glenthorne, 157 Highbury-new-park, N.</i>
1892	†Lidstone, George James, F.A.S. (VICE-PRESIDENT), <i>Equitable Life Assurance Soc., Mansion-house-street, E.C.</i>	1890	†Marks, Geoffrey, (VICE-PRESIDENT), <i>National Mutual Life Assur. Soc., 39 King-street, Cheapside, E.C.</i>
1901	†Little, James Fulton, <i>Departamento de Seguros de la Secretaria de Hacienda, Mexico.</i>	1900	†Marr, Vyvyan, F.F.A., <i>United Kingdom Temperance and General Provident Institution, 196 Strand, W.C.</i>
1899	Low, George Maeritchie, F.F.A., <i>Scottish Equitable Life Assur. Society, 28 St. Andrew-square, Edinburgh.</i>	1909	†Maudling, Reginald George. <i>St. Stephen's House, Victoria Embankment, Westminster, S.W.</i>
1899	†Lutt, Harold Edward William, <i>Northern Assurance Company, Limited, 1 Moorgate-street, E.C.</i>	1902	†May, Basil, <i>British Equitable Assurance Co., Ltd., 1, 2 & 3 Queen-street-place, E.C.</i>
1898	†Macanlay, Thomas Bassett, F.A.S., <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>	1897	†May, George Ernest, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1874	McClintock, Emory, F.A.S., <i>Mutual Life Insurance Company of New York, New York, U.S.A.</i>	1906	†May, Walter Thomas, <i>Liverpool and London and Globe Insur. Co., Ltd., 1 Dale-street, Liverpool.</i>
1894	†McDonald, John, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1908	†Melville, Henry Edward, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>
1883	†McGowan, James, B.A., <i>c/o Standard Bank of South Africa, 10 Clement-s-lane, E.C.</i>	1897	†Miller, Neville, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>
1885	Mackenzie, Alexander George, F.F.A., 29 Chester-terrace, Regent's-park, N.W.	1905	†Milligan, Charles Livingstone, <i>Alliance Ass. Co., Ltd. (Provident Life Fund), 50 Regent-street, W.</i>
1907	†Mackenzie, Michael Alexander, <i>University of Toronto, Toronto, Canada.</i>	1899	†Moir, Henry, F.F.A., F.A.S., <i>Home Life Insurance Co., 256 Broadway, New York, U.S.A.</i>
1911	†Maclean, Joseph Brotherton, F.F.A., <i>Mutual Life Insurance Co. of New York, 32 Nassau-street, New York, U.S.A.</i>	1890	†Molyneux, Arthur Ernest, <i>Provident Clerks' and General Mutual Life Assurance Assoc., 27 & 29 Moorgate-street, E.C.</i>
1900	†Macnaghten, Stuart Edye, A.C.A., <i>Standard Life Assurance Co., 3 George-street, Edinburgh.</i>	1901	†Moorhouse, Alfred, <i>Friends' Provident Institution, Bradford, Yorkshire.</i>

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1897	†Moors, Elphinstone McMahon, M.A., <i>University of Sydney, Australia.</i>	Under the Charter.	Priestley, John George, 44 <i>St. German's-road, Forest hill, S.E.</i>
1896	†Moran, Joseph Flack, <i>Reversionary Interest Society, 30 Coleman-street, E.C.</i>	1891	†Pulley, William Pritchard, <i>Norwich Union Life Insur. Soc., 71 & 72 King William-st., E.C.</i>
1900	†Morgan, Benjamin Charles, M.A., <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>	1903	†Rae, Joseph, <i>Finance Department, Town-hall, Upper-street, N.</i>
1895	†Muter, Percy, <i>New Zealand Government Life Insurance Department, Wellington, New Zealand.</i>	1899	†Raisin, Arthur Herbert, <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.</i>
1888	†Nash, Willie Oscar, <i>Law Reversionary Interest Soc., Limited, Thanet-house, 231 & 232 Strand (opposite the Law Courts), W.C.</i>	1909	†Raynes, Harold Ernest, <i>Legal and General Life Assurance Society, 10 Fleet-street, E.C.</i>
1906	†Neill, Samuel Bennett, F.S.S., <i>China Mutual Life Insurance Co., Shanghai, China.</i>	1897	†Rees, Martin, <i>Law Reversionary Interest Soc., Limited, Thanet-house, 231 & 232 Strand (opposite the Law Courts), W.C.</i>
1883	Neison, Francis G. P., F.S.S., <i>2 Brick-court, Temple, E.C.</i>	1901	†Reeve, Charles Ernest, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>
1888	†Newman, Philip Lewin, B.A., <i>Yorkshire Insur. Co., Ltd., York.</i>	1902	†Richmond, George William, <i>Scottish Widows' Fund and Life Assur. Society, 9 St. Andrew-square, Edinburgh.</i>
1865	Newton, Algernon, M.A., <i>c/o The Manager, London County and Westminster Bank, 1 Stratford-place, W.</i>	1904	†Rietschel, Hermann Julius, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1910	†Nicholl, Charles Carlyon, B.A., F.F.A., <i>Equitable Life Assce. Society, Mansion-house-street, E.C.</i>	1898	†Robinson, George Frederick, <i>Legal and General Life Assur. Society, 10 Fleet-street, E.C.</i>
1887	†Nightingale, Harry Ethelston, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>	1905	†Robinson, Hugh Thomas Kay, <i>Clergy Mutual Assur. Soc., 2 & 3 The Sanctuary, S.W.</i>
1903	†Norris, Charles Arthur, <i>National Mutual Life Association of Australasia, Limited, Melbourne, Australia.</i>	1888	†Rusher, Edward Arthur, F.S.S., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1901	†Norton, William Ernest, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>	1882	†Ryan, Sir Gerald Hemmington, (PRESIDENT), <i>Phoenix Assurance Co., Ltd., 19 Lombard-street, E.C.</i>
1905	†Oakley, Henry John Percy, <i>North British and Mercantile Insurance Company, 61 Threadneedle-street, E.C.</i>	1898	†Salmon, Richard George, F.S.S., <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1864	Pearson, Arthur, <i>Betchworth-house, The Bank, Highgate, N.</i>	1883	Saunders, Harris Charter Lindon, F.R.A.S., "Marquise," Twickenham.
1905	†Penman, William, Jr., <i>Atlas Assurance Company, Ltd., 92 Cheapside, E.C.</i>	1886	†Schooling, Frederick, F.A.S., (TREASURER), <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1891	†Phelps, William Peyton, M.A., (HON. SEC.), <i>Equity and Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.</i>		

FELLOWS.

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Date of becoming a Fellow.		Date of becoming a Fellow.	
1901	†Searle, George Morley, <i>Sun Life Assurance Society,</i> 63 Threadneedle-street, E.C.	1898	Stirling, Robert, F.F.A., <i>Law Union & Rock Insur. Co.,</i> Ltd., 126 Chancery-Lane, W.C.
1901	†Sharman, William Charles, <i>Prudential Assurance Company,</i> Holborn-bars, E.C.	1892	†Straker, Edward Robert, <i>Phoenix Assurance Co., Ltd.,</i> 70 Lombard-street, E.C.
1905	†Sherriff, Francis Henry, <i>Provident Clerks' and General</i> <i>Mutual Life Assurance Assoc.,</i> 27 & 29 Moorgate-street, E.C.	1878	†Straker, Frank Arthur, <i>Legal and General Life Assur.</i> <i>Society,</i> 10 Fleet-street, E.C.
1896	†Sim, William Abernethy, F.F.A., <i>Scottish Union and National</i> <i>Insurance Co.,</i> 3 King William- street, E.C.	1902	†Strong, William Richard, <i>London Guarantee & Accident</i> <i>Co.,</i> 42-45 New Broad-street, E.C.
1911	†Simmonds, Reginald Claud, <i>Alliance Assurance Co., Ltd.,</i> Bartholomew-lane, E.C.	1884	†Stuart, John Moody, F.F.A., <i>Leeds Permanent Benefit Build-</i> <i>ing Society,</i> Victoria-buildings, Park-lane, Leeds.
1875	†Smither, Arthur, <i>Green Bank, Lewes.</i>	1900	†Sutherland, John, M.A., <i>Australasian Temperance and</i> <i>General Mutual Life Assurance</i> <i>Society,</i> Swanston-street, Mel- bourne, Australia.
1911	†Sneddon, Andrew William, <i>Australian Mutual Provident</i> <i>Society,</i> South Sea House, 37 Threadneedle-st., E.C.	1906	†Symmons, Frank Percy, <i>Prudential Assurance Company,</i> Holborn-bars, E.C.
1881	†Somerville, William Finlay, <i>Liverpool and London and Globe</i> <i>Insur. Co., Ltd.,</i> 1 Dale-street, Liverpool.	1889	†Tarn, Arthur Wyndham, <i>Guardian Assurance Company,</i> 28 King-street, Covent-garden, W.C.
1877	†Sorley, James, F.S.S., F.R.S.E., 82 Onslow-gardens, S.W.	1887	Teece, Richard, F.F.A., F.A.S., F.S.S., <i>Australian Mutual Provident</i> <i>Society,</i> Sydney, Australia.
1898	†Spencer, John (LIBRARIAN), <i>English and Scottish Law Life</i> <i>Assurance Assoc.,</i> 33 St. James's- square, S.W.	1889	†Thiselton, Herbert Cecil, F.F.A., F.A.S., <i>Commercial Union Assurance</i> <i>Co.,</i> 24, 25 & 26 Cornhill, E.C.
1894	†Sprague, Alfred Ernest, D.Sc., M.A., F.F.A., <i>Edinburgh Life Assurance Co.,</i> 26 George-street, Edinburgh.	1901	†Thodey, Robert, <i>Australian Mutual Provident</i> <i>Society,</i> Sydney, Australia.
1857	Sprague, Thomas Bond, M.A., LL.D., Hon. F.F.A., F.S.S., F.R.S.E. (PAST-PRESIDENT, 1882-86), 29 Buckingham-ter., Edinburgh.	1893	†Thomas, Ernest Charles, <i>Gresham Life Assurance Society,</i> Limited, St. Mildred's-house, Poultry, E.C.
1906	†Spurgeon, Ernest Frank, (TUTOR, Part II), <i>Prudential Assurance Company,</i> Holborn-bars, E.C.	1899	†Thomas, Robert Arthur Caradoc, <i>Phoenix Assurance Co., Ltd.,</i> 70 Lombard-street, E.C.
1896	†Stahlschmidt, Rev. Louis, F.C.I., <i>St. John's College, Agra, India.</i>	1910	†Thompson, John Spencer, M.A., F.F.A., F.A.S., <i>Mutual Life Insurance Co. of</i> <i>New York, New York, U.S.A.</i>
	Under the Charter Stevens, Charles, <i>Aberdeen Ho., Preston, Brighton.</i>	1905	†Thompson, Thomas Percy, B.A., (TUTOR, Part I), <i>Phoenix Assurance Co., Ltd.,</i> 70 Lombard-street, E.C.
1888	Stewart, John, F.F.A., <i>City of Glasgow Life Assur. Co.,</i> 30 Renfield-street, Glasgow.		
1906	†Stewart, Lionel William, <i>Cedar Lodge, Little Ealing, W.</i>		

FELLOWS.

Those marked † are Fellows by Examination

Date of becoming a Fellow.		Date of becoming a Fellow.	
1895	†Thomson, Herbert Archer, M.A., 3 Kings Bench-walk, Temple, E.C.	1904	†Weatherill, Henry, National Debt Office, E.C.
1893	†Thorne, Alfred Charles, Equity & Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.	1880	†Whittall, Wm. Joseph Hutchings, F.A.S., 18 Airlie-gardens, Campden-hill, W.
1891	†Tilt, Robert Ruthven, General Reversionary & Investment Co., Ltd., 26 Pall-mall, S.W.	1905	†Wilson, John Sydney, Mutual Life & Citizens' Assur. Co., Ltd., Brisbane, Australia.
1902	†Tinner, Thomas, Comptroller's Depart., London County Council, Spring-gardens, S.W.	1864	Wilson, Robert, 44 Talfourd-rd., Camberwell, S.E.
1881	†Todd, George, M.A., Alliance Assurance Co., Ltd., 6 New Bridge-street, E.C.	1888	†Wilson, Robert, Jr., General Assurance Company, 103 Cannon-street, E.C.
1894	†Todhunter, Ralph, M.A., University Life Assur. Soc., 25 Pall-mall, S.W.	Under the Charter.	Winsor, Thomas Boorman, F.R.G.S., F.R.N.S., 81 Shooter's-hill-road, Blackheath, S.E.
1911	†Townley, E. William, United Kingdom Temperance and General Provident Institution, 196 Strand, W.C.	1899	†Winter, Arthur Thomas, Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.
1899	†Trouncer, Harold Moltke, M.A., London Life Association, Ltd., 81 King William-street, E.C.	1897	†Wintle, Lancelot Andrewes, Alliance Assurance Co., Ltd., 6 New Bridge-street, E.C.
1878	Turnbull, Andrew Hugh, F.F.A., F.R.S.E., 18 Whitehouse-loan, Edinburgh.	1904	†Wood, Arthur Barton, B.A., F.A.S., Sun Life Assurance Co. of Canada, Montreal, Canada.
1909	†Turner, Sidney, B.A., 20 Minster-road, Cricklewood, N.W.	1884	†Woods, Ernest, F.A.S., Guardian Assurance Company, 11 Lombard-street, E.C.
1912	†Vaughan, Hubert, Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.	1902	†Woolmer, Alfred Henry, Star Assurance Society, 32 Moorgate-street, E.C.
1889	Wallace, Thomas, F.F.A., North British & Mercantile Insurance Co., 64 Princes-street, Edinburgh.	1902	†Workman, William Arthur, Legal and General Life Assur. Soc., 10 Fleet-street, E.C.
1905	†Wandless, John Robert, Canada Life Assurance Co., 14 King William-street, E.C.	1902	†Worthington, William, Royal Insur. Co., Ltd., Liverpool.
1906	†Wares, Harold Wallace, Yorkshire Insurance Co., Ltd., Bank-buildings, Princes-street, E.C.	1875	†Wyatt, Frank Bertrand, F.A.S., (PAST-PRESIDENT 1906-8), Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary, S.W.
1888	†Warner, Samuel George, Law Union & Rock Insur. Co., Ltd., 126 Chancery-lane, W.C.	1906	†Young, Arthur Stanley, F.S.S., H.H. The Nizam's Government, Hyderabad, The Deccan, India.
1893	†Watson, Alfred William, F.S.S., National Insurance Joint Committee, Whitehall, S.W.	1874	Young, Thomas Emley, B.A., F.R.A.S., (PAST-PRESIDENT, 1896-8), 108 Evering-road, Stoke Newington, N.
1895	†Watson, James Douglas, F.A.S., (LIBRARIAN), Star Assurance Society, 32 Moorgate-street, E.C.		

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.		Date of becoming an Associate.	
1900	² Adams, Cecil Francis, <i>The Co-operative Assurance Co., Ltd., 129 Pitt-street, Sydney, Australia.</i>	1885	Barton, Arthur, <i>Royal Insurance Company, Ltd., Maidstone.</i>
1908	² Addey, Leonard, <i>Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary, s.w.</i>	1908	² Beatty, Samuel, M.A., <i>University of Toronto, Toronto, Canada.</i>
1869	² Adey, Theodore Henry, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>	1901	² Benjamin, Stanley O., <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1908	³ Alder, Milton Cromwell, <i>Mutual Life & Citizens' Assurance Co., Limited, Sydney, Australia.</i>	1908	² Bennett, Samuel, <i>Registrar of Friendly Societies, and Government Actuary, Perth, West Australia.</i>
1911	² Allen, Sidney, A.C.A., <i>c/o Messrs. M. O. Beale, Greaves and Co., 11 Ironmonger-lane, E.C.</i>	1881	Birks, Edmund Alfred, <i>Yorkshire Ins. Co., Ltd., York.</i>
1908	² Anderson, Robert Duncan, <i>120 Burnt Ash Hill, Lee, s.e.</i>	1906	² Blake, Francis Seymour, <i>Comptroller's Dept., London County Council, Spring-gardens, s.w.</i>
1899	² Ansell, George Frederic, <i>National Debt Office, E.C.</i>	1906	² Blehl, Ernest M., A.M., A.A.S., <i>Philadelphia Life Insurance Co., North American Building, Philadelphia, Pa., U.S.A.</i>
1904	² Ashley, Charles Henry, <i>British Widows' Assurance Co., 1 Old-street, E.C.</i>	1898	(²) Blount, Edward Thos. Joseph, F.F.A., F.S.S., <i>Standard Life Assurance Co., 3 George-street, Edinburgh.</i>
1883	² Ashley, John Geo., M.A., <i>War Office, s.w.</i>	1910	³ Blyth, Robert Oswald, M.A., F.F.A., <i>Scottish Amicable Life Assurance Society, 35 St. Vincent-place, Glasgow.</i>
1881	² Ayling, Charles Stephen, <i>Commercial Union Assur. Co., 26 New Bridge-street, E.C.</i>	1906	² Boag, Harold, F.S.S., <i>33 Albert-drive, Low Fell, Gateshead.</i>
1905	² Bain, William Algernon, B.A., A.A.S., <i>Manufacturers Life Insurance Co., Toronto, Canada.</i>	1873	² Boon, Gerald Inglis, <i>14 Bullingham-mansions, Kensington, w.</i>
1909	² Baker, Sydney Harry, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1906	² Borrajo, Edward Joseph William, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1903	³ Ball, Sidney Robertson, <i>English and Scottish Law Life Assurance Association, 33 St. James's-square, s.w.</i>	1908	² Bradbury, Algernon Charles, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1905	² Barford, Frederick William, M.A., <i>Commonwealth Bureau of Census and Statistics, Melbourne, Australia.</i>	1889	(²) Bremner, Thomas William, F.F.A., <i>Mutual Life of New York Building, Martin-place, Sydney, Australia.</i>
1903	² Barnett, Isaac, <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>	1907	³ Brown, Arthur Ewart, <i>Metropolitan Life Assurance Society, 13 Moorgate-street, E.C.</i>
1904	² Barrett, William Goodsman, <i>United Kingdom Temperance and General Provident Institution, 196 Strand, w.c.</i>	1910	² Brown, B. G. H., <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.		Date of becoming an Associate.	
1896	⁽²⁾ Brown, George Andrew, <i>Clerical, Medical & General Life Assurance Society, 1 King William-street, E.C.</i>	1911	⁽²⁾ Cockburn, Arthur, F.F.A., <i>Caledonian Insurance Co., Ltd., 19 George-street, Edinburgh.</i>
1899	² Brown, Harold, <i>Scottish Union and National Insurance Co., 35 St. Andrew-square, Edinburgh.</i>	1904	² Collier, Charles Aubrey, <i>6 Old Palace-yard, S.W.</i>
1910	² Brown, Peter Gordon, <i>Ecclesiastical Commission, Millbank, S.W.</i>	1871	Cook, Arthur James, M.J.I., F.S.S., <i>6 Dorset-square, Regent's-park, N.W.</i>
1882	Burke, David, F.S.S., <i>Royal Victoria Life Insur. Co., Montreal, Canada.</i>	1910	² Cook, Henry Milton, B.A., <i>Mutual Life Assurance Co. of Canada, Waterloo, Ontario, Canada.</i>
1906	² Burrows, George Eastoe, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1899	³ Cook, William Playfair, <i>Guardian Assurance Company, 11 Lombard-street, E.C.</i>
1895	³ Butterfield, William Thos., F.C.A., <i>9 Market-street, Bradford, Yorkshire.</i>	1897	² Coop, Charles Rowland, <i>United Kingdom Temperance and General Provident Institution, 28 High-street, Birmingham.</i>
1911	² Cammack, Edmund Ernest, F.A.S., <i>Aetna Life Insur. Co., Hartford, Conn., U.S.A.</i>	1905	² Cooper, John James, <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>
1911	² Carey, Norman Lewis, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, S.W.</i>	1910	² Cooper, John Lewis, <i>Liverpool and London and Globe Insur. Co., Ltd., 1 Dale-street, Liverpool.</i>
1908	² Carpenter, Thomas B. Boyd, <i>Clergy Mutual Assur. Society, 2 & 3 The Sanctuary, S.W.</i>	1891	² Coote, Ernest Charles, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>
1876	Carter, Eric Mackay, <i>33 Waterloo-street, Birmingham.</i>	1871	Coutts, Edwin Arthur, <i>North British and Mercantile Insurance Company, 12 Low-pavement, Nottingham.</i>
1904	⁽²⁾ Cathles, Lawrence MacLagan, F.F.A., <i>Southwestern Life Insurance Co., Dallas, Texas, U.S.A.</i>	1900	² Covington, Oliver Henry, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1910	² Chandler, Frederick Joseph, <i>Eagle Insurance Co., 79 Pall-mall, S.W.</i>	1908	² Coward, Charles Ernest, B.A., <i>Estate Duty Office, Somerset House, W.C.</i>
1905	² Chubb, William, <i>Superintendent of Insurance, Treasury Dept., Quebec, Canada.</i>	1907	⁽²⁾ Cowan, Hugh Francis, F.F.A., <i>30 Dick-place, Edinburgh.</i>
1910	² Clarke, Harold Thomas, B.A., <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, S.W.</i>	1912	² Cowdy, Henry Leslie, <i>Scottish Union & National Insur. Co., 3 King William-street, E.C.</i>
1908	³ Clemens, Frederic Broadbent, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1910	² Cox, Harry, <i>Pearl Life Assurance Company, Ltd., London-bridge, E.C.</i>
1898	² Coates, Thomas Linnaeus, <i>Mutual Life Insur. Co. of New York, 16, 17 & 18 Cornhill, E.C.</i>	1884	Craig, Robert Alexander, <i>Abstainers' and General Assur. Co., Edmund-street, Birmingham.</i>
		1908	² Dark, Thomas Arthur, A.A.S., <i>Excelsior Life Insurance Co., Toronto, Canada.</i>

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Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.		Date of becoming an Associate.	
1909	⁽²⁾ Davidson, William, F.F.A., <i>Scottish Amicable Life Assur. Society, 35 St. Vincent-place, Glasgow.</i>	1868	Eaton, Henry William, <i>Liverpool & London & Globe Insurance Co., Ltd., William-street, New York, U.S.A.</i>
1908	² Dawson, Herbert John, B.A., <i>Royal Military College, Kingston, Ontario, Canada.</i>	1904	² Eeroyd, Cuthbert W., <i>Friends' Provident Institution, Ocean Chambers, 44 Waterloo-street, Birmingham.</i>
1906	³ Defries, Frederick, <i>China Mutual Life Insurance Co., Ltd., Shanghai, China.</i>	1910	² Edwards, Herbert Alfred, <i>28 Plasket-rd., Upton Manor, E.</i>
1901	² Diamond, George Frederick, <i>City Mutual Life Assur. Society, Hunter-st., Sydney, Australia.</i>	1905	² Elderton, Robert Lapidge, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>
1901	⁽²⁾ Donald, Alexander Graham, M.A., F.F.A., <i>Scottish Provident Institution, 6 St. Andrew-square, Edinburgh.</i>	1907	² Eldridge, Ernest Edward Booth, <i>Midland and Textile Insurance Co., 39-41 New Broad-street, E.C.</i>
1881	Donaldson, John, <i>Australian Widows' Fund Life Assurance Society, Collins-street-west, Melbourne, Australia.</i>	1910	² Emmerson, Walter Hector Ross, <i>London and Lancashire Life and General Assurance Association, Ltd., 66 & 67 Cornhill, E.C.</i>
1899	² Dougharty, Harold, F.S.S., F.C.I.S., <i>London & Lancashire Life & General Assurance Association, Ltd., 66 & 67 Cornhill, E.C.</i>	1872	² Evans, William, F.F.A., F.R.S.E., <i>38 Morningside-park, Edinburgh.</i>
1902	² Donst-Smith, Ernest Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1896	² Featherstonehangh, William Irwin, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>
1881	Dovey, William Roadly, F.F.A., F.A.S., <i>255 George-street, Sydney, Australia.</i>	1903	² Ferguson, Colin C., B.A., <i>Great West Life Assurance Co., Winnipeg, Manitoba, Canada.</i>
1906	² Downes, Sidney Cecil, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1897	² Findlay, Alexander Wynaud, L.L.D., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1870	Dowson, John, <i>26 Wellesley-road, Liverpool.</i>	1911	² Finlayson, G. D., <i>Government Insurance Department, Ottawa, Canada.</i>
1910	² Drake, Charles Clifford Hall, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1902	² FitzGerald, Charles Reginald, <i>State Mutual Life Assur. Co., Worcester, Mass., U.S.A.</i>
1908	³ Duffell, James Henry, <i>Royal London Mutual Insurance Society, Ltd., & Royal London Auxiliary Insurance Co., Ltd., Royal London House, Finsbury-square, E.C.</i>	1901	² FitzGerald, William George, B.A., <i>Northern Life Assurance Co. of Canada, London, Ontario, Canada.</i>
1901	² Earle, Arthur Percival, <i>Travellers Life Assurance Co. of Canada, Montreal, Canada.</i>	1911	² Fortington, Harold Augustus, <i>Alliance Assurance Co., Ltd., 130 Colmore-row, Birmingham.</i>
		1890	⁽²⁾ Fox, Charles Edward, F.F.A., <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>

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Date of becoming an Associate.		Date of becoming an Associate.	
1886	(2) Fox, Morris, F.A.S., <i>New Zealand Government Life Insurance Dept., Wellington, New Zealand.</i>	1905	(2) Gould, W. H., M.A., <i>c/o Messrs. Suffern & Son, 149 Broadway, New York, U.S.A.</i>
1894	(2) Fraser, Thomas John, <i>Australian Alliance Assurance Company, Melbourne, Australia.</i>	1908	(2) Graham, George, Jr., F.F.A., <i>Actuary, State Insur. Depart., Springfield, Illinois, U.S.A.</i>
1907	(3) Fulford, William John, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1908	(3) Granger, Charles Keith, F.F.A., <i>City of Glasgow Life Assurance Co., 30 Renfield-street, Glasgow.</i>
1909	(2) Fyfe, Austyn James Claude, F.F.A., <i>Northern Assurance Co., Ltd., 1 Union-terrace, Aberdeen.</i>	1902	(2) Gray, Robert Alexander, B.A., <i>324 Markham-street, Toronto, Canada.</i>
1901	(2) Gaff, William Robertson, C.A., F.F.A., <i>54 New Broad-street, E.C.</i>	1868	Greig, John Andrew, <i>Sun Life Assurance Society, 60 Charing-cross, S.W.</i>
1873	(2) Gage, Uriah Woodard, <i>North British & Mercantile Insur. Co., 61 Threadneedle-st., E.C.</i>	1909	(2) Hall, Arthur F., <i>North American Life Assurance Co., Toronto, Canada.</i>
1896	(2) Galwey, Charles Edmund, <i>New Zealand Government Life Insurance Dept., Wellington, New Zealand.</i>	1903	(2) Hall, John Bertram, A.A.S., <i>Dominion Life Assurance Co., Suite 407, Kent-building, Toronto, Canada.</i>
1911	(2) Gawler, Oswald, <i>Registrar-General's Office, Perth, West Australia.</i>	1905	(2) Hallman, M. S., F.A.S., <i>Mutual Life Assurance Company of Canada, Waterloo, Ontario, Canada.</i>
1885	(2) Gayford, Herbert Stannard, <i>Northern Assurance Company, Ltd., 1 Moorgate-street, E.C.</i>	1905	(2) Hammond, Reginald, <i>British Equitable Assur. Co., Ltd., 1, 2 & 3 Queen-street-place, E.C.</i>
1899	(3) Gibb, James Burnett, F.F.A., A.A.S., <i>Penn Mutual Life Insce. Co. of Philadelphia, 923 Chestnut-st., Philadelphia, Pa., U.S.A.</i>	1910	(2) Handford, John James William, <i>Scottish Office, S.W.</i>
1909	(2) Gilliland, William Henry, B.A., <i>Government Insurance Department, Ottawa, Canada.</i>	1869	Hann, Robert George, F.A.S., <i>Equitable Life Assur. Soc. of the United States, 120 Broadway, New York.</i>
1897	(2) Goggs, Frank Sidney, <i>Scottish Metropolitan Assurance Co., Ltd., 25 St. Andrew-sq., Edinburgh.</i>	1894	(2) Hardeastle, Edward Edgington, M.A., F.A.S., <i>Union Central Life Office, Cincinnati, Ohio, U.S.A.</i>
1882	Goldman, Leopold, F.S.S., <i>North American Life Assurance Co., Toronto, Canada.</i>	1900	(2) Harding, Harry Burnard, <i>Commercial Union Assur. Co., 26 New Bridge-street, E.C.</i>
1897	(2) Goodwyn, John, <i>Royal Chambers, 3 Castlereagh-street, Sydney, Australia.</i>	1909	(2) Harley, Brian, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>
		1908	(2) Harnack, Frederick William, <i>Sceptre Life Association, Ltd., 40 Finsbury-pavement, E.C.</i>

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1909	² Harrington, Eustace Woods, <i>Northern Assurance Company, Ltd., 1 Moorgate-street, E.C.</i>	1899	³ Hudson, Alfred James, <i>Northern Assurance Company, Ltd., 1 Moorgate-street, E.C.</i>
1896	³ Harris, Frederick Joseph, <i>Australian Mutual Provident Society, Sydney, Australia.</i>	1908	² Humphreys, John Alfred, <i>Star Assurance Society, 32 Moorgate-street, E.C.</i>
1909	² Harvey, Percy Norman, <i>Atlas Assurance Company, Ltd., 92 Cheapside, E.C.</i>	1907	² Humphry, Edmund William, <i>Life Association of Scotland, 28 Bishopsgate, E.C.</i>
1910	² Hawes, Ernest Edward, <i>North British & Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>	1875	Hunt, Richard Aldington, F.S.S., <i>Wesleyan & General Assur. Soc., Steelhouse-lane, Birmingham.</i>
1897	² Hayercraft, William Melhuish, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1893	⁽²⁾ Hunter, Arthur, F.F.A., F.A.S., <i>New York Life Insurance Co., 346 & 348 Broadway, New York, U.S.A.</i>
1897	² Hazell, Stanley, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>	1902	² Hunter, Robertson G., F.A.S., <i>Germania Life Insur. Co., 50 Union-square, New York, U.S.A.</i>
1895	² Heness, Leonard Thomas, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1887	² Hunter, Samuel, <i>9 Grace-park-gardens, Dublin.</i>
1878	Henry, Alfred, F.C.A., <i>Throgmorton-house, Copthall-avenue, E.C.</i>	1911	² Hurley, John Cromwell, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1909	² Hines, Walter Robert, <i>Norwich Union Life Insurance Society, Norwich.</i>	1909	² Hustwitt, William Edmund, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1911	² Holgate, Benjamin, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>	1910	² Jackson, Herbert Moore, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1894	² Hollingworth, Albert Charles, <i>Australian Mutual Provident Society, South Sea House, 37 Threadneedle-street, E.C.</i>	1889	⁽²⁾ Jacobs, Frederick Job, <i>Post Office, Queen Victoria Markets, Sydney, Australia.</i>
1907	³ Holness, Archibald Stephen, <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.</i>	1876	² James, George Trevolyan, <i>33 St. James's-square, S.W.</i>
1909	⁽²⁾ Hope, Francis Moffat, F.F.A., <i>Occidental Life Insurance Co., Grosse Building, 6th and Spring-streets, Los Angeles, California, U.S.A.</i>	1871	Jellicoe, George Rogers, <i>Eagle Insurance Company, 79 Pall-mall, S.W.</i>
1898	² Howell, Chas. Edward, M.A., LL.D., <i>Standard Life Assurance Company, 59 Dawson-street, Dublin.</i>	1883	Jerman, Richard, <i>22, St. David's-hill, Exeter.</i>
		1896	² Jobson, Alexander, <i>Challis House, Martin Place, Sydney, Australia.</i>

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Date of becoming an Associate.		Date of becoming an Associate.	
1894	² Johnston, Frederick H., F.A.S., <i>Prudential Life Insurance Co. of America, Newark, N.J., U.S.A.</i>	1907	² Laing, John Morrison, B.A., A.A.S., <i>Mutual Life Assurance Co. of Canada, Waterloo, Ontario, Canada.</i>
1903	³ Jones, Leonard Alexander Mouat, <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>	1893	² Laing, William Claud, <i>North British and Mercantile Insurance Company, 61 Threadneedle-street, E.C.</i>
1903	² Jones, Wallace Mouat, <i>General Reversionary & Investment Company, Limited, 26 Pall-mall, S.W.</i>	1908	² Laird, John Melvin, B.A., <i>Connecticut General Life Insurance Co., Hartford, Conn., U.S.A.</i>
1898	² Kaufman, Henry N., A.A.S., <i>Phoenix Mutual Life Insurance Co., Hartford, Connecticut, U.S.A.</i>	1897	² Lane, Arthur Vere, B.A., <i>Legal & General Life Assurance Society, 217 West George-street, Glasgow.</i>
1911	² Keable, Henry Batten, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1907	² Langstaff, Milton Palmer, <i>Dominion Life Assurance Co., Waterloo, Ontario, Canada.</i>
1876	Kearry, Joseph, <i>44 Charlwood-street, Belgrave-road, S.W.</i>	1905	² Latham, Bertrand, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1910	² Keevil, Norman Alexander Clement, <i>Blagdon, Station-road, New Barnet.</i>	1906	² Latham, Percy James, <i>Altamira, Stonebridge Park, S.W. (Reinstated, 1911).</i>
1897	² Kemp, Julian Ernest Sandford, <i>Eagle Insurance Company, 79 Pall-mall, S.W.</i>	1906	⁽²⁾ Latta, Alexander, F.F.A., <i>Guardian Assurance Company, 28 King-st., Covent-garden, W.C.</i>
1902	² Kilgour, David Errett, M.A., F.A.S., <i>North American Life Assurance Co., Toronto, Canada.</i>	1899	² Lawton, George Herbert, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>
1910	⁽²⁾ Kilpatrick, Hugh Grainger, F.F.A., <i>Liverpool and London and Globe Insurance Company, Limited, 1 Cornhill, E.C.</i>	1905	³ Leigh, Samuel George, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
1909	³ King, Albert Edward, <i>Equitable Life Assurance Society, Mansion House-street, E.C.</i>	1879	Leitch, Alexander, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>
1882	² King, William Alfred, <i>Northern Assurance Co., Ltd., 1 Moorgate-street, E.C.</i>	1897	² Le Maitre, Frank William, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1908	⁽²⁾ Kyd, James Gray, F.F.A., <i>Northern Assurance Co., Ltd., 1 Moorgate-street, E.C.</i>	1885	Leveaux, Arthur Michael, F.S.S., <i>Registry of Friendly Societies, Central Office, 28 Abingdon-street, Westminster, S.W.</i>
1909	² Lafford, Harry George, <i>Legal and General Life Assurance Society, 10 Fleet-street, E.C.</i>	1868	Litchfield, Edward, <i>c/o Messrs. Knox & Service, 41 St. Vincent-place, Glasgow.</i>

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1909	² Lohan, John Joseph, <i>National Mutual Life Association of Australasia, Melbourne, Australia.</i>	1909	² Marples, Percy Morris, M.A., B.Sc., 82 Kingsbury-road, Gravelly-hill, Birmingham.
1876	² Lucey, Herbert (EDITOR), <i>General Assurance Company, 103 Cannon-street, E.C.</i>	1911	² Marshall, Arthur William, <i>Blenheim Lodge, St. Ann's-hill, Nottingham.</i>
1890	⁽²⁾ Lugton, Hugh, F.F.A., <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>	1911	² Marshall, Cecil George, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1900	³ McArthur, Harry de C., <i>Box 282, Dunedin, New Zealand.</i>	1896	² Martin, Sidney George, <i>National Mutual Life Assoc. of Australasia, Ltd., Brisbane, Australia.</i>
1911	² McCormack, Percy Hicks, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>	1897	² Mascall, Alfred John, <i>Standard Life Assurance Co., Barbados, West Indies.</i>
1867	Macdonald, William Rae, F.F.A., <i>Scottish Metropolitan Assurance Co., Limited, 25 St. Andrew-square, Edinburgh.</i>	1900	² Maunder, George Harvard, <i>National Mutual Life Assur. Society, 39 King-st., Cheapside, E.C.</i>
1882	³ McDougald, Alfred, <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.</i>	1902	⁽²⁾ Maxwell Benjamin Bell, F.F.A., <i>Scottish Equitable Life Assur. Society, 28 St. Andrew-square, Edinburgh.</i>
1905	² Macfarlane, James Allan, <i>Monarch Life Assurance Co., Winnipeg, Manitoba, Canada.</i>	1899	² Meade, Gerald Willoughby, <i>North British & Mercantile Insurance Company, 61 Threadneedle-street, E.C.</i>
1884	Mackay, Alexander, <i>Law Union & Rock Insur. Co., Ltd., 126 Chancery-lane, W.C.</i>	1896	² Merfield, Percy Henry, <i>Phoenix Assurance Co., Ltd. (Law Life Office), 187 Fleet-street, E.C.</i>
1905	³ McKeehmie, James Baldwin, M.A., F.A.S., <i>Manufacturers Life Insurance Company, Toronto, Canada.</i>	1909	² Mol, Wilhelmus Johannes Bartholomeus, <i>Algemeene Maatschappij van Levensverzekering en Lijfrente, Soerabaya, Dutch East Indies.</i>
1905	² McPhail, Frederick Charles, <i>Colonial Mutual Life Assurance Soc., Ltd., Melbourne, Australia.</i>	1905	² Monilaws, William Barrington, <i>Lukut Estate, Port Dickson, Federated Malay States.</i>
1910	² MacTavish, Archie Neill, B.A., <i>Government Insurance Department, Ottawa, Canada.</i>	1879	Monilaws, William Macgeorge, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>
1883	² Makeham, William Reed, <i>Alliance Assurance Co., Ltd. (Imperial Life Assurance Fund), 47 Chancery-lane, W.C.</i>	1905	² Monkhouse, Charles Cosmo, B.A., <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, S.W.</i>
1880	Manwaring, Henry, <i>National Debt Office, E.C.</i>	1877	Moon, James, J.P., <i>Prudential Assurance Company, 30 Dale-street, Liverpool.</i>
1911	² Marathey, Ganesh Sadashe, M.A., 351 Shanwar Peit, Poona City, <i>Bombay Presidency, India.</i>		
1910	² Marlin, James Harold, <i>Ocean Accident and Guarantee Corporation, 36-44 Moorgate-street, E.C.</i>		

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1877	Moon, John, <i>Parkhurst, Didsbury, Manchester.</i>	1895	² Pagden, Lionel King, <i>Union Life Branch of the Commercial Union Assur. Co., 1 & 2 Royal Exchange-buildings, E.C.</i>
1903	² Moore, George Cecil, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>	1864	Panton, Edward Henry, <i>2 Benson-road, Forest Hill, S.E.</i>
1905	² Moore, George Edward, <i>Mutual Life & Citizens' Assur. Co., Ltd., Sydney, Australia.</i>	1901	³ Papps, Percy Charles Herbert, <i>F.A.S., Mutual Benefit Life Insur. Co., Newark, New Jersey, U.S.A.</i>
1898	² Moore, Joseph Patrick, <i>Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.</i>	1895	² Paradice, William Henry, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1871	² Moore, Roderick Mackenzie, <i>39 Drewstead-road, Streatham, S.W.</i>	1869	Park, David Francis, C.A., F.F.A., <i>Kenly Green, Boarhills, Fife, N.B.</i>
1900	² Nash, Alfred Charles, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, S.W.</i>	1907	² Parker, John Gowans, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>
1909	² Nathan, Eric Burnett, <i>London & Lancashire Life & General Assce. Assn., Ltd., 66 & 67, Cornhill, E.C.</i>	1905	² Paton, Albert George, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>
1897	² Newling, Sidney Wallis, B.A., <i>Woodleigh, South Woodford, Essex.</i>	1898	(²) Pearce, Henry John, F.F.A., <i>(AUDITOR), Scottish Amicable Life Assurance Society, 1 Threadneedle-st., E.C.</i>
1905	² Newnham, Ernest Whiffin, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1899	² Peele, Thomas, <i>37 Roseberry-street, Moss Side, Manchester.</i>
1903	² Nicholls, Arthur William, <i>Australian Mutual Provident Society, Brisbane, Australia.</i>	1911	² Pequegnat, A. Eugene, <i>Mutual Life Assurance Co. of Canada, Waterloo, Ontario, Canada.</i>
1884	Nicoll, John, F.F.A., <i>Life Association of Scotland, 82 Princes-street, Edinburgh.</i>	1909	² Perry, Sidney James, <i>Northern Assurance Co., Ltd., 1 Moorgate-street, E.C.</i>
1883	Orr, Lewis Potter, F.F.A., <i>Scottish Life Assur. Co., Ltd., 19 St. Andrew-sq., Edinburgh.</i>	1909	³ Peter, James Calthorpe, <i>London and Lancashire Life and General Assurance Assoc., Ltd., 66 & 67 Cornhill, E.C.</i>
1908	³ Osborne, William Arthur, <i>Guardian Assurance Company, 11 Lombard-street, E.C.</i>	1900	³ Peters, Charles Furness, <i>L'pool. Victoria Legal Friendly Society, St. Andrew-street, E.C.</i>
1908	² Owen, David John, B.A., <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>	1907	² Phillips, Thomas Ashley, <i>Minnesota Mutual Life Insur. Co., St. Paul, Minn., U.S.A.</i>
1906	(²) Padday, Percy King, F.F.A., <i>Scottish Metropolitan Assurance Co., Ltd., 8 King-st., Cheapside, E.C.</i>	1908	³ Pickup, John Richardson, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>

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1902	² Pigrome, George Davey, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1908	² Robertson, Bernard, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1899	² Pipe, Sidney Herbert, F.A.S., <i>5 Temple Building, Toronto,</i> <i>Canada.</i>	1904	³ Robertson, Frederiek William, F.F.A., <i>Scottish Widows' Fund & Life</i> <i>Assurance Society, 9 St. Andrew-</i> <i>square, Edinburgh.</i>
1909	² Pollard, Edward Cecil, <i>General Accid., Fire and Life</i> <i>Assurance Corporation, Ltd.,</i> <i>General Buildings, Aldwych,</i> <i>W.C.</i>	1904	³ Robertson, James Leask, F.F.A., <i>Edinburgh Life Assurance Co.,</i> <i>26 George-street, Edinburgh.</i>
1903	² Portch, Albert Garfield, F.A.S., <i>Franklin Life Insurance Co.,</i> <i>Springfield, Illinois, U.S.A.</i>	1878	Robertson, William, F.F.A., <i>29 Stafford-street, Edinburgh.</i>
1911	² Priestman, Basil, <i>Abstainers & General Insurance</i> <i>Co., Ltd., Craven House, Kings-</i> <i>way, W.C.</i>	1876	Robinson, Andrew, <i>Sunningdale-park, Sunningdale,</i> <i>Berks.</i>
1869	Pringle, James, C.A., F.F.A., <i>42 Drumsheugh-gardens, Edin-</i> <i>burgh.</i>	1911	² Robinson, Archie, <i>Standard Life Assurance Co.,</i> <i>83 King William-street, E.C.</i>
1884	Pullar, James, F.F.A., <i>Colonial Mutual Life Assurance</i> <i>Society, Melbourne, Australia.</i>	1885	Ronald, Thomas Robert, <i>Montrose, Weybridge, Surrey.</i>
1881	Purves, Thomas Peter, <i>New York Life Insurance Com-</i> <i>pany, Sydney, Australia.</i>	1903	² Rowland, Stanley Jackson, <i>Equitable Life Assurance Soc.,</i> <i>Mansion House-street, E.C.</i>
1904	(²) Rankin, John Adam, F.F.A., <i>Edinburgh Life Assurance Co.,</i> <i>26 George-street, Edinburgh.</i>	1897	² Ryley, Edmund, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1910	³ Reeve, Gilfrid Montier, <i>Guardian Assurance Company,</i> <i>11 Lombard-street, E.C.</i>	1911	² Sanders, Bertram G. T., <i>Standard Life Assurance Co.,</i> <i>83 King William-street, E.C.</i>
1898	² Reid, Edward E., B.A., <i>London Life Insurance Co.,</i> <i>London, Ontario, Canada.</i>	1896	² Sanderson, Frank, M.A., LL.D., F.F.A., F.A.S., <i>Canada Life Assurance Company,</i> <i>Toronto, Canada.</i>
1901	⁵ Rhodes, Francis, B.A., <i>Royal Insur. Co., Ltd., Liverpool.</i>	1905	³ Savery, Robert S. B., <i>Gresham Life Assurance Society,</i> <i>Ltd., 30 Rue du Provence, Paris,</i> <i>France.</i>
1887	(²) Richardson, Josephus Hargreaves, F.F.A., F.A.S., <i>New Zealand Government Life</i> <i>Insurance Department, Wel-</i> <i>lington, New Zealand.</i>	1909	² Sivory, Donald Stuart, B.A., <i>37 Churchill-road, Boscombe.</i>
1879	Roberts, Thomas B., <i>Australian Alliance Assurance</i> <i>Company, Collins-street, Mel-</i> <i>bourne, Australia.</i>	1884	Schooling, John Holt, <i>Fotheringay-house, Montpelier-</i> <i>row, Twickenham.</i>
		1899	² Schouten, Pieter, <i>Verzekering Maatschappij,</i> <i>"Arahem," Stations-plein, 17,</i> <i>Arahem, Holland.</i>
		1906	(²) Scott, Albert George, <i>English and Scottish Law Life</i> <i>Assur. Association, 33 St.</i> <i>James's-square, S.W.</i>

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1873	Scott, Ernest Willem, M.A.S., <i>Algemeene Maatschappij van Levensverzekering en Lijfrente, Damrak, 74, Amsterdam.</i>	1898	² Smith, Robert Parker, <i>Royal Insurance Company, Ltd., Liverpool.</i>
1904	³ Searle, Arthur Joseph, <i>English & Scottish Law Life Assurance Association, 33 St. James's-square, S.W.</i>	1906	² Smither, Herbert Buxton, <i>University Life Assurance Soc., 25 Pall-mall, S.W.</i>
1861	² Searle, Thomas John, <i>Mansion - house - chambers, Bucklersbury, E.C.</i>	1905	² Somerville, Walter Harold, <i>Mutual Life Assur. Co. of Canada, Waterloo, Ontario, Canada.</i>
1903	² Searls, Edwin Richard, <i>Northern Assurance Company, Ltd., 1 Moorgate-street, E.C.</i>	1871	Spencer, Robert James, F.S.S., <i>75 King's-road, Southsea.</i>
1909	(2) Sellar, Alexander Smith, F.F.A., <i>95 Culverley-road, Catford, S.E.</i>	1910	² Spiegel, Ellis William Ralfs, <i>70 Coniston-rd., Muswell-hill, N.</i>
1909	² Sen, Jogesh Chandra, M.A., B.L., <i>15 Sitaram Ghos's-st., Calcutta, India.</i>	1866	Stark, William Emery, <i>Safe Deposit, Chapel - walks, Manchester.</i>
1909	² Sharp, Harold Gregory, <i>Friends' Provident Institution, 17 Gracechurch-street, E.C.</i>	1910	² Stephenson, Herbert Roy, <i>Manufacturers Life Insurance Co., Toronto, Canada.</i>
1900	² Sharpe, Edgar Cecil Engledue, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>	1878	Stevenson, Charles, <i>9 Albert-square, Manchester.</i>
1907	(2) Shearer, Gilbert Edward, F.F.A., <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>	1880	Stock, Edward James, <i>National Mutual Life Assoc. of Australasia, Melbourne, Aus- tralia.</i>
1894	³ Sheppard, Herbert Norman, B.A., F.A.S., <i>Home Life Insurance Company, 256 Broadway, New York, U.S.A.</i>	1909	² Stocks, Joseph, <i>Law Union and Rock Insur. Co., Ltd., 126 Chancery-lane, W.C.</i>
1897	² Shimmell, James Edward, F.S.S., <i>British Legal and United Prov. Assur. Co., Ltd., 78 New Oxford- st., W.C.</i>	1906	² Story, Cyril Lionel William Steane, <i>Norwich Union Life Insurance Society, 71 & 72 King William- street, E.C.</i>
1896	² Shlager, Joseph, <i>Equitable Life Assurance Society of the United States, Mansion- house-chambers, Adderley-street, Cape Town, South Africa.</i>	1905	² Strong, Allan Wilmot, <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>
1903	² Shovelton, Sydney Taverner, M.A., <i>8 Holly-park-gardens, Finchley, N.</i>	1909	² Strong, William Boughton, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1905	² Shute, Oxenham Bent, <i>National Provincial Bank of England, 53 Baker-street, W.</i>	1896	² Stuckey, Jos. James, M.A., <i>Salisbury Chambers, 49a King William-street, Adelaide, South Australia.</i>
1864	Smith, Howard Samuel, F.F.A., F.C.A., F.S.S., <i>Bank-chambers, 11 Waterloo- street, Birmingham.</i>	1905	² Stuckey, Reginald Robert, <i>Australian Mutual Provident Soc., Adelaide, South Australia.</i>
		1911	² Sturgeon, Robert W., <i>Royal Insur. Co., Ltd., Liverpool.</i>

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.		Date of becoming an Associate.	
1905	³ Sturt, Herbert Rothsay, <i>Phoenix Assurance Co., Ltd.,</i> 70 Lombard-street, E.C.	1907	² Underwood, Reginald Edward, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, S.W.</i>
1911	² Stutfield, Martin, <i>Consolidated Assur. Company,</i> 9 Fleet-street, E.C.	1884	Vian, William Collett, <i>Railway Passengers' Assurance Company, 64 Cornhill, E.C.</i>
1904	⁽²⁾ Tatlock, John, M.A., F.R.A.S., F.A.S., 141 Broadway, New York, U.S.A.	1884	Vincent, Frederick James, F.S.S., <i>Pearl Life Assurance Co., Ltd.,</i> London-bridge, E.C.
1910	² Tayler, Harold Hosking, <i>Pearl Life Assurance Co., Ltd.,</i> London-bridge, E.C.	1899	² Vokins, George Alfred, <i>Prudential Assurance Company,</i> Holborn-bars, E.C.
1893	² Taylor, Arthur, <i>Guardian Assurance Company,</i> 28 King-street, Covent-garden, W.C.	1908	² Walker, Dwight A., <i>Equitable Life Assurance Society of the United States, 120 Broadway, New York, U.S.A.</i>
1908	³ Thompson, John Henry Reginald, <i>Prudential Assurance Company,</i> Holborn-bars, E.C.	1879	Wall, Walter George, 34 Kingsland-road, Birkenhead.
1906	³ Thomson, Frederick Robert T., <i>Law Union & Rock Insur. Co., Ltd.,</i> 126 Chancery-lane, W.C.	1911	² Walters, Arthur Hawksley, A.C.A., 15 George-street, Mansion-house, E.C.
1909	⁽²⁾ Thomson, Gordon William, F.F.A., <i>San Francisco Life Insurance Company, 57 Post-street, San Francisco, California.</i>	1905	² Wansbrough, Thomas Percival, <i>English and Scottish Law Life Assurance Assoc., and British Law Fire Insurance Co., 37 Queen Victoria-street, E.C.</i>
1904	⁽²⁾ Thomson, John Walter, F.F.A., F.S.S., <i>Scottish Life Assur. Co., 19 St. Andrew-square, Edinburgh.</i>	1906	⁽²⁾ Wardrop, James Charles, (AUDITOR), <i>Life Association of Scotland,</i> 25 Bishopsgate, E.C.
1883	² Titmuss, Walter George, <i>Alliance Assur. Co., Ltd. (Provident Life Fund), 50 Regent-street, W.</i>	1907	² Warren, Cyril Ferdinand, <i>Prudential Assurance Company,</i> Holborn-bars, E.C.
1905	² Touzel, Philip Duncan, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>	1903	² Watherston, Charles F., B.A., <i>War Office, S.W.</i>
1902	³ Traversi, Antonio Thomas, <i>Friendly Societies' Department,</i> Wellington, New Zealand.	1909	² Watson, Andrew Daniel, <i>Government Insurance Department, Ottawa, Canada.</i>
1905	² Tully, Arthur Patrick Thomas, <i>Gresham Life Assurance Society, Ltd., Omar Agha Han, Galata, Constantinople.</i>	1883	² Watson, John Robertson, <i>British Law Fire Insurance Co.,</i> 105 West George-st., Glasgow.
		1908	² Watt, William Arthur, M.A., F.A.S., <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>

ASSOCIATES.

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 Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.		Date of becoming an Associate.	
1894	² Watt, George, <i>Royal Insur. Co., Ltd., Liverpool.</i>	1870	² Wilson, Henry Edward, <i>Northern Assurance Co., Ltd., 1 Moorgate-street, E.C.</i>
1900	(²) Watt, James, W.S., F.F.A., <i>24 Rothesay-terrace, Edinburgh.</i>	1873	² Windett, Charles, <i>Profits & Income Insurance Co., Ltd., 9 Fleet-street, E.C.</i>
1902	² Weatherill, Charles, <i>Scottish Office, S.W.</i>	1905	² Winstanley, Charles William, <i>North British & Mercantile Insurance Co., 61 Threadneedle- street, E.C.</i>
1911	² Webb, Lloyd, <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>	1911	² Wisdom, Sidney Herbert, <i>Estate Duty Office, Somerset- house, W.C.</i>
1894	(²) Weeks, Rufus Wells, F.A.S., <i>New York Life Insurance Co., 346 & 348 Broadway, New York, U.S.A.</i>	1911	² Wolfenden, Edgar Sydney, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1909	² Wenn, Albert Edward <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1903	² Wood, William Archibald Porter, B.A., F.A.S., <i>Canada Life Assurance Co., Toronto, Canada.</i>
1909	³ Wenyon, Herbert John, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1909	² Woodall, Edward Arthur, <i>National Mutual Life Assurance Soc., 39 King-st., Cheapside, E.C.</i>
1898	³ Whigham, Charles Frederick, F.F.A., C.A., <i>22 Old Broad-street, E.C.</i>	1883	Woodhouse, Lister, A.C.A., F.S.S., <i>City Comptroller, Westminster City-hall, Charing Cross-road, W.C.</i>
1908	² White, Osborn Denyer, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>	1877	² Woods, Arthur Biddle, <i>Law Union & Rock Insur. Co., Ltd., 126 Chancery-lane, W.C.</i>
1909	² White, Wilfred Clare, M.A., A.A.S., <i>Federal Life Assurance Co., Hamilton, Ontario, Canada.</i>	1866	Woods, Bernard, <i>Metropolitan Life Assurance Society, 13 Moorgate-street, E.C.</i>
1897	² Wickens, Charles H., <i>Commonwealth Bureau of Census and Statistics, Melbourne, Vic- toria, Australia.</i>	1879	Wormun, Thornton Selden, <i>Law Union & Rock Insur. Co., Ltd., 126 Chancery-lane, W.C.</i>
1896	² Wilkinson, Edward Berkeley, <i>24 Maxilla-gardens, N. Kensington, W.</i>	1903	² Worth, Bertram Oliver, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>
1903	² Wilkinson, William Magnay, <i>Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.</i>	1910	² Yeldham, William James, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1904	² Williams, Frederick Alfred, F.S.S., F.A.S., <i>"La Nacional" Compania de Seguros sobre la Vida, Apartado, 1420, Mexico D.F.</i>		

STUDENTS.

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Date of becoming a student.		Date of becoming a student.	
1892	¹ Aaron, David Hyam, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1908	¹ Bazell, Harry, <i>North British and Mercantile Insurance Co., Ltd., 61 Threadneedle-street, E.C.</i>
1903	¹ Acmm, Wilfred Harry, <i>15 Lordship-lane, Wood Green, N.</i>	1910	² Bedford, Fredetick, <i>Star Assurance Society, 32 Moorgate-street, E.C.</i>
1911	¹ Aiyar, S. Narayana, M.A., L.T., <i>17 Pycroft's-road, Triplicane, Madras, India.</i>	1898	¹ Bennell, Samuel Thomas, <i>Witham-house, Brisbane-road, Ilford.</i>
1908	¹ Aldridge, Wilfred Henry, <i>Britannic Assurance Co., Ltd., Broad-st.-corner, Birmingham.</i>	1906	¹ Bennett, Henry Gordon, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1911	¹ Alison, Sidney Herbert, B.Sc., <i>Comptroller's Department, London County Council, Spring-gardens, S.W.</i>	1911	¹ Bewsher, Robert, <i>49 Gunterstone - road, West Kensington, W.</i>
1906	² Allen, Arthur Ormiston, M.A., B.Sc., <i>2 St. Michael's Villas, Cardigan-road, Leeds.</i>	1895	¹ Bigby, Robert Frederick Mitchell, <i>General Assurance Company, 103 Cannon-street, E.C.</i>
1904	¹ Allison, Sinclair E., A.A.S., <i>Reliance Life Insurance Co., Pittsburgh, Pa., U.S.A.</i>	1910	¹ Bill, Austin Frederick, B.A., <i>Government Annuities Branch, Department of Trade and Commerce, Ottawa, Canada.</i>
1908	¹ Armon, Thomas, A.C.I.S., <i>Pearl Life Assurance Co., Ltd., London-bridge, E.C.</i>	1900	¹ Bingeman, Milton H., <i>Great West Life Assurance Co., Winnipeg, Manitoba, Canada.</i>
1886	Arnold, Thomas, Jr., <i>British Equitable Assur. Co., Ltd., 1, 2 & 3 Queen-street-place, E.C.</i>	1905	¹ Blackadar, E. Gordon, B.A., A.A.S., <i>Government Annuities Branch, Department of Trade and Commerce, Ottawa, Canada.</i>
1902	¹ Askwith, Thomas Nowell, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>	1908	¹ Blake, Leslie Sarjant, <i>10 Oaslow-road, Richmond, Surrey.</i>
1905	¹ Atkins, Francis Cuthbert, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1887	Blossom, James, <i>24 Grange-crescent, Sheffield.</i>
1904	¹ Ayseough, Ivan, <i>Equity and Law Life Assurance Soc., 18 Lincoln's-inn-fields, W.C.</i>	1892	¹ Boddy, Henry Mitchell, F.S.S., <i>Manufacturers Life Insurance Co., Vancouver, B.C.</i>
1899	¹ Baber, Walter Crosbie, A.A.S., <i>c/o Baber Brothers & Co., P.O. Box 293, Montreal, Canada.</i>	1906	¹ Bolt, Jan Cornelis, <i>Middelland-strait 102, Rotterdam, Holland.</i>
1907	⁽¹⁾ Bannatyne, Arthur Gordon, B.A., <i>Alliance Assurance Co., Ltd., 151 West George-st., Glasgow.</i>	1902	¹ Bowerman, Judah Philip, <i>Southern States Mutual Life Insur. Co., Charleston, Kanawha County, West Virginia, U.S.A.</i>

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Date of becoming a Student.		Date becoming a Student.	
1897	¹ Bowles, Francis Marsh, <i>Pearl Life Assurance Co., Ltd.,</i> <i>London-bridge, E.C.</i>	1907	¹ Chandler, Francis Philip, <i>London Assurance Corporation,</i> <i>7 Royal Exchange, E.C.</i>
1891	¹ Boyd, Henry Norris, F.F.A., <i>City of Glasgow Life Assurance</i> <i>Company, 21 St. Andrew-square,</i> <i>Edinburgh.</i>	1907	¹ Charles, Ashley Hyde, <i>13 South Moulton-street, w.</i>
1903	² Bradshaw, Frank Law, <i>Royal Exchange Assur. Corpor-</i> <i>ation, Royal Exchange, E.C.</i>	1908	¹ Chase, Harold Philip, <i>Star Assurance Society, 32</i> <i>Moorgate-street, E.C.</i>
1899	¹ Brady, John Francis, <i>Mutual Life & Citizens' Assurance</i> <i>Co., Ltd., Sydney, Australia.</i>	1897	¹ Cherry, John Arnold, Barrister- at-Law, <i>Bombay Port Trust, Bombay,</i> <i>India. (Reinstated, 1905.)</i>
1906	¹ Breeds, Arthur Heywood, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1903	¹ Cheshire, Harold Frank, <i>47 Havelock-road, Hastings.</i>
1908	¹ Brenton, William Percy, <i>Radlett, Herts.</i>	1897	¹ Clinton, George, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1894	¹ Brough, Frank, <i>Federal Life Assurance Company,</i> <i>Hamilton, Ontario, Canada.</i>	1901	¹ Cockerton, John Leonard, <i>Pioneer Life Assurance Co., Ltd.,</i> <i>67 Dale-street, Liverpool.</i>
1891	¹ Brown, William Heron, <i>Gresham Life Assur. Soc., Ltd.,</i> <i>St. Mildred's-house, Poultry, E.C.</i>	1895	¹ Cogar, William Edward, <i>New York Life Insurance Co.</i> <i>Trafalgar-square, W.C.</i>
1907	² Bullwinkle, Leonard Albert, <i>c/o T. G. Ackland, Esq., 5 & 6</i> <i>Clement's-inn, Strand, W.C.</i>	1899	¹ Collins, Patrick A., <i>Mutual Life & Citizens' Assur.</i> <i>Co., Ltd., Sydney, Australia.</i>
1903	¹ Capon, Frank Christopher, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1902	¹ Collins, William Ernest, <i>Fell.</i> <i>Inst. Accts. S.A., 97 King</i> <i>William-street, Adelaide, South</i> <i>Australia.</i>
1902	¹ Capon, Geoffrey William, <i>Norwich Union Life Insurance</i> <i>Society, Norwich.</i>	1909	¹ Cook, Henry Milton, <i>Standard Life Assurance Com-</i> <i>pany, Dalhousie-sq., Calcutta,</i> <i>India.</i>
1908	¹ Carpmael, Charlton, <i>Star Assurance Society, 32</i> <i>Moorgate-street, E.C.</i>	1905	¹ Countts, Kenneth Vawdrey, <i>Clergy Mutual Assurance Soc.,</i> <i>2 & 3 The Sanctuary, s.w.</i>
1907	¹ Cashman, Thomas, <i>North British and Mercantile</i> <i>Insurance Co., 61 Threadneedle-</i> <i>street, E.C.</i>	1894	Cox, Edward William, <i>Canada Life Assurance Co.,</i> <i>Toronto, Canada.</i>

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Date of becoming a student.		Date of becoming a student.	
1894	¹ Cox, Herbert Coplin, <i>Canada Life Assurance Co., Toronto, Canada.</i>	1896	¹ de Ville, Francis, <i>Clergy Pensions Institution, 11 Norfolk-street, Strand, W.C.</i>
1905	¹ Cox, Stanley Nelson, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1890	¹ Docker, Leslie, <i>North British and Mercantile Insurance Co., 61 Threadneedle- street, E.C.</i>
1907	¹ Crang, James Simon, <i>29 King Edward-road, Waltham- stow, E.</i>	1906	³ Doucet, Gerald Danby, <i>London & Lancashire Life and General Assurance Associ'n., Limited, 66 & 67 Cornhill, E.C.</i>
1887	¹ Cross, Henry John, <i>Wilton House, Chepstow-rise, Croydon.</i>	1906	² Doyle, Joseph Patrick, <i>Mutual Life & Citizens' Assur- ance Company, Ltd., Sydney, Australia.</i>
1907	¹ Currie, James Thorn, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>	1908	¹ Eames, George Stanley, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1907	¹ Curtis, Augustus Thomas George, <i>Pearl Life Assurance Co., Ltd., London-bridge, E.C.</i>	1905	¹ Eastcott, William Merrill, <i>Sun Life Assur. Co. of Canada, Ottawa, Canada.</i>
1904	¹ Cushing, Robertson Macanlay, <i>Sun Life Assurance Company of Canada, Montreal, Canada.</i>	1910	¹ Edwards, Arthur John Charles, <i>National Debt Office, E.C.</i>
1904	¹ Dalrymple, Alfred George, <i>Canada Life Assurance Company, Toronto, Canada.</i>	1892	¹ Edwards, Edward Samuel, M.A., <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1897	¹ Dalton, John, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>	1892	¹ Eedy, Arthur Malcolm, <i>Mutual Life & Citizens' Assur- ance Company, Ltd., Sydney, Australia.</i>
1889	¹ Davies, Hugh Myddleton, <i>Royal Insur. Co., Ltd., Liverpool.</i>	1901	¹ Eggleton, Harold Edward, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1900	¹ Davies, William Allison, A.S.A.A., <i>Assistant Borough Treasurer, Town Hall, Birkenhead.</i>	1906	³ Emery, Walter Sydney, <i>Mutual Life & Citizens Assur- ance Company, Ltd., Sydney, Australia.</i>
1891	¹ Dawson, Frank Aubrey, <i>Ecclesiastical Insurance Office, Limited, 11 Norfolk-street, Strand, W.C.</i>	1907	³ Epps, George Selby Washington, B.A., <i>English and Scottish Law Life Assurance Association, 33 St. James's-square, S.W.</i>
1902	³ Deck, James Gilbert, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>		
1902	¹ Denmark, Robert John, <i>Norwich Union Life Insurance Society, Norwich.</i>		
1901	¹ Dent, Ernest Edward, <i>London and Lancashire Life and General Assurance Associ'n., Limited, 66 & 67 Cornhill, E.C.</i>		

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Date of becoming a student.		Date of becoming a student.	
1911	¹ Evans, Arthur William, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1900	¹ Garner, James, <i>9 Arlington-gardens, Gunnersbury, W.</i>
1908	¹ Evans, Cyril Ormond, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>	1893	Glasson, George Cornish, <i>Alliance Assurance Co., Ltd., Corn-street, Bristol.</i>
1906	¹ Fairlie, James, M.A., <i>Franklin Life Insurance Co., Springfield, Illinois, U.S.A.</i>	1902	¹ Gleave, Charles Sheldon, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
1892	¹ Farrell, John, <i>Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.</i>	1899	¹ Goddard, Egbert, <i>Coombe Brook, Coombe, Surrey. (Reinstated, 1908).</i>
1907	¹ Fidler, William Edward, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>	1894	¹ Golding, Arthur, <i>40 Allerton-road, Stoke Newington, S.</i>
1910	² Field, John Morton, <i>Commercial Union Assurance Co., Ltd., 24-26 Cornhill, E.C.</i>	1905	¹ Goodall, Ernest Victor, <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>
1908	² Fielder, Tom Lionel, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>	1903	¹ Gopp, John Ive, <i>14 Church-hill-road, Waltham-stow, E.</i>
1901	¹ Fisher, John William, B.A., A.A.S., <i>Crown Life Insurance Co., Toronto, Canada.</i>	1907	¹ Grant, Frederick John, <i>Star Assurance Soc., 30 Cross-street, Manchester.</i>
1896	¹ Fisk, George William Victor, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1909	¹ Gravatt, Hubert Charles Alfred, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1904	¹ Fletcher, Andrew W. A. C., <i>Standard Life Assurance Co., 3 George-street, Edinburgh.</i>	1907	¹ Green, John Spencer, <i>United London and Scottish Insurance Co., Ltd., 18 New Bridge-street, E.C.</i>
1905	¹ Flynn, Benedict Devine, F.A.S., <i>Travelers Insurance Company, Hartford, Conn., U.S.A.</i>	1886	Greening, Herbert Joseph, <i>Abstainers' & General Insurance Company, Edmund-street, Birmingham.</i>
1905	¹ Forbes, James, A.A.S., <i>Great West Life Assurance Co., Winnipeg, Manitoba, Canada.</i>	1910	¹ Gupta, Saratkumar Datta, M.A., F.S.S., F.R.E.S., <i>Prices Enquiry, Government of India, Treasury Buildings, Calcutta, India.</i>
1906	¹ Foster, Joseph, <i>33 Westwood-street, Moss Side, Manchester.</i>		

STUDENTS.

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Date of becoming a Student.		Date of becoming a Student.	
1907	¹ Guthrie, Isles Hampden, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>	1911	¹ Hawkins, Lionel Frank, <i>c/o T. G. Ackland, Esq., 5 & 6 Clement's-lane, Strand, W.C.</i>
1901	¹ Hamilton, George Powell, <i>North American Life Assurance Company, McLean Block, 10 Douglas-street, Guelph, Ontario, Canada.</i>	1907	³ Henry, Alfred, <i>5 Branstone-road, Kew-gardens.</i>
1902	¹ Hammant, Francis Clive, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1906	¹ Hilbery, Reginald William, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>
1905	¹ Hammond, Harry Pierson, B.A., A.A.S., F.S.S., <i>Insurance Dept. of the State of Connecticut, Hartford, Conn., U.S.A.</i>	1902	¹ Hodge, Cecil Wilfred, <i>c/o Messrs. Frank L. H. Collins and Sons, 4b Frederick's-place, Old Jewry, E.C.</i>
1892	Hancock, Arthur Tom, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's- square, S.W.</i>	1908	¹ Holgate, Thomas, <i>Refuge Assurance Co., Oxford- street, Manchester.</i>
1901	¹ Harper, Henry, <i>83 Waverley-road, Small Heath, Birmingham.</i>	1911	¹ Holmes, Horace, <i>24 Dunbar-rd., Toronto, Canada.</i>
1889	¹ Harris, Henry, <i>Sunnyside, Mill Hill, N.W.</i>	1910	¹ Horner, Bernard Stuart, <i>Northern Assurance Company, Ltd., 1 Moorgate-street, E.C.</i>
1908	¹ Harris, Sydney Ewart, <i>Sun Life Assurance Company of Canada, Canada-house, 4 & 5 Norfolk-street, Strand, W.C.</i>	1902	¹ Houston, Charles Cornelius, <i>Metropolitan Asylums Board, Victoria-embankment, E.C.</i>
1908	¹ Harrison, Alfred Lowther, <i>Northern Assurance Company, Ltd., 1 Moorgate-street, E.C.</i>	1901	¹ Howell, Archibald Rennie, B.A., <i>Gresham Life Assurance Society, Ltd., Montreal, Canada.</i>
1908	¹ Harrison, Wilfrid, <i>Eastburn, Hexham-on-Tyne.</i>	1907	¹ Howell, Percy, <i>26 Clifford-gardens, Kensal-rise, N.W.</i>
1896	Haskins, George Frederick, A.C.A., <i>18 Walbrook, E.C.</i>	1898	Hughes, Arthur J., <i>China Mutual Life Insur. Co., Shanghai, China.</i>
		1902	¹ Hughes, Charles, A.A.S., <i>The Workmen's Compensation Service and Information Bureau, 1 Liberty-street, New York, U.S.A.</i>

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Date of becoming a student.		Date of becoming a student.	
1902	¹ Hugill, Herbert, " <i>Briarfield</i> ," <i>Keighley</i> .	1896	¹ Jones, Richard Foxley, A.C.I.S., <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
1904	² Humphreys, Harry Lewis, <i>National Mutual Life Assurance Society, 39 King-street, Cheap-side, E.C.</i>	1909	² Jones, Richard McNair, B.A., <i>26 Disraeli-gardens, Wandsworth, S.W.</i>
1891	Hunt, Arthur Leonard, <i>Bryn, Somerville-road, Sutton Coldfield.</i>	1906	¹ Kearns, William Norman, <i>Royal Insur. Co., Ltd., Liverpool.</i>
1907	¹ Hutchings, Leonard Hollinworth, <i>Phoenix Assurance Company, Limited, 70 Lombard-street, E.C.</i>	1905	¹ Kenchington, Frank, B.A., <i>North British and Mercantile Insurance Company, 61 Threadneedle-street, E.C.</i>
1902	⁽¹⁾ Jackson, Charles William, M.A., F.A.S., <i>Greensboro Life Insurance Company, Greensboro, N.C., U.S.A.</i>	1906	¹ Kime, Virgil Morrison, <i>American Central Life Insurance Co., Indianapolis, Ind., U.S.A.</i>
1890	² Jackson, Samuel, F.F.A., <i>Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew-square, Edinburgh.</i>	1894	² Kingsbury, James William, <i>Australian Mutual Provident Society, South Sea House, 37 Threadneedle-street, E.C.</i>
1896	¹ Jepps, John Blacklee, <i>Star Assurance Society, 32 Moorgate-street, E.C.</i>	1895	¹ Knight, Alfred Murray, <i>Bank-house, Chapel-st., Devonport.</i>
1905	¹ Johns, Arthur Humphreys, <i>Colonial Mutual Life Assurance Society, Melbourne, Australia.</i>	1908	¹ Kubota, Takajiro, <i>Teikoku Life Assurance Company, Tokio, Japan.</i>
1904	¹ Johnson, Frank Henry, <i>Phoenix Assurance Co., Limited (Law Life Office), 187 Fleet-street, E.C.</i>	1902	¹ Lang, Frederick John, <i>Royal London Mutual Insur. Soc., Ltd., & Royal London Auxiliary Insur. Co., Ltd., Royal London House, Finsbury-square, E.C.</i>
1898	¹ Johnston, Arthur Edward, <i>3 Cumnor-road, Sutton.</i>	1907	¹ Latham, Fergus Norman Wilkinson, <i>3 Wyresdale-rd., Bolton, Lancs.</i>
1903	² Jones, Ernest Stephens, <i>National Debt Office, E.C.</i>	1907	¹ Ledger, Robert John, <i>Grove-lodge, Grove-rd., Epsom.</i>
1908	¹ Jones, Ernest Washington, <i>63 Bertram-road, Hendon, N.W.</i>	1894	Leonard, Maurice, <i>Frith Hill Cottage, Great Missenden, Bucks.</i>

STUDENTS.

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 Those marked (i) have been exempted under the Bye-laws from Sections (1) and (2) of Part I of the current Syllabus.

Date of becoming a Student.		Date of becoming a Student.	
1906	¹ Le Rossignol, Leonard F., <i>English and Scottish Law Life Assurance Association, 33 St. James's-square, S.W.</i>	1888	¹ McConway, James Robert, <i>15 Henthorn-road, New Ferry, Cheshire.</i>
1908	² Lever, Ernest H., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1906	¹ McCulloch, James Arthur, <i>Ecclesiastical Commission, Millbank, S.W.</i>
1906	¹ Lewis, David Hugh, <i>Refuge Assurance Company, Oxford-street, Manchester.</i>	1903	¹ Macdonald, Charles Strange, M.A., <i>Confederation Life Association, Toronto, Canada.</i>
1904	² Lewty, Francis Arthur, <i>Equity and Law Life Assurance Society, 18 Lincoln's-inn-fields, W.C.</i>	1910	¹ McLean, Percy Stewart, <i>North American Life Assurance Co., Toronto, Canada.</i>
1889	¹ Lighton, Harold John, <i>Law Union & Rock Insurance Co., Ltd., 126 Chancery-lane, W.C.</i>	1907	¹ Macleod, John, <i>43 Grosvenor-terrace, Bootham, Yorks.</i>
1911	¹ Linton, Morris Albert, B.S., M.A., A.A.S., <i>Provident Life and Trust Co., 409 Chestnut-street, Philadelphia, Pa., U.S.A.</i>	1907	¹ Macorquodale, F. D., <i>Prudential Life Insurance Co., Winnipeg, Manitoba, Canada.</i>
1908	¹ Lithgow, James Hector Farncombe, A.A.S., <i>Manufacturers Life Insurance Co., Toronto, Canada.</i>	1903	¹ Manly, George William, B.A., <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>
1895	¹ Littell, Lewis Lloyd, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>	1908	² Mann, Frederick Christmas, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>
1904	¹ Littlefair, James Taylor, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>	1905	¹ Marshall, John Edwin, <i>Prudential Assurance Company, 47 Earl-street, Coventry.</i>
1890	Love, Robert, <i>Ecclesiastical Insurance Office, Ltd., 11 Norfolk-street, Strand, W.C.</i>	1903	¹ Martin, Frederick Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
		1906	¹ Martin, William Alexander, <i>National Mutual Life Association of Australasia, Melbourne, Australia.</i>

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Date of becoming a student.		Date of becoming a student.	
1904	¹ Matheson, Donald, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>	1902	¹ Muckle, Charles P., <i>Union Life Assurance Company, Toronto, Canada. (Reinstated. 1907.)</i>
1906	Maunder, Henry Ernest, <i>10 Orchard-house, County-grove, Camberwell, S.E.</i>	1903	¹ Myers, Harry Duxbury, A.S.A.A., <i>United Counties' Bank-chambers, North-street, Keighley.</i>
1890	¹ Meikle, Henry George Watson, F.F.A., <i>Oriental Government Security Life Assurance Co., Limited, Bombay, India.</i>	1907	¹ Nash, Kenneth Oscar, <i>The Barn, Thames Ditton.</i>
1892	¹ Meyers, Henry Wilson, <i>National Mutual Life Association of Australasia, 5 Cheapside, E.C.</i>	1906	¹ Needell, Brian, <i>North British and Mercantile Insurance Company, 61 Thread-needle-street, E.C.</i>
1907	¹ Miller, Arthur Axel, A.C.A., <i>32 Kyverdale-road, N.</i>	1903	¹ Neill, William Adam Hoyes, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>
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1899	² Minns, Ernest Edwin, <i>Norwich Union Life Insurance Society, Norwich.</i>	1911	¹ Oatway, William Hubert, <i>Eagle Insurance Company, 79 Pall Mall, S.W.</i>
1907	¹ Monilaws, Stanley Hope, <i>St. Leonard's Estate, Port Dickson, Federated Malay States.</i>	1911	¹ O'Brien, Henry, <i>c/o T. G. Ackland, Esq., 5 & 6 Clement's-inn, Strand, W.C.</i>
1902	² Moore, Hubert Fred, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>	1902	⁽¹⁾ O'Connor, William, M.A., M.D., <i>Mutual Life Insurance Company of New York, Toronto, Canada.</i>
1902	Morton, Francis, <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>	1910	¹ Olitiers, Edward, <i>c/o M. M. Dawson, Esq., 111 Broadway, New York, U.S.A.</i>
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Date of becoming a student.		Date of becoming a student.	
1911	¹ Orrel, Harry, <i>Refuge Assurance Company, Oxford-street, Manchester.</i>	1908	¹ Pickworth, Edgar Broughton, <i>65 Hinde-road, Harrow.</i>
1897	¹ Osborn, Nathaniel Banner Francis, <i>11 Bruce-grove, Tottenham, N.</i>	1907	¹ Pocock, Horace George Grooby, <i>Alliance Assurance Company, Ltd. (Imperial Life Assurance Fund), 47 Chancery-lane, W.C.</i>
1901	¹ Papworth, Frederick William, <i>A.S.A.A., 54 Elderton-road, Westcliffe-on-Sea.</i>	1898	Poort, Willem Anthonie, Phil. Nat. Doct., <i>Algemeene Friesche Iereus-verzekerings Maatschappij Leeuwarden, Leeuwarden, Holland.</i>
1904	¹ Parker, Walter Montgomery, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1907	² Preston, John Edwin, B.A., <i>Yorkshire Insur. Co., Ltd., York.</i>
1895	¹ Pascoe, William Yeoman Bennett, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1908	¹ Proddow, William Norman, <i>Pearl Life Assurance Company, Ltd., London Bridge, E.C.</i>
1897	¹ Patrick, James, <i>49 Birch-road, Oxtou, Birkenhead. (Reinstated, 1905.)</i>	1907	¹ Prout, Herbert John, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1907	¹ Pattison, George Benjamin, A.A.S., <i>Peoria Life Insurance Company, Peoria, Ill., U.S.A.</i>	1908	¹ Purry, William Baldwin, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1896	² Penny, Charles Augustus, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1901	¹ Ramsay, Cecil Byron, <i>Mutual Life Insur. Co. of New York, 16, 17 & 18 Cornhill, E.C.</i>
1901	¹ Petter, Herbert, <i>Britannic Assurance Co., Ltd., Broad-st.-corner, Birmingham.</i>	1898	¹ Reynell, Guy Courtenay, <i>National Mutual Life Assurance Society, 39 King-street, Cheap-side, E.C.</i>
1908	² Phillips, Ernest William, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1908	¹ Richardson, George Rowley, <i>Legal and General Life Assurance Society, 10 Fleet-street, E.C.</i>
1904	¹ Phillips, Walter, A.C.I.S., A.S.A.A., <i>c/o Messrs. Arthur Guinness, Son & Co., Ltd., St. James's-gate, Dublin.</i>	1904	¹ Ridgway, Wulfric, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>

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Date of becoming a Student.		Date of becoming a Student.	
1902	¹ Robertson, Aubrey Charles, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>	1908	¹ Shepherdson, Herbert Jepson, B.A., B.Sc., <i>Royal Insurance Company, Limited, Liverpool.</i>
1903	¹ Robinson, Ernest William, <i>Australian Metropolitan Life Assurance Company Ltd., Metro- politan-buildings, cr. Adelaide and Albert-streets, Brisbane, Australia.</i>	1907	¹ Shine, John Nugent, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1893	¹ Roll, Frederick James, <i>Pearl Life Assurance Co., Ltd., London-bridge, E.C.</i>	1908	¹ Shurrock, Christopher William, <i>Royal Exchange Assurance Cor- poration, Royal Exchange, E.C.</i>
1893	¹ Roodenburch, Bartholomeus Adrianus, <i>Verzekeringsbank Victoria, 126 Keizersgracht, Amsterdam.</i>	1892	¹ Simpson, William Murray, <i>North British and Mercantile Insurance Company, 61 Thread- needle-street, E.C.</i>
1901	¹ Rountree, Arthur FitzGerald, A.C.A., <i>c/o Messrs. Ralli Bros., Calcutta, India.</i>	1905	¹ Sinclair, Coll Claude, B.A., <i>Great West Life Assurance Co., Winnipeg, Manitoba, Canada.</i>
1906	¹ Ruddle, Francis, <i>Consolidated Assurance Co., 9 Fleet-street, E.C.</i>	1891	¹ Sindall, Alfred John, <i>London and Lancashire Life and General Assurance Associa- tion, Limited, 66 & 67 Cornhill, E.C.</i>
1907	¹ Rushton, Thomas Arthur, <i>85 Fleet-street, E.C.</i>	1888	² Slimon, William James. F.F.A., <i>10 Mayfield-terrace, Edinburgh.</i>
1899	¹ Rutter, Edward Valentine, <i>Phoenix Assurance Company, Limited, 70 Lombard-street, E.C.</i>	1907	¹ Smith, Frederick James, <i>Refuge Assurance Company, Oxford-street, Manchester.</i>
1897	¹ Scott, Alexander Lewis, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>	1907	¹ Smith, Reginald Thomas, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>
1911	¹ Shaw, David Winfred, M.A., <i>Reliance Life Insurance Co., Pittsburg, Pa., U.S.A.</i>	1907	¹ Smith, Sydney Arthur George, <i>Fire Offices Committee, Appli- ances Department, 3 York-street, Manchester.</i>
		1903	¹ Smith, William, B.A., <i>Australasian Temperance and General Mutual Life Assurance Society, Ltd., Swanston-street, Melbourne, Australia.</i>

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Date of becoming a student.		Date of becoming a student.	
1904	¹ Spring, Stanley Harold, <i>London Guarantee and Accident Company, 42-45 New Broad-street, E.C.</i>	1907	¹ Taylor, Frederick George, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1910	¹ Spurgeon, Carey Bradford, <i>Phoenix Assurance Company, Limited, 70 Lombard-street, E.C.</i>	1907	¹ Taylor, Herbert George Brooks, <i>National Mutual Life Assoc. of Australasia, 5 Cheapside, E.C.</i>
1901	¹ Steffensen, Johan F., <i>Forsikringsraadet, 1 Christiansgade, Copenhagen.</i>	1908	¹ Thomlinson, Harry, <i>Yorkshire Insur. Co., Ltd., York.</i>
1886	² Stirling, James, F.F.A., <i>Law Union and Rock Insur. Co., Ltd., 126 Chancery-lane, W.C.</i>	1905	¹ Thompson, Joseph William, <i>Norwich Union Life Insurance Society, Norwich.</i>
1908	¹ Stiver, Claud Frank, <i>Manufacturers Life Insurance Co., Toronto, Canada.</i>	1904	¹ Thompson, William George, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>
1910	¹ Stockman, George Donovan, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, S.W.</i>	1906	¹ Thomson, Ernest H. W., <i>64 Cornhill, E.C.</i>
1888	¹ Stott, Walter, <i>Royal Insur. Co., Ltd., Liverpool.</i>	1902	¹ Thwaites, Frederick George, <i>Norwich Union Life Insurance Society, Norwich.</i>
1904	² Strong, Gordon Gilbert, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1907	¹ Tomlinson, Benjamin, <i>Pearl Life Assurance Co., Ltd., London-bridge, E.C.</i>
1904	¹ Sturt, Arthur James, <i>Phoenix Assurance Company, Limited, 70 Lombard-street, E.C.</i>	1897	¹ Townshend, Edward Villiers, <i>Scottish Widows' Fund and Life Assurance Society, 48 Castle-street, Liverpool.</i>
1906	¹ Sutton, Maurice William, <i>127 Knollys-road, Streatham, S.W.</i>	1911	¹ Townshend, Harry, <i>208 Evering-rd., Upper Clapton, N.E.</i>
		1910	¹ Trachtenberg, Henry Lyon, B.A., <i>7 Lauderdale-road, W.</i>
		1907	¹ Turner, John Gilmour, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>

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Date of becoming a Student.		Date of becoming a Student.	
1905	¹ Tutill, Hubert Linzee, <i>English & Scottish Law Life Assurance Association, 33 St. James's-square, S.W.</i>	1893	¹ Welman, Arthur Joseph, <i>Legal & General Life Assurance Soc., 158 Leadenhall-street, E.C.</i>
1891	Tyler, Edgar Alfred, F.S.A.A., F.C.I.S., F.S.S., <i>9 Old Jewry-chambers, Bank, E.C.</i>	1905	² Welsh, Willis, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1906	² Tyler, Victor William, <i>Alliance Assurance Company, Ltd., Bartholomew-lane, E.C.</i>	1908	¹ Williams, Caradoc, <i>Alliance Assurance Co., Ltd. St. James's-street, S.W.</i>
1907	¹ Vineberg, Harris Elias, <i>American Bankers Insurance Co., Tacoma Building, Chicago Ill., U.S.A.</i>	1895	¹ Williams, Henry Samuel Walter, <i>Liverpool & London & Globe Insurance Co., Ltd., Melbourne, Australia.</i>
1906	¹ Warhurst, James, <i>Alliance Assur. Co., Ltd. (Provident Life Fund), 68 Fountain-street, Manchester.</i>	1900	^(A) Williams, Lewis, B.A., <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>
1908	¹ Warner, Arthur Joseph, <i>Equitable Life Assurance of the United States, 346 Strand, W.C.</i>	1906	¹ Williamson, Wallace White, <i>Norwich Union Life Insurance Society, Norwich.</i>
1910	¹ Warwick, Robert Webster, M.A., <i>Government Insurance Dept., Ottawa, Canada.</i>	1907	¹ Wilson, William Clement, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>
1903	¹ Watson, Alexander R. D., <i>89 Queen-street, Auckland, New Zealand.</i>	1901	² Wilton, Herbert George, <i>Norwich Union Life Insurance Society, Norwich.</i>
1906	¹ Watson, John A., <i>Guardian Assurance Company, Sherborne-lane, King William-street, E.C.</i>	1908	² Wolfenden, Hugh Herbert, <i>Grimsby, Ontario, Canada.</i>
1907	¹ Welch, Leslie Gordon, <i>Phoenix Assurance Company, Limited (Law Life Office), 187 Fleet-street, E.C.</i>	1895	¹ Wood, David James, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>
1905	¹ Wellington, Frank, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>	1901	¹ Wood, Roland Stuart, <i>Liverpool & London & Globe Insur. Co., Ltd., 1 Cornhill, E.C.</i>

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1900 ¹ Woolston, Paul Livingston, B.S.,
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1886 Yeatman, Alexander Alfred,
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Dr. Gottfried Schaertlin,
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* * It is requested that any inaccuracy in the foregoing list may be pointed
out to the ASSISTANT SECRETARY.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

A new method of approximating to the values of Last Survivor Annuities on two or more lives, and to the values of Joint-Life Annuities when the advantages of Makeham's Law are not available. By GEORGE J. LIDSTONE, Fellow and one of the Vice-Presidents of the Institute of Actuaries; Fellow of the Actuarial Society of America; Actuary and Secretary of The Equitable Life Assurance Society.

[Read before the Institute, 27 November 1911.]

PART I.

General Explanations.

1. THE complete calculation and tabulation of annuity values depending on two or more lives present considerable practical difficulties, owing to the multiplicity of age-combinations to be dealt with. When the mortality table follows Makeham's Law the difficulty is completely avoided so far as joint life annuities are concerned by the method of equal ages and uniform seniority, but this in no way meets the case of last survivor annuities, which necessitate a special calculation increasing in complexity with the number of lives involved. When the mortality table does not follow Makeham's Law, the difficulty extends to joint life annuities as well as last survivor annuities. In the case of a great standard table, like the O^M, the labour of calculation and expense of printing a complete set of two-life annuities may be justified, but this will rarely be so with a table of less

general importance, while in any case the complete tabulation of three-life and four-life annuities will be quite impracticable; so that at the present time even for O^M Mortality there are no tables that enable such annuity values to be readily calculated.

2. The method now about to be described will be found to meet this want to a very considerable degree, though from the nature of the case it does not possess either the mathematical exactness or the universal applicability that distinguish the elegant method of uniform seniority, as applied to joint life annuities when Makeham's Law holds. On the other hand the method applies, within its limits of range and accuracy, equally to last survivor annuities and joint-life annuities and the allied functions; and provided the mortality table be smoothly graduated it applies whatever the law of mortality involved, so long as the same law extends to all the lives, a proviso which unfortunately excludes the case of combinations of male and female lives subject to different mortality. It may be added that the method holds equally with aggregate tables and with select tables, whatever the period elapsed since selection.

3. A certain amount of mathematical work—not really of a difficult description—is required in demonstrating the theoretical basis of the method, and in working out its details; but this may be taken for granted in the practical application of the method, which is most simple. If α represent the annuity-value on lives of equal age, being the arithmetical average of the actual ages involved, the required annuity value takes the simple form—

$$\alpha + C_1 \times \text{1st correction} + C_2 \times \text{2nd correction}.$$

4. Here C_1 and C_2 are certain coefficients which depend entirely on the actual disparities of age, as compared with a standard set of disparities, so that these coefficients are the same for all mortality tables and rates of interest, and for all functions to which the method applies, and accordingly they can be tabulated once and for all. The quantities 1st correction and 2nd correction are of the nature of differences of a special kind, and like α have to be tabulated for each particular basis and rate of interest required. Where the differences of age are small, the coefficient C_2 is so small that the third term may often be omitted.

5. The greater the range of ages covered by the tables the greater will be their utility: on the other hand, as the range is extended the accuracy of the approximation is diminished, so that for practical purposes a compromise must be made between wideness of range and closeness of approximation. It will be seen in the detailed investigation that the following range of ages can be covered with a *maximum* error of the amount stated in the second decimal place of the annuity value, and a much smaller *average* error.

	Range of Disparities of Age	No. of Age- Combinations	Maximum Error in 2nd Decimal Place
2 lives—joint ...	30 years	31	1-2
„ —last survivor	„	31	1-2
3 lives—joint ...	20 years	231	2-3
„ —last survivor	„	231	7
4 lives	„	1771	See Pars. 21-3, 27

6. As only three sets of annuity-values have to be calculated* in order to find the tabular values which provide for all these combinations, it will be seen that the method, though not of universal applicability, is one of very great power, and covers a large area of practical requirements. Fortunately, too, in many practical instances—as in the case of husband and wife, brothers and sisters—the ages involve only moderate disparity of age, and then the approximation is very close. Further, although joint life and last survivor annuities are probably the most practically important cases, the method applies equally (see par. 8) to any benefit—whole term, temporary, intercepted or deferred, and whether constant or variable—on any status of the form r or $\left[\frac{r}{p} \right]$: thus *wayz ...* in particular it applies to the corresponding values of P, A, E, V and W, and the same coefficients apply throughout.

7. In Part II of this paper (paragraphs 8-27) the method is worked out theoretically and practically, and illustrative tables and examples are given; and in Part III some further developments and special applications are considered.

* It may be noted also that owing to the way in which the three sets are chosen, the last survivor annuity-values to be calculated take a simple form, involving fewer terms than in the general case.

PART II.

Theory and Practice of the Method.

8. Let $F(x, y, z \dots)$ be any symmetrical function of any number of variables, *i.e.*, a function in which the variables are all similarly involved; this is the case with any annuity-value (whole term, temporary, intercepted or deferred) of the form $a_{\overline{x}|r \text{ or } [r]}$ and with the corresponding values of P, A, E and V . Then if $x=y=z \dots$ and $a+b+c \dots = 0$, so that x is the mean value of the variables $(x+a), (y+b), (z+c) \dots$, it is shown in the Appendix that we have an expression of the following form for $F(x+a, x+b, x+c \dots)$, viz., $F(x+a, x+b, x+c \dots) = A + B\Sigma a^2 + C\Sigma a^3 + D\Sigma a^4 \dots$ where $A = F(x, x, x \dots)$. This simple formula is the basis of the method of calculation now about to be described, and it will be found unnecessary to consider the analytical form of $B, C, D \dots$, although this is indicated in the Appendix.

9. In order to fix our ideas it will now be supposed that the variables represent the ages of lives, and that the function F represents an annuity-value, though the line of argument will be precisely the same for other functions. The formula shows that a rough approximation will be the first term A , *i.e.*, the annuity-value for lives of equal ages being the mean of the actual ages involved, and this approximation may actually be sufficient when the individual ages differ by only a few years. A closer approximation may be found by taking in the second term of the formula, involving the value of B , which may be found approximately from any two annuity-values on lives of different sets of ages both giving the same *average* age. Similarly, a further approximation may be found by taking in the third term, and the values of the corrections may be found approximately from any three annuity-values, all on lives of the same average age. Beyond this point we can hardly go without sacrificing the simplicity of calculation that is necessary if the method is to be practically useful and effective. It will be found, however, that without introducing more than a trifling error in the second place of decimals, the 2-term approximation will often cover a considerable range of ages, while the 3-term approximation will enable the range to be considerably increased.

Two Lives.

10. If two lives only are involved, the ages will be of the form $x \pm \frac{1}{2}a$, where a is the range or difference of ages, and the terms of the formula involving odd powers all vanish. Hence the formula takes the shape

$$\begin{aligned} F(x \pm \tfrac{1}{2}a) &= A + Ba^2 + Da^4 + \dots \\ &= A + (B + D + \dots)a^2 + D(a^4 - a^2) + \dots \end{aligned}$$

11. If the corrections be found from the annuity-values on (x, x) , $(x \pm t)$ and $(x \pm nt)$, calling these α , β and γ , and taking $2t$ years as the unit of measurement of the range or difference of ages, it must be assumed that the terms after D vanish, and we have

$$\left. \begin{aligned} \alpha &= A \\ \beta &= A + B + D \\ \gamma &= A + n^2B + n^4D \end{aligned} \right\} \text{whence } \left\{ \begin{aligned} A &= \alpha \\ (B + D) &= \beta - \alpha \\ D &= (\gamma - \beta - n^2 - 1) \beta - \alpha \end{aligned} \right. \div (n^4 - n^2)$$

Hence, the annuity on $(x \pm kt)$ will be

$$\begin{aligned} &\alpha + k^2(\beta - \alpha) + (k^4 - k^2)(\gamma - \beta - \overline{n^2 - 1} \beta - \alpha) \div (n^4 - n^2) \\ &= \alpha + k^2(\beta - \alpha) + \frac{k^4 - k^2}{n^2} \frac{(\gamma - \beta - \overline{n^2 - 1} \beta - \alpha)}{n^2 - 1} \\ &= \alpha + k^2(\beta - \alpha) + \frac{k^2 - k^4}{n^2} \left(\beta - \alpha - \frac{\gamma - \beta}{n^2 - 1} \right) \end{aligned}$$

12. The coefficients involving k^2 and k^4 depend only on the actual range of ages as compared with the standard range. They can thus be calculated and tabulated once and for all, and, accordingly, it is desirable to put them in such a form (though not apparently the most natural) as will give the simplest calculation for the variable terms involving α , β and γ . The form $A + B.k^2 + D.k^4$ would have the advantage of giving slightly smaller values of the third term for small values of α , that is, for lives of nearly equal age which occur most commonly in practice, so that on the whole it would increase to some extent the number of cases in which the third term is negligible. But the form which has been adopted, namely, $A + (B + D)k^2 + D(k^4 - k^2)$, gives a considerably simpler calculation for the tabular value to be multiplied

by k^2 , and it has also the advantage that the coefficient $k^4 - k^2$ is zero for $k=0$ and $k=1$ instead of growing steadily between those limits and its average value between those limits is $-\frac{2}{15} = -\cdot133$ as compared with $\cdot2$, the average value of k^4 .

13. After various trials it was found convenient to use the values for equal ages, 20 years' disparity and 30 years' disparity. Thus $t=10$, $n=\frac{3}{2}$, and $(\beta-\alpha) - \frac{\gamma-\beta}{n^2-1}$ becomes

$$(\beta-\alpha) - \frac{\gamma-\beta}{1\cdot25} = (\beta-\alpha) - \cdot8(\gamma-\beta), \text{ which is easily calculated.}$$

Table A, p. 34, gives the values of the coefficients, which of course are the same for all pairs of ages having the given difference, and for all mortality tables and rates of interest.

14. As an example, having given the joint-life and last-survivor annuities for ages 50:50, 60:40, and 65:35, OM $2\frac{1}{2}$ per-cent, find the values for ages 62:38. Here the difference of age is 24 years and the standard difference is 20 years, so that

$$k=24\div20=1\cdot2, \quad k^2=1\cdot44 \text{ and } \frac{k^2-k^4}{n^2} = -\cdot282$$

$$\alpha = a_{50:50} = 11\cdot174$$

$$\beta - \alpha = -1\cdot461 \quad (\beta - \alpha) - \cdot8(\gamma - \beta)$$

$$\beta = a_{60:40} = 9\cdot713 \quad = -1\cdot461 + 1\cdot152$$

$$\gamma - \beta = -1\cdot440 \quad = -\cdot309$$

$$\gamma = a_{65:35} = 8\cdot273$$

Required approximate value

$$* \quad = 11\cdot174 - 1\cdot461 \times 1\cdot44 + -\cdot309 \times -\cdot282$$

$$* \quad = 11\cdot174 - 2\cdot104 + \cdot087 = 9\cdot157 \quad \left. \vphantom{\begin{matrix} * \\ * \end{matrix}} \right\} \text{error } \cdot009$$

$$\text{True value, } a_{62:38}, \quad = 9\cdot166$$

$$\alpha = a_{50:50} = 18\cdot514$$

$$\beta - \alpha = +1\cdot213 \quad (\beta - \alpha) - \cdot8(\gamma - \beta)$$

$$\beta = a_{60:40} = 19\cdot727 \quad = 1\cdot213 - \cdot959$$

$$\gamma - \beta = +1\cdot199 \quad = +\cdot254$$

$$\gamma = a_{65:35} = 20\cdot926$$

Required approximate value

$$* \quad = 18.514 + 1.213 \times 1.44 + .254 \times -.282$$

$$* \quad = 18.514 + 1.742 - .077 = 20.189$$

True value, $a_{62:38}$

$$= 20.181 \quad \left. \vphantom{= 20.181} \right\} \text{error } .008$$

These examples have been worked out in full so as to illustrate the construction of Table B as well as the calculation of the annuity-value, but when Tables A and B are already prepared, the only additional work required is that indicated in the lines marked *.

15. In order to illustrate the tabulation of values for the practical application of the method, Tables B and C, pp. 34-5, have been prepared on the basis of $O^M 2\frac{1}{2}$ per-cent, the former giving the values for joint-life annuities and the latter for last survivor annuities. As the two joint-life annuity-values are completely tabulated in the published Tables, Table B gives the values, according to the present method, for quinary ages only, this being sufficient as an illustration. The values have to be tabulated for half-ages because the mean age will involve a half-year when the difference of age is an odd number of years, and the tabular values for half-ages are the averages of the two values for integral ages between which they fall. By means of these Tables, in conjunction with the table of coefficients (Table A) we may with great ease calculate annuity-values for any pair of ages not differing by more than 30 years; so that a wide area of practical requirements may be covered by means of Tables, the preparation of which involves the calculation of only three sets of annuity-values with some simple subsidiary calculations. The degree of accuracy attained may be judged by means of the following extensive examples based on Tables B and C, and it may fairly be presumed, in view of the theoretical basis of the method, that the magnitude of the errors would be similar for other mortality tables and rates of interest: though it may be that when the disparity of age is considerable, the error would be greater if the Table were less intimately related to one following Makeham's Law. The errors would, of course, almost certainly be greater if the Table were not smoothly graduated.

EXAMPLES.

Joint Lives—Mean Ages 20. $a_{20:20} = 21.213$. $0^M 2\frac{1}{2}$ per-cent.

Actual Ages	True Value	2-term approx.	Error	3rd term	Error in 3-term approx.
19 : 21	21.202	21.203	+ .001	— .000	.001
17 $\frac{1}{2}$: 22 $\frac{1}{2}$	21.145	21.150	.005	— .003	.002
16 : 24	21.041	21.051	.010	— .006	.004
14 $\frac{1}{2}$: 25 $\frac{1}{2}$	20.891	20.907	.016	— .009	.007
13 : 27	20.699	20.716	.017	— .011	.006
11 $\frac{1}{2}$: 28 $\frac{1}{2}$	20.467	20.481	.014	— .009	.005
10 : 30	20.199	20.199	.000	.000	.000

Joint Lives—Mean Ages 35. $a_{35:35} = 16.481$.

34 : 36	16.468	16.469	+ .001	— .001	.000
32 $\frac{1}{2}$: 37 $\frac{1}{2}$	16.399	16.405	.006	— .004	+ .002
31 : 39	16.272	16.285	.013	— .009	.004
29 $\frac{1}{2}$: 40 $\frac{1}{2}$	16.092	16.110	.018	— .014	.004
28 : 42	15.858	15.880	.022	— .017	.005
26 $\frac{1}{2}$: 43 $\frac{1}{2}$	15.578	15.595	.017	— .014	.003
25 : 45	15.254	15.254	.000	.000	.000
23 $\frac{1}{2}$: 46 $\frac{1}{2}$	14.890	14.859	— .031	+ .029	— .002
22 : 48	14.491	14.407	— .084	.079	— .005
20 $\frac{1}{2}$: 49 $\frac{1}{2}$	14.060	13.902	— .158	.158	.000

Joint Lives—Mean Ages 50. $a_{50:50} = 11.174$.

Actual Ages	True Value	2-term approx.	Error	3rd term	Error in 3-term approx.
49 : 51	11.158	11.159	+ .001	— .001	.000
47½ : 52½	11.072	11.083	.011	— .008	+ .003
46 : 54	10.916	10.940	.024	— .019	.005
44½ : 55½	10.695	10.733	.038	— .029	.009
43 : 57	10.415	10.458	.043	— .034	.009
41½ : 58½	10.085	10.119	.034	— .028	.006
40 : 60	9.713	9.713	.000	.000	.000
38½ : 61½	9.308	9.243	— .065	.059	— .006
37 : 63	8.876	8.705	— .171	.160	— .011
35½ : 64½	8.425	8.103	— .323	.318	— .005

Joint Lives—Mean Ages 65. $a_{65:65} = 5.902$.

64 : 66	5.888	5.890	+ .002	— .001	+ .001
62½ : 67½	5.816	5.828	.012	— .009	.003
61 : 69	5.683	5.712	.029	— .020	.009
59½ : 70½	5.500	5.543	.043	— .032	.011
58 : 72	5.270	5.319	.049	— .038	.011
56½ : 73½	5.006	5.044	.038	— .030	.008
55 : 75	4.713	4.713	.000	.000	.000
53½ : 76½	4.404	4.330	— .074	+ .065	— .009
52 : 78	4.083	3.893	— .190	.177	— .013
50½ : 79½	3.761	3.493	— .358	.351	— .007

Joint Lives—Mean Ages 80. $a_{80:80} = 2.171$.

79 : 81	2.163	2.165	+ .002	— .001	+ .001
77½ : 82½	2.123	2.132	.009	— .006	.003
76 : 84	2.048	2.069	.021	— .014	.007
74½ : 85½	1.948	1.979	.031	— .022	.009
73 : 87	1.823	1.859	.036	— .027	.009
71½ : 88½	1.685	1.712	.027	— .021	.006
70 : 90	1.535	1.535	.000	.000	.000
68½ : 91½	1.384	1.330	— .054	+ .045	— .009
67 : 93	1.232	1.096	— .136	.124	— .012
65½ : 94½	1.085	.834	— .251	.246	— .005

Last Survivor of Two Lives—Mean Ages 20. $a_{20:20} = 28.589$

Actual Ages	True Value	2-term approx.	Error	3rd term	Error in 3-term approx.
19 : 21	28.595	28.594	-.001	.000	-.001
17½ : 22½	28.625	28.622	-.003	.003	.000
16 : 24	28.683	28.673	-.010	.007	-.003
14½ : 25½	28.761	28.748	-.013	.011	-.002
13 : 27	28.864	28.848	-.016	.013	-.003
11½ : 28½	28.982	28.970	-.012	.011	-.001
10 : 30	29.117	29.117	-.000	.000	.000

Last Survivor of Two Lives—Mean Ages 35. $a_{35:35} = 24.199$

34 : 36	24.209	24.207	-.002	.001	-.001
32½ : 37½	24.255	24.249	-.006	.004	-.002
31 : 39	24.343	24.329	-.014	.009	-.005
29½ : 40½	24.463	24.444	-.019	.014	-.005
28 : 42	24.619	24.596	-.023	.017	-.006
26½ : 43½	24.800	24.785	-.015	.014	-.001
25 : 45	25.010	25.010	.000	.000	.000
23½ : 46½	25.239	25.271	+ .032	-.029	+ .003
22 : 48	25.485	25.570	.085	-.079	.006
20½ : 49½	25.745	25.904	.159	-.157	.002

Last Survivor of Two Lives—Mean Ages 50. $a_{50:50} = 18.514$

49 : 51	18.527	18.526	-.001	+ .001	.000
47½ : 52½	18.599	18.589	-.010	.007	-.003
46 : 54	18.728	18.708	-.020	.015	-.005
44½ : 55½	18.913	18.880	-.033	.024	-.009
43 : 57	19.144	19.108	-.036	.028	-.008
41½ : 58½	19.418	19.390	-.028	.023	-.005
40 : 60	19.727	19.727	.000	.000	.000
38½ : 61½	20.063	20.118	+ .055	-.048	+ .007
37 : 63	20.422	20.564	.142	-.132	.010
35½ : 64½	20.798	21.064	.266	-.262	.004

Last Survivor of Two Lives—Mean Ages 65. $a_{65:65} = 11.816$.

Actual Ages	True Value	2-term approx.	Error	3rd term	Error in 3-term approx.
64 : 66	11.835	11.833	-.002	.001	-.001
62½ : 67½	11.934	11.919	-.015	.009	-.006
61 : 69	12.112	12.082	-.030	.022	-.008
59½ : 70½	12.365	12.319	-.046	.034	-.012
58 : 72	12.683	12.632	-.051	.040	-.011
56½ : 73½	13.059	13.018	-.041	.032	-.009
55 : 75	13.481	13.481	.000	.000	.000
53½ : 76½	13.940	14.017	+.077	-.069	+.008
52 : 78	14.428	14.630	.202	-.188	.014
50½ : 79½	14.936	15.316	.380	-.374	.006

Last Survivor of Two Lives—Mean Ages 80. $a_{80:80} = 5.665$

79 : 81	5.684	5.681	-.003	+.001	-.002
77½ : 82½	5.781	5.766	-.015	.008	-.007
76 : 84	5.951	5.926	-.025	.018	-.007
74½ : 85½	6.198	6.157	-.041	.029	-.012
73 : 87	6.508	6.464	-.044	.034	-.010
71½ : 88½	6.878	6.842	-.036	.027	-.009
70 : 90	7.295	7.295	.000	.000	.000
68½ : 91½	7.755	7.820	+.065	-.058	+.007
67 : 93	8.250	8.420	.170	-.157	.013
65½ : 94½	8.774	9.091	.317	-.313	.004

16. These examples show that for mean ages 10 to 80 the maximum error is about .012 when the disparity is not more than 20 years, and about .014 when the disparity is between 20 and 30 years. Though it is trying the method rather high to take disparities of age greater than the extreme range, viz. 30 years, on which the Tables are based it will be found that the approximation is quite respectable up to a disparity of 34 years. For example, taking a disparity of 32 years, the error in the joint-life annuity is +.009 for mean age 35, +.019 for mean age 50, +.023 for mean age 65, and +.023 for mean age 80. Again, taking a disparity of 34 years, the error is +.022 for 35, +.049 for 50, +.064 for 65, and +.047 for 80.

17. The examples suggest that the extreme range of 30 years on which the Tables are based might perhaps be extended without serious increase of error. The following additional examples based on the values for equal ages, 24 years' disparity and 36 years' disparity (thus preserving the convenient value $\frac{3}{2}$ for n) will illustrate this point. The examples have been calculated for disparities of 16 and 32 years, being respectively $\frac{2}{3}$ and $\frac{4}{3}$ of the extended standard, viz., 24 years, because these proportions are found to produce the maximum errors. The results are as follow for joint-life annuities, and are practically identical for last-survivor annuities:

<i>Mean age 35.</i>	Actual ages 27 : 43, error +.011.
	„ „ 19 : 51, „ -.012.
<i>Mean age 50.</i>	„ „ 42 : 58, „ +.022.
	„ „ 34 : 66, „ -.024.
<i>Mean age 65.</i>	„ „ 57 : 73, „ +.028.
	„ „ 49 : 81, „ -.031.

Three Lives.

18. When more than two lives are involved, the terms including odd powers will not as a rule vanish, and the first three terms will therefore be

$$\begin{aligned} F(x+a, x+b, x+c) &= A + B\Sigma a^2 + C\Sigma a^3 \\ &= A + (B+C)\Sigma a^2 + C(\Sigma a^3 - \Sigma a^2) \end{aligned}$$

which latter form will be used for the reasons already indicated in the case of two lives. It will be found that for any given range of ages, say $3t$, the first coefficient Σa^2 has its maximum value when the ages are of the form $y, y+3t, y+3t$ or $y, y, y+3t$, which give respectively $a=-2t, b=t, c=t$ and $a=-t, b=-t, c=2t$. These give identical values for Σa^2 , namely, $6t^2$, and the same numerical values with the sign changed for Σa^3 , namely, $\mp 6t^3$. Let us put $a_{x;x;x}=\alpha, a_{x-2t;x+t;x+t}=\beta, a_{x-t;x-t;x+2t}=\gamma$, which give these values of a, b and c ; and similarly with the corresponding last survivor annuities. Then we have

$$\left. \begin{aligned} \alpha &= A \\ \beta &= A + 6t^2B - 6t^3C \\ \gamma &= A + 6t^2B + 6t^3C \end{aligned} \right\} \text{whence } \left\{ \begin{aligned} A &= \alpha \\ B &= \left(\frac{\beta + \gamma}{2} - \alpha \right) \div 6t^2 = \left(\beta - \alpha + \frac{\gamma - \beta}{2} \right) \div 6t \\ C &= (\gamma - \beta) \div 12t^3 \end{aligned} \right.$$

19. Hence the annuity-value for three lives of average age x , but with other differences of age and therefore other values of a, b and c , will be

$$\begin{aligned} & \alpha + \frac{\Sigma a^2}{6t^2} \left(\beta - \alpha + \frac{\gamma - \beta}{2} \right) + \frac{\Sigma a^3}{12t^3} (\gamma - \beta) \\ &= \alpha + \frac{\Sigma a^2}{6t^2} (\beta - \alpha) + \left(\frac{\Sigma a^2}{12t^2} + \frac{\Sigma a^3}{12t^3} \right) (\gamma - \beta) \end{aligned}$$

20. Taking $3t=18$, Table D gives the values of the coefficients $\frac{\Sigma a^2}{6t^2}$ and $\left(\frac{\Sigma a^2}{12t^2} + \frac{\Sigma a^3}{12t^3} \right)$ for all combinations of ages within a range of 20 years. It will be found that differences of age p and q , *i.e.*, actual ages $y, y+p, y+q$, give the same values for $\Sigma a^2:6t^2$ as $(q-p)$ and q , *i.e.*, actual ages $y, y+q-p, y+q$. Table E gives the values of $\alpha, (\beta-\alpha)$ and $(\gamma-\beta)$ for three joint lives on the basis $O^M 2\frac{1}{2}$ per-cent, and Table F the corresponding values for the last survivor of three lives. It would involve a good deal of heavy numerical work to test the accuracy of the approximation by a number of examples on that basis, as the true annuity-values are not tabulated in the published volume of tables. The following comparisons have, however, been worked out on the $O^M 5$ Table, $2\frac{1}{2}$ per-cent, and it is thought that the degree of error involved would be similar for other mortality tables and rates of interest.

EXAMPLES.

Three Joint Lives. $x : x + m : x + n$. $O^{M51} 2\frac{1}{2}$ per-cent.

Youngest Age	Mean Ages 20. $a_{20 : 20 : 20} = 17.527$.						
	m	n	True Value	2-term Approximation	Error	3-term Approximation	Final Error
$18\frac{1}{3}$	0	5	17.473	17.486	.013	17.476	.003
$16\frac{2}{3}$	0	10	17.306	17.362	.056	17.312	.006
15	0	15	17.016	17.155	.139	17.022	.006
$13\frac{1}{3}$	0	20	16.602	16.866	.264	16.594	-.008
$16\frac{2}{3}$	5	5	17.478	17.486	.008	17.480	.002
15	5	10	17.375	17.403	.028	17.379	.004
$13\frac{1}{3}$	5	15	17.154	17.238	.084	17.159	.005
$11\frac{2}{3}$	5	20	16.805	16.990	.185	16.807	.002
$13\frac{1}{3}$	10	10	17.345	17.362	.017	17.348	.003
$11\frac{2}{3}$	10	15	17.201	17.238	.037	17.204	.003
10	10	20	16.934	17.032	.098	16.935	.001
10	15	15	17.141	17.155	.014	17.143	.002
$8\frac{1}{3}$	15	20	16.964	16.990	.026	16.964	.000
$6\frac{2}{3}$	20	20	16.883	16.866	-.017	16.880	-.003
Mean Ages 35. $a_{35 : 35 : 35} = 13.814$.							
$33\frac{1}{3}$	0	5	13.738	13.756	.018	13.742	.004
$31\frac{2}{3}$	0	10	13.500	13.581	.081	13.511	.011
30	0	15	13.093	13.290	.197	13.105	.012
$28\frac{1}{3}$	0	20	12.523	12.883	.360	12.504	-.019
$31\frac{2}{3}$	5	5	13.743	13.756	.013	13.748	.005
30	5	10	13.597	13.640	.043	13.606	.009
$28\frac{1}{3}$	5	15	13.285	13.407	.122	13.297	.012
$26\frac{2}{3}$	5	20	12.800	13.058	.258	12.803	.003
$28\frac{1}{3}$	10	10	13.557	13.581	.024	13.561	.004
$26\frac{2}{3}$	10	15	13.351	13.407	.056	13.360	.009
25	10	20	12.978	13.116	.138	12.981	.003
25	15	15	13.268	13.290	.022	13.273	.005
$23\frac{1}{3}$	15	20	13.020	13.058	.038	13.021	.001
$21\frac{2}{3}$	20	20	12.908	12.883	-.025	12.903	-.005
Mean Ages 50. $a_{50 : 50 : 50} = 9.045$.							
$48\frac{1}{3}$	0	5	8.959	8.981	.022	8.966	.007
$46\frac{2}{3}$	0	10	8.692	8.786	.094	8.711	.019
45	0	15	8.244	8.463	.219	8.265	.021
$43\frac{1}{3}$	0	20	7.637	8.011	.374	7.606	-.031
$46\frac{2}{3}$	5	5	8.966	8.981	.015	8.972	.006
45	5	10	8.801	8.852	.051	8.816	.015
$43\frac{1}{3}$	5	15	8.454	8.593	.139	8.475	.021
$41\frac{2}{3}$	5	20	7.929	8.205	.276	7.932	.003
$43\frac{1}{3}$	10	10	8.755	8.786	.031	8.765	.010
$41\frac{2}{3}$	10	15	8.527	8.593	.066	8.543	.016
40	10	20	8.120	8.270	.150	8.126	.006
40	15	15	8.435	8.463	.028	8.445	.010
$38\frac{1}{3}$	15	20	8.165	8.205	.040	8.166	.001
$36\frac{2}{3}$	20	20	8.044	8.011	-.033	8.032	-.012

*Three Joint Lives. $x : x + m : x + n$.—Continued.***O^{M5} 21₂** per-cent.

Youngest Age	Mean Ages 65. $a_{65 : 65 : 65} = 4.441.$						
	<i>m</i>	<i>n</i>	True Value	2-term Approximation	Error	3-term Approximation	Final Error
63 $\frac{1}{3}$	0	5	4.373	4.391	.018	4.380	.007
61 $\frac{2}{3}$	0	10	4.163	4.242	.079	4.187	.024
60	0	15	3.823	3.993	.170	3.848	.025
58 $\frac{1}{3}$	0	20	3.382	3.645	.263	3.348	-.034
61 $\frac{2}{3}$	5	5	4.379	4.391	.012	4.385	.006
60	5	10	4.249	4.292	.043	4.266	.017
58 $\frac{1}{3}$	5	15	3.981	4.093	.112	4.007	.026
56 $\frac{2}{3}$	5	20	3.591	3.794	.203	3.594	.003
58 $\frac{1}{3}$	10	10	4.212	4.242	.030	4.226	.014
56 $\frac{2}{3}$	10	15	4.037	4.093	.056	4.056	.019
55	10	20	3.731	3.844	.113	3.738	.007
55	15	15	3.966	3.993	.027	3.980	.014
53 $\frac{1}{3}$	15	20	3.765	3.794	.029	3.765	.000
51 $\frac{2}{3}$	20	20	3.674	3.645	-.029	3.661	-.013
Mean Ages 80. $a_{80 : 80 : 80} = 1.442.$							
78 $\frac{1}{3}$	0	5	1.408	1.418	.010	1.413	.005
76 $\frac{2}{3}$	0	10	1.304	1.345	.041	1.320	.016
75	0	15	1.140	1.224	.084	1.157	.017
73 $\frac{1}{3}$	0	20	.939	1.054	.115	.916	-.023
76 $\frac{2}{3}$	5	5	1.411	1.418	.007	1.415	.004
75	5	10	1.346	1.369	.023	1.357	.011
73 $\frac{1}{3}$	5	15	1.216	1.272	.056	1.232	.016
71 $\frac{2}{3}$	5	20	1.033	1.127	.094	1.034	.001
73 $\frac{1}{3}$	10	10	1.328	1.345	.017	1.338	.010
71 $\frac{2}{3}$	10	15	1.213	1.272	.059	1.255	.042
70	10	20	1.097	1.151	.054	1.102	.005
70	15	15	1.208	1.224	.016	1.218	.010
68 $\frac{1}{3}$	15	20	1.113	1.127	.014	1.114	.001
66 $\frac{2}{3}$	20	20	1.071	1.054	-.017	1.061	-.010

Last Survivor of Three Lives. $x : x + m : x + n$.

$0^{\text{M}5} 2\frac{1}{2}$ per-cent.

Youngest Age	Mean Ages 20. $a_{20:20:20}=29\cdot683$.						
	m	n	True Value	2-term Approximation	Error	3-term Approximation	Final Error
$18\frac{1}{3}$	0	5	29·713	29·715	·002	29·715	·002
$16\frac{2}{3}$	0	10	29·824	29·813	·011	29·813	·011
15	0	15	29·989	29·975	·014	29·976	·013
$13\frac{1}{3}$	0	20	30·181	*30·201	·020	30·202	·021
$16\frac{2}{3}$	5	5	29·721	29·715	·006	29·715	·006
15	5	10	29·792	29·780	·012	29·780	·012
$13\frac{1}{3}$	5	15	29·923	29·910	·013	29·910	·013
$11\frac{2}{3}$	5	20	30·104	*30·104	·000	30·105	·001
$13\frac{1}{3}$	10	10	29·823	29·813	·010	29·813	·010
$11\frac{2}{3}$	10	15	29·929	29·910	·019	29·910	·019
10	10	20	30·080	30·072	·008	30·072	·008
10	15	15	29·991	29·975	·016	29·975	·016
$8\frac{1}{3}$
$6\frac{2}{3}$
Mean Ages 35. $a_{35:35:35}=25\cdot619$.							
$33\frac{1}{3}$	0	5	25·678	*25·675	·003	25·673	·005
$31\frac{2}{3}$	0	10	25·856	*25·845	·011	25·834	·022
30	0	15	26·120	*26·128	·008	26·099	·021
$28\frac{1}{3}$	0	20	26·434	26·523	·089	26·463	·029
$31\frac{2}{3}$	5	5	25·682	*26·675	·007	26·674	·008
30	5	10	25·803	*25·788	·015	25·783	·020
$28\frac{1}{3}$	5	15	26·029	*26·014	·015	25·997	·032
$26\frac{2}{3}$	5	20	26·313	26·353	·040	26·313	·000
$28\frac{1}{3}$	10	10	25·861	*25·845	·016	25·842	·019
$26\frac{2}{3}$	10	15	26·036	*26·014	·022	26·007	·029
25	10	20	26·284	26·297	·013	26·276	·008
25	15	15	26·148	*26·128	·020	26·125	·023
$23\frac{1}{3}$	15	20	26·342	26·353	·011	26·347	·005
$21\frac{2}{3}$	20	20	26·499	*26·523	·024	26·526	·027
Mean Ages 50. $a_{50:50:50}=20\cdot103$.							
$48\frac{1}{3}$	0	5	20·194	*20·194	·000	20·187	·007
$46\frac{2}{3}$	0	10	20·468	*20·466	·002	20·433	·035
45	0	15	20·866	20·921	·055	20·835	·031
$43\frac{1}{3}$	0	20	21·344	21·557	·213	21·380	·036
$46\frac{2}{3}$	5	5	20·202	*20·194	·008	20·190	·012
45	5	10	20·387	*20·375	·012	20·359	·028
$43\frac{1}{3}$	5	15	20·723	*20·739	·016	20·688	·035
$41\frac{2}{3}$	5	20	21·168	21·283	·115	21·164	·004
$43\frac{1}{3}$	10	10	20·491	*20·466	·025	20·457	·034
$41\frac{2}{3}$	10	15	20·761	*20·739	·022	20·717	·044
40	10	20	21·145	21·193	·048	21·130	·015
40	15	15	20·948	*20·921	·027	20·913	·035
$38\frac{1}{3}$	15	20	21·262	21·283	·021	21·266	·004
$36\frac{2}{3}$	20	20	21·522	*21·557	·035	21·566	·044

*Last Survivor of Three Lives. $x : x + m : x + n$.—Continued.***0^{M5} 2 $\frac{1}{2}$** per-cent.

Youngest Age	Mean Ages 65. $a_{\overline{65:65:65}} = 13.318.$						
	<i>m</i>	<i>n</i>	True Value	2-term Approximation	Error	3-term Approximation	Final Error
63 $\frac{1}{3}$	0	5	13.447	*13.445	— .002	13.431	— .016
61 $\frac{2}{3}$	0	10	13.801	*13.829	.028	13.760	— .041
60	0	15	14.322	14.467	.145	14.284	— .038
58 $\frac{1}{3}$	0	20	14.942	15.358	.416	14.983	.041
61 $\frac{2}{3}$	5	5	13.456	*13.445	— .011	13.437	— .019
60	5	10	13.704	*13.700	— .004	13.667	— .037
58 $\frac{1}{3}$	5	15	14.149	14.211	.062	14.102	— .047
56 $\frac{2}{3}$	5	20	14.727	14.975	.248	14.722	— .005
58 $\frac{1}{3}$	10	10	13.855	*13.829	— .026	13.809	— .046
56 $\frac{2}{3}$	10	15	14.223	*14.211	— .012	14.164	— .059
55	10	20	14.733	14.848	.115	14.715	— .018
55	15	15	14.497	*14.467	— .030	14.450	— .047
53 $\frac{1}{3}$	15	20	14.940	14.975	.035	14.939	— .001
51 $\frac{2}{3}$	20	20	15.319	*15.358	.039	15.378	.059
	Mean Ages 80. $a_{\overline{80:80:80}} = 6.686.$						
	<i>m</i>	<i>n</i>	True Value	2-term Approximation	Error	3-term Approximation	Final Error
78 $\frac{1}{3}$	0	5	6.814	*6.820	.006	6.799	— .015
76 $\frac{2}{3}$	0	10	7.156	7.222	.066	7.120	— .036
75	0	15	7.649	7.893	.244	7.623	— .026
73 $\frac{1}{3}$	0	20	8.248	8.830	.582	8.276	.028
76 $\frac{2}{3}$	5	5	6.825	*6.820	— .005	6.808	— .017
75	5	10	7.068	*7.087	.019	7.038	— .030
73 $\frac{1}{3}$	5	15	7.504	7.623	.119	7.462	— .042
71 $\frac{2}{3}$	5	20	8.061	8.427	.366	8.054	— .007
73 $\frac{1}{3}$	10	10	7.238	*7.222	— .016	7.193	— .045
71 $\frac{2}{3}$	10	15	7.606	*7.623	.017	7.554	— .052
70	10	20	8.113	8.294	.181	8.097	— .016
70	15	15	7.910	*7.893	— .017	7.868	— .042
68 $\frac{1}{3}$	15	20	8.375	8.427	.052	8.373	— .002
66 $\frac{2}{3}$	20	20	8.805	*8.830	.025	8.859	.054

21. It is hardly to be expected that in the case of 3-life annuities, the approximation will hold good for so long a range as in the case of 2-life annuities, because in the former case we neglect terms involving *fourth* and higher orders of diff. cos., whereas in the latter case the first neglected term involves diff. cos. of the *sixth* order. Still, it will be seen from the foregoing examples that as regards 3-life annuities on joint lives, the approximation is quite good up to a range of 20 years, which will cover no less than 231 age-combinations,* all based on the direct calculation of only three combinations.

22. In the case of the last survivor annuities on three lives, the approximation is not always so close, although it is thought that it should be sufficient for many, if not most, practical purposes—especially having regard (a) to the much larger annuity-values involved, and (b) to the great practical convenience and saving of labour secured by dealing direct with the last survivor annuity-value, $a_{\overline{xyz}}$, as a single tabulated function, instead of having to calculate each of its constituent parts in the form

$$a_x + a_y + a_z - (a_{xy} + a_{xz} + a_{yz}) + a_{xyz}.$$

23. A curious feature, which the writer cannot satisfactorily explain, is that in the majority of cases (marked with * in the examples) the first approximation is actually closer than the second. Where, however, this is not the case, the first approximation sometimes involves a quite inadmissible error, and accordingly it will be better to use the second approximation throughout, unless further investigation reveals some means of determining in advance the class of cases for which the first approximation gives the better results.

Four Lives.

24. When four lives are involved, the formula will be of the same type as for three lives, and we shall have

$$F(x+a, x+b, x+c, x+d) = A + (B+C)\Sigma a^2 + C(\Sigma a^3 - \Sigma a^2)$$

For any given range of ages, say $4t$, it is convenient to take the annuity-values for ages of the form $y, y+4t, y+4t,$

* If n be the extreme range, the number of 3-life combinations is $\frac{1}{2}(n+1)(n+2)$.

$y + 4t$ or $y, y, y, y + 4t$, which give respectively $a = -3t$, $b = t$, $c = t$, $d = t$, and $a = -t$, $b = -t$, $c = -t$, $d = 3t$. These give the same values for Σa^2 , namely, $12t^2$ and the same numerical value with the signs changed for Σa^3 , namely, $\mp 24t^3$. Let us put

$$\alpha = a_{x;x;x;x}$$

$$\beta = a_{x-3t;x+t;x+t;x+t}$$

$$\gamma = a_{x-t;x-t;x-t;x+3t}$$

which involve the required values of a , b and c and similarly with the corresponding last survivor annuities. Then we shall find—by the same method as in the case of three lives—that the annuity-value for four lives of average age x , but with other differences of age and therefore other values of a , b , c and d will be

$$\alpha + \frac{\Sigma a^2}{12t^2}(\beta - \alpha) + \left(\frac{\Sigma a^2}{24t^2} + \frac{\Sigma a^3}{48t^3}\right)(\gamma - \beta)$$

25. Taking $4t = 20$, Table G gives the values of the coefficients $\frac{\Sigma a^2}{12t^2}$ and $\left(\frac{\Sigma a^2}{24t^2} + \frac{\Sigma a^3}{48t^3}\right)$ for selected combinations of ages of sufficient extent to enable a representative set of examples to be calculated. The complete tables would contain 1,771 age combinations* and would thus be very lengthy: and it is doubtful whether 4-life functions occur sufficiently often in practice to justify the labour and expense of calculating and printing a complete table, especially as the results are not found to be altogether satisfactory.

26. Table H gives the values of α , $(\beta - \alpha)$ and $(\gamma - \beta)$ for four joint lives on the basis $O^M 2\frac{1}{2}$ per-cent, and Table J the corresponding values for the last survivor of four lives. For the reasons stated in the case of three lives (par. 20) the following comparisons are based on the $O^M 5\%$ Table at the same rate of interest.

* If n be the extreme range, the number of 4-life combinations is $\frac{1}{6}(n+1)(n+2)(n+3)$. Thus, in Table G, where the unit is 4 years and the extreme range, 20 years, is 5 units, the number of cases is $\frac{1}{6} \times 6 \times 7 \times 8 = 56$, of which 55 are given and the case of equal ages is omitted.

EXAMPLES.

Four Joint Lives. $x : x + m : x + n : x + p$. $O^{M5} 2\frac{1}{2}$ per-cent.

Young- est Age	Mean Ages 25. $a_{25:25:25:25} = 14.542$							
	m	n	p	True Value	2-term Approximation	Error	3-term Approximation	Final Error
24	0	0	4	14.511	14.521	.010	14.512	.001
22	0	0	12	14.234	14.349	.115	14.237	.003
21	0	8	8	14.387	14.428	.041	14.386	-.001
19	0	8	16	14.081	14.228	.147	14.077	-.004
17	0	16	16	13.946	14.086	.140	13.920	-.026
16	0	16	20	13.762	13.950	.188	13.721	-.041
15	0	20	20	13.633	13.829	.196	13.570	-.063
22	4	4	4	14.513	14.521	.008	14.515	.002
20	4	4	12	14.344	14.407	.063	14.344	.000
18	4	4	20	13.849	14.121	.272	13.852	.003
18	4	12	12	14.290	14.349	.059	14.287	-.003
16	4	12	20	13.946	14.121	.175	13.936	-.010
14	4	20	20	13.791	13.950	.159	13.749	-.042
19	8	8	8	14.440	14.456	.016	14.437	-.003
17	8	8	16	14.232	14.314	.082	14.231	-.001
15	8	16	16	14.160	14.228	.068	14.152	-.008
13	8	20	20	13.914	14.028	.114	13.891	-.023
16	12	12	12	14.324	14.349	.025	14.321	-.003
14	12	12	20	14.087	14.178	.091	14.085	-.002
13	12	16	20	14.078	14.142	.064	14.072	-.006
13	16	16	16	14.177	14.200	.023	14.175	-.002
12	16	16	20	14.090	14.121	.031	14.085	-.005
11	16	20	20	14.032	14.057	.025	14.030	-.002
10	20	20	20	14.007	14.007	.000	14.007	.000
Mean Ages 45. $a_{45:45:45:45} = 9.197$								
44	0	0	4	9.155	9.168	.013	9.156	.001
42	0	0	12	8.772	8.935	.163	8.789	.017
41	0	8	8	8.982	9.042	.060	8.988	.006
39	0	8	16	8.565	8.769	.204	8.572	.007
37	0	16	16	8.389	8.575	.186	8.359	-.030
36	0	16	20	8.148	8.390	.242	8.092	-.056
35	0	20	20	7.980	8.225	.245	7.887	-.093
42	4	4	4	9.158	9.168	.010	9.160	.002
40	4	4	12	8.921	9.013	.092	8.931	.010
38	4	4	20	8.261	8.623	.362	8.272	.011
38	4	12	12	8.849	8.935	.086	8.854	.005
36	4	12	20	8.389	8.623	.234	8.382	-.007
34	4	20	20	8.185	8.390	.205	8.128	-.057
39	8	8	8	9.055	9.080	.025	9.056	.001
37	8	8	16	8.769	8.886	.117	8.778	.009
35	8	16	16	8.672	8.769	.097	8.669	-.003
33	8	20	20	8.347	8.497	.150	8.319	-.028
36	12	12	12	8.894	8.935	.041	8.899	.005
34	12	12	20	8.574	8.701	.127	8.580	.006
33	12	16	20	8.562	8.652	.090	8.560	-.002
33	16	16	16	8.696	8.730	.034	8.698	.002
32	16	16	20	8.577	8.623	.046	8.576	-.001
31	16	20	20	8.501	8.536	.035	8.501	.000
30	20	20	20	8.468	8.468	.000	8.468	.000

Four Joint Lives. $x : x + m : x + n : x + p$.—Continued. $0^{M5} 2\frac{1}{2}$ per-cent.

Young-est Age	Mean Ages 65. $a_{65 : 65 : 65 : 65} = 3.555.$							
	m	n	p	True Value	2-term Approximation	Error	3-term Approximation	Final Error
64	0	0	4	3.524	3.534	.010	3.526	.002
62	0	0	12	3.243	3.365	.122	3.268	.025
61	0	8	8	3.396	3.443	.047	3.407	.011
59	0	8	16	3.095	3.246	.151	3.115	.020
57	0	16	16	2.974	3.105	.131	2.961	-.013
56	0	16	20	2.813	2.972	.159	2.774	-.039
55	0	20	20	2.702	2.853	.151	2.628	-.074
62	4	4	4	3.526	3.534	.008	3.529	.003
60	4	4	12	3.351	3.422	.071	3.367	.016
58	4	4	20	2.887	3.140	.253	2.906	.019
58	4	12	12	3.297	3.365	.068	3.311	.014
56	4	12	20	2.974	3.140	.166	2.979	.005
54	4	20	20	2.837	2.972	.135	2.798	-.039
59	8	8	8	3.449	3.471	.022	3.455	.006
57	8	8	16	3.241	3.330	.089	3.258	.017
55	8	16	16	3.170	3.246	.076	3.180	.010
53	8	20	20	2.945	3.049	.104	2.930	-.015
56	12	12	12	3.330	3.365	.035	3.341	.011
54	12	12	20	3.102	3.197	.095	3.116	.014
53	12	16	20	3.093	3.161	.068	3.100	.007
53	16	16	16	3.188	3.218	.030	3.196	.008
52	16	16	20	3.104	3.140	.036	3.109	.005
51	16	20	20	3.051	3.077	.026	3.054	.003
50	20	20	20	3.028	3.028	.000	3.028	.000
Mean Ages 85. $a_{85 : 85 : 85 : 85} = .590.$								
84	0	0	4	.580	.584	.004	.582	.002
82	0	0	12	.493	.532	.039	.506	.013
81	0	8	8	.539	.556	.017	.546	.007
79	0	8	16	.449	.496	.047	.460	.011
77	0	16	16	.415	.454	.039	.415	.000
76	0	16	20	.370	.413	.043	.359	-.011
75	0	20	20	.341	.377	.036	.316	-.025
82	4	4	4	.581	.584	.003	.583	.002
80	4	4	12	.525	.550	.025	.535	.010
78	4	4	20	.391	.464	.073	.400	.009
78	4	12	12	.510	.532	.022	.517	.007
76	4	12	20	.415	.464	.049	.420	.005
74	4	20	20	.377	.413	.036	.365	-.012
79	8	8	8	.557	.564	.007	.560	.003
77	8	8	16	.492	.522	.030	.502	.010
75	8	16	16	.472	.496	.024	.478	.006
73	8	20	20	.407	.446	.039	.414	.007
76	12	12	12	.519	.532	.013	.525	.006
74	12	12	20	.451	.481	.030	.459	.008
73	12	16	20	.449	.470	.021	.453	.004
73	16	16	16	.477	.488	.011	.482	.005
72	16	16	20	.452	.464	.012	.455	.003
71	16	20	20	.437	.445	.008	.439	.002
70	20	20	20	.430	.430	.000	.430	.000

Last Survivor of Four Lives. $x : x + m : x + n : x + p$.

$0^{M.5} 2\frac{1}{2}$ per-cent.

Young- est Age	Mean Ages 25. $a_{25 : 25 : 25 : 25} = 29.204$.							
	<i>m</i>	<i>n</i>	<i>p</i>	True Value	2-term Approximation	Error	3-term Approximation	Final Error
24	0	0	4	29.230	29.231	.001	29.228	-.002
22	0	0	12	29.433	29.447	.014	29.416	-.017
21	0	8	8	29.354	29.348	-.006	29.337	-.017
19	0	8	16	29.588	29.600	.012	29.559	-.029
17	0	16	16	29.773	29.779	.006	29.734	-.039
16	0	16	20	29.910	29.950	.040	29.888	-.022
15	0	20	20	30.033	30.102	.069	30.031	-.002
22	4	4	4	29.231	29.231	.000	29.229	-.002
20	4	4	12	29.376	29.375	-.001	29.358	-.018
18	4	4	20	29.683	29.734	.051	29.661	-.027
18	4	12	12	29.464	29.447	-.017	29.430	-.034
16	4	12	20	29.713	29.734	.021	29.683	-.030
14	4	20	20	29.912	29.950	.038	29.895	-.017
19	8	8	8	29.321	29.312	-.009	29.307	-.014
17	8	8	16	29.494	29.492	-.002	29.469	-.025
15	8	16	16	29.611	29.600	-.011	29.579	-.032
13	8	20	20	29.833	29.851	.018	29.814	-.019
16	12	12	12	29.461	29.447	-.014	29.439	-.022
14	12	12	20	29.657	29.662	.005	29.637	-.020
13	12	16	20	29.716	29.707	-.009	29.688	-.028
13	16	16	16	29.655	29.635	-.020	29.628	-.027
12	16	16	20	29.745	29.734	-.011	29.724	-.021
11	16	20	20	29.825	29.815	-.010	29.808	-.017
10	20	20	20	29.878	29.878	.000	29.878	.000
Mean Ages 45. $a_{45 : 45 : 45 : 45} = 23.011$.								
44	0	0	4	23.066	23.063	-.003	23.056	-.010
42	0	0	12	23.430	23.476	.046	23.390	-.040
41	0	8	8	23.291	23.286	-.005	23.254	-.037
39	0	8	16	23.704	23.769	.065	23.652	-.052
37	0	16	16	24.037	24.113	.076	23.985	-.052
36	0	16	20	24.286	24.441	.155	24.264	-.022
35	0	20	20	24.513	24.734	.221	24.534	.021
42	4	4	4	23.065	23.063	-.002	23.058	-.007
40	4	4	12	23.328	23.338	.010	23.289	-.039
38	4	4	20	23.856	24.028	.172	23.820	-.036
38	4	12	12	23.484	23.476	-.008	23.428	-.056
36	4	12	20	23.930	24.028	.098	23.885	-.045
34	4	20	20	24.307	24.441	.134	24.286	-.021
39	8	8	8	23.237	23.218	-.019	23.204	-.033
37	8	8	16	23.546	23.563	.017	23.499	-.047
35	8	16	16	23.769	23.769	.000	23.710	-.059
33	8	20	20	24.170	24.251	.081	24.145	-.025
36	12	12	12	23.504	23.476	-.028	23.454	-.050
34	12	12	20	23.860	23.890	.030	23.818	-.042
33	12	16	20	23.970	23.976	.006	23.922	-.048
33	16	16	16	23.872	23.838	-.034	23.819	-.053
32	16	16	20	24.042	24.028	-.014	24.000	-.042
31	16	20	20	24.185	24.183	-.002	24.162	-.023
30	20	20	20	24.303	24.303	.000	24.303	.000

*Last Survivor of Four Lives. $x : x + m : x + n : x + p$.—Continued.***O^{M5}** $2\frac{1}{2}$ per-cent.

Young- est Age	Mean Ages 65.			$a_{65 : 65 : 65 : 65} = 14.263$				
	<i>m</i>	<i>n</i>	<i>p</i>	True Value	2-term Approximation	Error	3-term Approximation	Final Error
64	0	0	4	14.344	14.347	.003	14.331	-.013
62	0	0	12	14.876	15.022	.146	14.827	-.049
61	0	8	8	14.686	14.712	.026	14.639	-.047
59	0	8	16	15.299	15.500	.201	15.236	-.063
57	0	16	16	15.823	16.061	.238	15.771	-.052
56	0	16	20	16.196	16.597	.401	16.198	.002
55	0	20	20	16.548	17.074	.526	16.622	.074
62	4	4	4	14.358	14.347	-.011	14.336	-.022
60	4	4	12	14.738	14.796	.058	14.685	-.053
58	4	4	20	15.490	15.922	.432	15.452	-.038
58	4	12	12	14.985	15.022	.039	14.914	-.071
56	4	12	20	15.652	15.922	.270	15.599	-.053
54	4	20	20	16.246	16.597	.351	16.246	.000
59	8	8	8	14.612	14.600	-.012	14.567	-.045
57	8	8	16	15.084	15.163	.079	15.019	-.065
55	8	16	16	15.448	15.500	.052	15.366	-.082
53	8	20	20	16.072	16.287	.215	16.048	-.024
56	12	12	12	15.053	15.022	-.031	14.973	-.080
54	12	12	20	15.600	15.696	.096	15.533	-.067
53	12	16	20	15.781	15.838	.057	15.715	-.066
53	16	16	16	15.654	15.612	-.042	15.569	-.085
52	16	16	20	15.922	15.922	.000	15.859	-.063
51	16	20	20	16.162	16.175	.013	16.128	-.034
50	20	20	20	16.371	16.371	.000	16.371	.000

Mean Ages 85. $a_{85 : 85 : 85 : 85} = 5.468$

Young- est Age	Mean Ages 85.			$a_{85 : 85 : 85 : 85} = 5.468$				
	<i>m</i>	<i>n</i>	<i>p</i>	True Value	2-term Approximation	Error	3-term Approximation	Final Error
84	0	0	4	5.537	5.555	.018	5.530	-.007
82	0	0	12	6.004	6.248	.244	5.953	-.051
81	0	8	8	5.863	5.930	.067	5.820	-.043
79	0	8	16	6.409	6.741	.332	6.342	-.067
77	0	16	16	6.916	7.317	.401	6.879	-.037
76	0	16	20	7.267	7.868	.601	7.264	-.003
75	0	20	20	7.619	8.359	.740	7.675	.056
82	4	4	4	5.546	5.555	.009	5.539	-.007
80	4	4	12	5.890	6.017	.127	5.850	-.040
78	4	4	20	6.552	7.174	.622	6.464	-.088
78	4	12	12	6.143	6.248	.105	6.084	-.059
76	4	12	20	6.741	7.174	.433	6.685	-.056
74	4	20	20	7.331	7.868	.537	7.338	.007
79	8	8	8	5.800	5.815	.015	5.766	-.034
77	8	8	16	6.237	6.394	.157	6.176	-.061
75	8	16	16	6.602	6.741	.139	6.539	-.063
73	8	20	20	7.208	7.549	.341	7.188	-.020
76	12	12	12	6.238	6.248	.010	6.174	-.064
74	12	12	20	6.756	6.942	.186	6.696	-.060
73	12	16	20	6.963	7.087	.124	6.901	-.062
73	16	16	16	6.861	6.856	-.005	6.790	-.071
72	16	16	20	7.131	7.174	.043	7.079	-.052
71	16	20	20	7.394	7.434	.040	7.363	-.031
70	20	20	20	7.636	7.636	.000	7.636	.000

27. It is not, perhaps, surprising to find that 4-life functions involve a heavier strain than the method—which after all is only an approximate one—can bear with complete success. It must certainly be admitted that, judged by the standard which has been reached in the case of 2-life and 3-life functions, the results in respect of four lives are unsatisfactory. Probably, however, they compare not unfavourably with the rough methods which would very often be adopted in practice; and bearing in mind that 1,771 age-combinations can be dealt with by means of the direct calculation of only three, the method may thus be of some utility in its application to 4-life annuities, when a rapid but not very close result is required.

PART III.

Further developments and special applications.

28. It was pointed out in paragraph 2 that the method just described is independent of any special analytical law governing the mortality table; and in particular it is independent of Makeham's law. When, however, the mortality table is intimately related to a Makeham Table and approximates to it more and more closely as the middle and older ages are reached—as is the case with the O^M Table—the ordinary law of Uniform Seniority will give an approximate value of a joint-life annuity, though useless for a last survivor annuity. (For examples, see Austin and Symmons Joint-life Tables, page iv). In fact it will be found that at the older ages the use of the uniform seniority based on the $O^{M(5)}$ Table will give approximate O^M joint-life annuity-values with a slightly smaller error than is shewn by the method under discussion, in the examples which have been considered. It is not clear that the approximation would be so close in general, *i.e.*, when the mortality tables were less closely and organically connected, but it is often the case that a table either deviates in a systematic way from a Makeham curve, approximating to it more closely in the middle and ends of the Table—as in the case of the O^M just referred to and the $O^{[a]}$ —or that without closely following a Makeham curve it is similar in general character. It will be interesting to consider whether this fact may be

used, in association with the method described in this paper, to obtain a closer approximation or an increased range of application. In dealing with this question we shall make use of a principle which may often be usefully applied to matters of interpolation. If we require the value of a function F which is not readily calculated and is therefore to be found by interpolation; and if f is another function which is readily calculated and moreover follows the same general curve as F so that it is an approximation to F ; then it is often useful to express F as $f + (F - f)$, where f is calculated directly and $(F - f)$ is made the subject of interpolation. If the functions are properly chosen, $(F - f)$ will be small and it may change much less rapidly than F , and therefore be successfully interpolated over a much wider interval.

29. In the present case, let $F(x+a, x+b, \dots)$ represent the required annuity-value on $(x+a)$, $(x+b) \dots$ and let $f(x+a, x+b, \dots)$ represent the annuity-value on lives of equal age $(x+w)$, $(x+w), \dots$ found, on the same basis of interest and mortality but on the *assumption* of uniform seniority, by the formula $nc^w = c^a + c^b + \dots$ (where c may have any suitable value). This annuity-value is a symmetrical function of a , b and c , and by the fundamental formula, writing F for $F(x, x, \dots)$, f for $f(x, x, x)$ we have

$$F(x+a, x+b \dots) = F + \Sigma a^2.B + \Sigma a^3.C + \Sigma a^4.D \dots$$

$$f(x+a, x+b \dots) = f + \Sigma a^2.B' + \Sigma a^3.C' + \Sigma a^4.D' \dots$$

$$\text{Difference} = (F - f) + \Sigma a^2.B'' + \Sigma a^3.C'' + \Sigma a^4.D'' \dots$$

writing B'' for $(B - B')$ and so on. Now $F = f$ or $F - f$ vanishes, for it represents the error due to the assumption of uniform seniority, and this error is zero when the lives are all of the same age. Thus we have

$$\begin{aligned} F(x+a, x+b \dots) = f(x+a, x+b \dots) \\ + \Sigma a^2.B'' + \Sigma a^3.C'' + \Sigma a^4.D'' \dots \end{aligned}$$

This is in the same form as previously found, namely,

$$\alpha + C_1 \times \text{1st correction} + C_2 \times \text{2nd correction} + \&c.,$$

the distinction being

- (i) that the equal ages for determining α are found by Uniform Seniority, instead of being the simple average of the actual ages, so that an interpolation will generally be required.
- (ii) that the 1st correction and 2nd correction are found from selected values of $(F-f)$ instead of F , so that the construction of the Tables will be slightly more laborious.
- (iii) that the corrections will usually be much reduced in magnitude, so that either a greater range or greater accuracy may be secured—especially if the curve of mortality does not widely deviate from a Makeham curve.

30. It has been pointed out that any suitable value of c may be used. It will usually be convenient to use that value, $c = \log_{10}^{-1} \cdot 039$, for which extensive tables of uniform seniority have been calculated, though in the case of 2-life annuities it might sometimes be convenient to choose such a value of c and of n (see par. 11-13) as would produce integral equal ages (on the assumption of uniform seniority) for the annuity-values α , β and γ .

31. A rather severe test of this further development has been made by applying it to the $O^{[af]}$ Table (interest $2\frac{1}{2}$ per cent). This table is a blend of two separate Makeham curves and departs very considerably from the typical form of a simple Makeham curve, to which the property of uniform seniority would apply. The case is thus quite an unfavourable one for the application of the extended method, yet it will be found that the results are not unsatisfactory, the errors in 2-life annuities being greatly reduced at the points where they are normally most considerable.

32. As in the case of the 2-life examples previously given (see pars. 13 *et seq.*) it was found convenient to base the work on the values for equal ages (α), 20 years' disparity (β), and 30 years' disparity (γ), and accordingly the coefficients can be taken from Table A. The examples taken were: mean ages, $x : x = 35 : 35$, $50 : 50$ and $65 : 65$, and actual ages $x \pm 7$ and $x \pm 13$, giving disparities of 14 and 26 years, which may be expected to produce relatively high errors. Owing to the 2-life annuities not being completely tabulated on the $O^{[af]}$ basis, the "true" values of the

annuities were found by interpolation by central second differences, in the manner illustrated in the Annuity Tables, p. 229, § 3-4-7. The $O^{[M]}$ or $O^{[M(5)]}$ uniform seniority Tables, based on $\log_{10} e = .039$ were used. The first example is worked out in some detail to enable the method to be easily followed.

Original method : mean ages 35 : 35.

$$\alpha = a_{[35][35]} = 17.056$$

$$\beta - \alpha = - .921 \quad (\beta - \alpha) - .8(\gamma - \beta)$$

$$\beta = a_{[25][45]} = 16.135 \quad = - .921 + .822$$

$$\gamma - \beta = -1.027 \quad = - .099$$

$$\gamma = a_{[20][50]} = 15.108$$

$$a_{[25][42]} = 17.056 - .921 \times .49 - .099 \times .111$$

$$= 17.056 - .451 - .011 = 16.594$$

$$\left. \begin{array}{l} \text{"True" value, by 2nd central} \\ \text{difference interpolation} = 16.587 \end{array} \right\} \text{error } .007.$$

Further development.

The values headed "U.S." are found on the assumption that the $O^{[M(5)]}$ or $O^{[M]}$ Table of uniform seniority holds for the $O^{[eff]}$ Table. Thus against $a_{[25][45]}$ in that column is the value of $a_{[28.99][38.99]}$, and against $a_{[20][50]}$ is the value of $a_{[43.91][43.91]}$,

True. U.S. Δ

$$\alpha = 17.056 \quad 17.056 \quad .000$$

$$\beta - \alpha = .195 \quad (\beta - \alpha) - .8(\gamma - \beta)$$

$$\beta = 16.135 \quad 15.940 \quad .195 \quad = 2\text{nd correction}$$

$$\gamma - \beta = .108 \quad = .195 - .086$$

$$\gamma = 15.108 \quad 14.805 \quad .303 \quad = .109$$

The interpolated value of Δ for $[28][42]$ is therefore

$$.195 \times .49 + .109 \times .111 = .108$$

The U.S. value is $a_{[37.07][37.07]} = 16.479$, and adding the interpolated value of Δ , viz., .108, the final result is 16.587, which is without error.

33. In this way the following Table was calculated, and as it showed a considerable reduction in the errors, some further examples were calculated for a larger range, viz., taking 26 and 39 years (thus preserving the value $n = \frac{3}{2}$, see par. 13) for the standard ranges, and 18 and 34 years' actual disparity. These results in the same way showed a considerable reduction of error. This is secured at the expense of some slight increase in labour, which, however, in practice would be less than appears from the examples, because the 1st correction and 2nd correction based on Δ would be tabulated in the form of Table B.

Standard Range of Ages 20-30 years.

$O^{[a]}$ $2\frac{1}{2}$ per-cent.

Mean Ages	Actual Ages	True Value	Approx. by Original Method	Error	Approx. by further development	Error
35 : 35	28 : 42	16.587	16.594	.007	16.587	.000
35 : 35	22 : 48	15.556	15.551	-.005	15.559	.003
50 : 50	43 : 57	12.100	12.106	.006	12.099	-.001
50 : 50	37 : 63	10.591	10.585	-.006	10.591	.000
65 : 65	58 : 72	6.901	6.933	.032	6.923	.022
65 : 65	52 : 78	5.389	5.354	-.035	5.367	-.022

Standard Range of Ages 26-39 years.

Mean Ages	Actual Ages	True Value	Approx. by Original Method	Error	Approx. by further development	Error
50 : 50	41 : 59	11.673	11.688	.015	11.669	-.004
50 : 50	33 : 67	9.300	9.290	-.010	9.303	.003
65 : 65	56 : 74	6.422	6.515	.093	6.479	.057
65 : 65	48 : 82	4.377	4.294	-.083	4.335	-.042

34. It seems probable that the further development would be even more successful as applied to a Table which more closely approximated to a Makeham Curve. To test this point, the following examples were calculated on the basis of the O^M Table with interest at $2\frac{1}{2}$ per-cent. It will be seen that the results are very satisfactory.

*Standard Range of Ages 32-48 years.***O^M 2½ per-cent.**

Mean Ages	Actual Ages	True Value	Approx. by Original Method	Error	Approx. by further development	Error
45 : 45	33 : 57	11·023	11·080	·057	11·023	·000
45 : 45	25 : 65	8·476	8·416	—·060	8·477	·001
60 : 60	48 : 72	5·748	5·830	·082	5·749	·001
60 : 60	40 : 80	3·760	3·674	—·086	3·761	·001
75 : 75	63 : 87	2·084	2·161	·077	2·085	·001
75 : 75	55 : 95	1·086	1·003	—·083	1·078	—·008

35. It would seem reasonable to expect that the further development would be equally successful as applied to 3-life and 4-life annuities, but the writer has not been able to undertake the labour of testing this, as it would be very considerable.

Joint-Life Endowment Assurances.

36. The method supplies a ready means of calculating the values of a , A and P , for joint-life endowment assurances, and since these usually run for terms of moderate length expiring before any of the lives reach the age of (say) 70, the third term in the approximation may usually be neglected. The following examples will illustrate this point. It may be added that Office premiums may be tabulated in the same way, provided that a uniform system of loading is adopted. In the following examples the standard disparity of age is 20 years, and the examples are calculated for a disparity of 14 years, *i.e.*, $\cdot 7$ of the standard, this being approximately the proportion that gives the maximum value to the coefficient of the neglected third term. The results are sufficient for all practical purposes.

*Two-life Endowment Assurances, O^M 2½ per-cent.**Standard interval 20 years. Third term neglected.*

$$a_{x-7 : x+7 : \bar{n}} = a_{x : x : \bar{n}} + \cdot 49(a_{x-10 : x+10 : \bar{n}} - a_{x : x : \bar{n}})$$

n	$x : x$	$x \pm 10$	$x \pm 7$	$a_{x : x : \bar{n}}$	$a_{x-10 : x+10 : \bar{n}}$	$a_{x-7 : x+7 : \bar{n}}$		
						Approx.	True.	Error
10	50 : 50	40 : 60	43 : 57	7·254	6·876	7·069	7·071	—·002
20	40 : 40	30 : 50	33 : 47	12·342	11·752	12·053	12·057	—·004
30	30 : 30	20 : 40	23 : 37	16·482	15·889	16·191	16·199	—·008
40	20 : 20	10 : 30	13 : 27	20·047	19·419	19·739	19·740	—·001

Valuation of Joint-Life Assurances.

37. It would appear that the method might be useful in connexion with the valuation of joint-life assurances in groups when the method of uniform seniority does not apply either exactly or with sufficient practical accuracy. Let the policies be classified according to average ages (or according to the sum of the individual ages.) Then in any group the annuity-value and assurance value for any individual policy will be of the form

$$\alpha + C_1\beta + C_2\gamma$$

where α , β and γ depend only on the average age, and are therefore constant for the group; while C_1 and C_2 are the coefficients which depend on the actual disparity of ages, and therefore may differ for each policy though they will remain unchanged during the whole duration of the policy. If then C_1 and C_2 multiplied by the sum assured say SC_1 and SC_2 be recorded on the valuation cards at the outset, and entered in the valuation schedules the weighted mean assurance or annuity-value for the group will be

$$\alpha + \left(\frac{\sum SC_1}{\sum S}\right)\beta + \left(\frac{\sum SC_2}{\sum S}\right)\gamma$$

38. It is thought that this method—which secures the advantages of a broad grouping by a simple process of calculation—should produce very satisfactory results; and bearing in mind that the tendency is for the great bulk of joint-life policies to be effected on lives differing in age by only a few years, it is probable that sufficiently close results would be secured by neglecting the third term in the approximation. There would then be only one constant to be recorded and the process would be of the greatest simplicity.

It might not improbably be found that the ratio $\frac{\sum SC_1}{\sum S}$, *i.e.*, the weighted mean value of C_1 , would tend to a fairly constant value, corresponding to a constant disparity of age, say $2t$ years. In that case, the group of policies on lives of average age x could be valued by the function for $x-t : x+t$. This is a method which is actually adopted by one office, having been introduced—it is believed—by Mr. Ralph

P. Hardy; and no doubt the underlying principle is as stated above. It would be of great interest if some actuary having access to a large body of joint life assurances would investigate the method here suggested, and communicate the results during the discussion on the paper. A point to be particularly borne in mind would be this—would the mean value of C_1 (and of C_2 if employed in the calculation), determined by “weighting” with the sums assured, apply with sufficient accuracy to the premiums, and to the bonus additions in a with-profit class?

CONCLUDING REMARKS.

39. The labour and time expended in preparing the tables of coefficients and the illustrative tables and examples given in this paper having been very considerable, it has not been practicable to furnish a complete set of tables for practical use or to make more extended comparisons. It is hoped however that if the method receive the approval of the profession, there will not be lacking those who will be prepared to take their share of the work by pursuing the investigation and calculating such extended tables for publication in the *Journal* or elsewhere; and it need hardly be said that the writer will be glad to give any assistance or further explanations that may be found necessary in the course of the work.

40. In conclusion, the writer desires to record his great indebtedness to Mr. A. E. King, A.I.A., who has kindly undertaken the responsibility of calculating the Tables and Examples and has given most valuable help in preparing the paper for press and in dealing with many points that arose in the course of the work. It is upon his shoulders that the bulk of the work has fallen, and without his willing aid the writer could not have undertaken the task.

APPENDIX.

41. Let $F(w, x, y, z \dots)$ be a function of any number of variables $w, x, y, z \dots$; and let the following easily-printed forms be used for the partial differential coefficients, namely,

$$\frac{dF}{dx} = D(x), \quad \frac{dF}{dw} = D(w), \quad \frac{d^2F}{dx^2} = D(x^2), \quad \frac{d^3F}{dx dy dz} = D(x, y, z) \text{ and}$$

so on. Then it is shown in treatises on the Differential Calculus that $F(w+a, x+b, y+c \dots)$ may be expanded in the following form :

$$\left. \begin{aligned} F(w+a, x+b, y+c, z+d \dots) = \\ F(w, x, y \dots) + \Sigma aD(w) + \frac{1}{2}\Sigma a^2D(w^2) + \Sigma abD(w, x) \\ + \frac{1}{6}\Sigma a^3D(w^3) + \frac{1}{2}\Sigma a^2bD(w^2, y) + \Sigma abcD(w, y, z) \\ + \frac{1}{24}\Sigma a^4D(w^4) + \frac{1}{6}\Sigma a^3bD(w^3, y) + \frac{1}{4}\Sigma a^2b^2D(w^2, y^2) \\ + \frac{1}{2}\Sigma a^2bcD(w^2, y, z) + \frac{1}{24}\Sigma abcdD(w, x, y, z) \\ + \text{terms involving diff. cos. of higher orders.} \end{aligned} \right\} (1)$$

Here the symbol Σ represents "the sum of all terms similar to"; so that for example if there are four variables $w+a, x+b, y+c, z+d$,

$$\begin{aligned} \Sigma abD(w, x) = abD(w, x) + acD(w, y) + adD(w, z) \\ + bcD(x, y) + bdD(x, z) + cdD(y, z) \end{aligned}$$

42. Now if F is a *symmetrical* function of the variables (that is if they are similarly involved) and if, further, $w=x=y=z \dots$, all the differential coefficients of the same order and type will be identically equal. For example, in such circumstances $D(w)=D(x)=D(y) \dots$: and $D(w^2)=D(x^2)=D(y^2) \dots$: and $D(w^2yz)=D(x^2xy)$ and so on. Hence using A, B, C, \dots : as abbreviations of the differential coefficients multiplied by their numerical coefficients, equation (1) becomes

$$\left. \begin{aligned} F(x+a, x+b, x+c \dots) = F(x, x, x \dots) + A\Sigma a + B\Sigma a^2 + C\Sigma ab \\ + D\Sigma a^3 + E\Sigma a^2b + F\Sigma abc + G\Sigma a^4 \\ + H\Sigma a^3b + G\Sigma a^2b^2 + H\Sigma a^2bc + J\Sigma abcd \\ + \dots \end{aligned} \right\} (2)$$

which holds for all values of $a, b, c \dots$ provided F is a symmetrical function of the variables.

43. It will now be shown that considerable further simplification arises if $\Sigma a=0$, that is if $a+b+c+\dots=0$, that is, if x is the mean value of the variables $(x+a)$, $(x+b)$, $x+c\dots$. In that case—

(i) The term involving A vanishes.

$$\begin{aligned} \text{(ii) } 2\Sigma ab &= a(b+c+\dots) + b(a+c+\dots) + c(a+b+d\dots) \\ &= a(-a) + b(-b) + \dots \\ &= -\Sigma a^2 \end{aligned}$$

Hence the terms involving B and C coalesce into one term, say $B'\Sigma a^2$

$$\begin{aligned} \text{(iii) } \Sigma a^2 b &= a^2(b+c+\dots) + b^2(a+c+\dots) + \\ &= a^2(-a) + b^2(-b) + \dots = -\Sigma a^3 \end{aligned}$$

$$\begin{aligned} \text{(iv) } 3\Sigma abc &= ab(-\overline{a+b}) + ac(-\overline{a+c}) + \dots \\ &= -[a^2b + ab^2 + a^2c + ac^2 + \dots] \\ &= -k\Sigma a^2b = k\Sigma a \end{aligned}$$

where k is a numerical coefficient the value of which is immaterial to the present enquiry.

Hence the terms involving D, E and F coalesce into one term, say $C'\Sigma a^3$.

44. The terms of the 4th, 5th and higher orders may be similarly dealt with, and we shall have finally

$$\begin{aligned} F(x+a, x+b, x+c\dots) &= F(x, x, x\dots) + B'\Sigma a^2 + C'\Sigma a^3 \\ &\quad + D'\Sigma a^4 + E'(\Sigma a^2)^2 + F'\Sigma a^5 + G'(\Sigma a^3)(\Sigma a^2) + \dots \end{aligned} \quad (3)$$

which, it may be repeated, holds only if $a+b+c+\dots=0$ and F is a symmetrical function of the variables.

TABLE A.
Table of coefficients.—Two Lives. $x : x + m$.
Standard range of ages, 20 and 30 years.

m	Coefficient of 1st correction k^2	Coefficient of 2nd correction $(k^2 - k^4) \div n^2$	m	Coefficient of 1st correction k^2	Coefficient of 2nd correction $(k^2 - k^4) \div n^2$
1	·002	·001	18	·810	·068
2	·010	·004	19	·902	·039
3	·022	·010	20	1·000	·000
4	·040	·017	21	1·102	— ·050
5	·062	·026	22	1·210	— ·113
6	·090	·036	23	1·322	— ·190
7	·122	·048	24	1·440	— ·282
8	·160	·060	25	1·562	— ·391
9	·202	·072	26	1·690	— ·518
10	·250	·083	27	1·822	— ·666
11	·302	·094	28	1·960	— ·836
12	·360	·102	29	2·102	— 1·030
13	·422	·108	30	2·250	— 1·250
14	·490	·111	31*	2·402	— 1·498
15	·562	·109	32*	2·560	— 1·775
16	·640	·102	33*	2·722	— 2·084
17	·722	·089	34*	2·890	— 2·428

* These values fall outside the extreme range of 30 years on which the tables are based and must therefore be used with caution. (See par. 16).

TABLE B.
Two-Joint Lives.

$0^M 2\frac{1}{2}$ per-cent.

$0^M 2\frac{1}{2}$ per-cent.

x	$a_{x:x}$	1st correction	2nd correction	x	$a_{x:x}$	1st correction	2nd correction
20	21·213	— 1·014	— 1·00	55	9·339	— 1·428	— ·338
25	19·709	— 1·055	— 1·09	60	7·559	— 1·333	— ·348
30	18·125	— 1·122	— 1·14	65	5·902	— 1·189	— ·341
35	16·481	— 1·227	— 1·53	70	4·428	— 1·013	— ·318
40	14·775	— 1·345	— 2·13	75	3·179	— ·823	— ·281
45	13·001	— 1·432	— 2·69	80	2·171	— ·636	— ·239
50	11·174	— 1·461	— 3·09	85	1·395	— ·460	— ·121

TABLE C.
Last Survivor of Two Lives.

$0^M 2\frac{1}{2}$ per-cent.

$0^M 2\frac{1}{2}$ per-cent.

x	$a_{x:\overline{x}}$	1st correction	2nd correction	x	$a_{x:\overline{x}}$	1st correction	2nd correction
20	28·589	+ ·528	·120	24	27·537	·601	·129
$\frac{1}{2}$	28·462	·537	·121	$\frac{1}{2}$	27·398	·611	·128
21	28·334	·546	·122	25	27·259	·621	·127
$\frac{1}{2}$	28·204	·555	·123	$\frac{1}{2}$	27·119	·630	·130
22	28·073	·564	·124	26	26·979	·640	·132
$\frac{1}{2}$	27·940	·574	·125	$\frac{1}{2}$	26·836	·647	·130
23	27·808	·583	·127	27	26·694	·654	·128
$\frac{1}{2}$	27·672	·592	·128	$\frac{1}{2}$	26·547	·666	·132

TABLE C—continued.
Last Survivor of Two Lives. $0^M 2\frac{1}{2}$ per-cent. $0^M 2\frac{1}{2}$ per-cent.

x	$a_{x:\overline{x}}$	1st correction	2nd correction	x	$a_{x:\overline{x}}$	1st correction	2nd correction
28	26400	677	137	57	15449	1450	318
$\frac{1}{2}$	26232	686	137	$\frac{1}{2}$	15224	1466	322
29	26104	694	137	58	14998	1481	325
$\frac{1}{2}$	25952	704	138	$\frac{1}{2}$	14772	1496	328
30	25799	714	140	59	14545	1512	332
$\frac{1}{2}$	25646	723	141	$\frac{1}{2}$	14318	1526	334
31	25492	732	142	60	14091	1541	337
$\frac{1}{2}$	25334	742	144	$\frac{1}{2}$	13863	1555	340
32	25176	752	145	61	13635	1569	344
$\frac{1}{2}$	25016	761	145	$\frac{1}{2}$	13407	1583	348
33	24857	770	145	62	13179	1597	351
$\frac{1}{2}$	24694	780	146	$\frac{1}{2}$	12952	1609	353
34	24532	789	147	63	12724	1621	355
$\frac{1}{2}$	24366	800	150	$\frac{1}{2}$	12497	1632	356
35	24199	811	152	64	12270	1643	357
$\frac{1}{2}$	24031	820	152	$\frac{1}{2}$	12043	1654	359
36	23863	830	153	65	11816	1665	363
$\frac{1}{2}$	23691	842	156	$\frac{1}{2}$	11590	1674	364
37	23519	854	160	66	11365	1683	364
$\frac{1}{2}$	23346	864	160	$\frac{1}{2}$	11142	1690	364
38	23172	873	161	67	10919	1697	363
$\frac{1}{2}$	22994	886	165	$\frac{1}{2}$	10696	1704	364
39	22816	898	169	68	10474	1711	365
$\frac{1}{2}$	22636	910	171	$\frac{1}{2}$	10254	1716	364
40	22455	921	173	69	10034	1721	364
$\frac{1}{2}$	22272	932	175	$\frac{1}{2}$	9817	1724	363
41	22089	944	177	70	9600	1728	362
$\frac{1}{2}$	21902	958	181	$\frac{1}{2}$	9385	1730	362
42	21715	971	185	71	9170	1733	361
$\frac{1}{2}$	21524	985	189	$\frac{1}{2}$	8959	1733	358
43	21334	999	193	72	8748	1733	356
$\frac{1}{2}$	21142	1012	196	$\frac{1}{2}$	8540	1732	355
44	20949	1026	199	73	8332	1732	354
$\frac{1}{2}$	20754	1040	203	$\frac{1}{2}$	8128	1730	352
45	20559	1054	207	74	7924	1727	349
$\frac{1}{2}$	20359	1070	213	$\frac{1}{2}$	7724	1722	345
46	20159	1087	219	75	7525	1717	341
$\frac{1}{2}$	19958	1100	222	$\frac{1}{2}$	7328	1712	339
47	19758	1114	224	76	7132	1707	337
$\frac{1}{2}$	19552	1132	230	$\frac{1}{2}$	6942	1700	334
48	19347	1149	236	77	6751	1692	330
$\frac{1}{2}$	19140	1164	240	$\frac{1}{2}$	6565	1682	324
49	18934	1180	244	78	6379	1673	319
$\frac{1}{2}$	18724	1196	249	$\frac{1}{2}$	6198	1663	316
50	18514	1213	254	79	6017	1653	312
$\frac{1}{2}$	18301	1230	260	$\frac{1}{2}$	5841	1642	308
51	18088	1248	265	80	5665	1630	304
$\frac{1}{2}$	17874	1264	268	$\frac{1}{2}$	5496	1616	298
52	17660	1280	272	81	5327	1601	292
$\frac{1}{2}$	17442	1298	277	$\frac{1}{2}$	5162	1586	287
53	17225	1315	282	82	4998	1572	282
$\frac{1}{2}$	17006	1332	288	$\frac{1}{2}$	4840	1556	277
54	16786	1350	293	83	4681	1539	272
$\frac{1}{2}$	16566	1366	297	$\frac{1}{2}$	4528	1522	268
55	16345	1383	301	84	4375	1504	264
$\frac{1}{2}$	16122	1399	304	$\frac{1}{2}$	4229	1486	262
56	15899	1415	307	85	4083	1467	261
$\frac{1}{2}$	15674	1432	312				

TABLE D.

Table of Coefficients.—Three Lives. $x : x + m : x + n$.

Standard range of ages, 18 years.

m	n	Average age = x +	Coefficient of 1st correction	Coefficient of 2nd correction	m	n	Average age = x +	Coefficient of 1st correction	Coefficient of 2nd correction
0	1	$\frac{1}{2}$.003	.002	2	2	$1\frac{1}{2}$.012	.005
	2	$\frac{1}{2}$.012	.007		3	$\frac{1}{2}$.022	.010
	3	1	.028	.016		4	2	.037	.018
	4	$\frac{1}{2}$.049	.030		5	$\frac{1}{2}$.059	.032
	5	$\frac{1}{2}$.077	.049		6	$\frac{1}{2}$.086	.050
	6	2	.111	.074		7	3	.120	.074
	7	$\frac{1}{2}$.151	.105		8	$\frac{1}{2}$.160	.104
	8	$\frac{1}{2}$.198	.143		9	$\frac{1}{2}$.207	.141
	9	3	.250	.187		10	4	.259	.185
	10	$\frac{1}{2}$.309	.240		11	$\frac{1}{2}$.318	.237
	11	$\frac{1}{2}$.373	.301		12	$\frac{1}{2}$.383	.297
	12	4	.444	.370		13	5	.454	.366
	13	$\frac{1}{2}$.522	.449		14	$\frac{1}{2}$.531	.444
	14	$\frac{1}{2}$.605	.538		15	$\frac{1}{2}$.614	.532
	15	5	.694	.636		16	6	.704	.630
	16	$\frac{1}{2}$.790	.746		17	$\frac{1}{2}$.799	.738
	17	$\frac{1}{2}$.892	.867		18	$\frac{1}{2}$.901	.859
	18	6	1.000	1.000		19	7	1.009	.991
	19	$\frac{1}{2}$	1.114	1.145		20	$\frac{1}{2}$	1.123	1.135
	20	$\frac{1}{2}$	1.235	1.303					
1	1	$\frac{1}{2}$.003	.001	3	3	2	.028	.012
	2	1	.009	.005		4	$\frac{1}{2}$.040	.017
	3	$\frac{1}{2}$.022	.012		5	$\frac{1}{2}$.059	.027
	4	$\frac{1}{2}$.040	.023		6	3	.083	.042
	5	2	.065	.039		7	$\frac{1}{2}$.114	.062
	6	$\frac{1}{2}$.096	.061		8	$\frac{1}{2}$.151	.088
	7	$\frac{1}{2}$.133	.089		9	4	.194	.120
	8	3	.176	.123		10	$\frac{1}{2}$.244	.160
	9	$\frac{1}{2}$.225	.164		11	$\frac{1}{2}$.299	.207
	10	$\frac{1}{2}$.281	.212		12	5	.361	.262
	11	4	.343	.269		13	$\frac{1}{2}$.429	.325
	12	$\frac{1}{2}$.410	.333		14	$\frac{1}{2}$.503	.398
	13	$\frac{1}{2}$.485	.408		15	6	.583	.479
	14	5	.565	.491		16	$\frac{1}{2}$.670	.571
	15	$\frac{1}{2}$.651	.584		17	$\frac{1}{2}$.762	.673
	16	$\frac{1}{2}$.744	.688		18	7	.861	.786
	17	6	.843	.803		19	$\frac{1}{2}$.966	.912
	18	$\frac{1}{2}$.948	.930		20	$\frac{1}{2}$	1.077	1.050
	19	$\frac{1}{2}$	1.059	1.069	4	4	2	.049	.019
	20	7	1.176	1.220		5	3	.065	.026

TABLE D.—*Continued.**Table of Coefficients.—Three Lives. $x : x + m : x + n$.**Standard range of ages, 18 years.*

<i>m</i>	<i>n</i>	Average age = $x +$	Coefficient of 1st correction	Coefficient of 2nd correction	<i>m</i>	<i>n</i>	Average age = $x +$	Coefficient of 1st correction	Coefficient of 2nd correction
4	6	31	.086	.036	6	14	62	.457	.266
	7	34	.114	.052		15	7	.528	.329
	8	4	.148	.074		16	1	.605	.401
	9	1	.188	.102		17	1	.688	.482
	10	1	.235	.137		18	8	.778	.574
	11	5	.287	.178		19	1	.873	.677
	12	1	.346	.228		20	1	.975	.790
	13	1	.410	.285					
	14	6	.481	.352	7	7	42	.151	.046
	15	1	.559	.428		8	5	.176	.053
	16	1	.642	.513		9	1	.207	.066
	17	7	.731	.608		10	1	.244	.084
	18	1	.827	.715		11	6	.287	.109
	19	1	.929	.834		12	1	.336	.140
	20	8	1.037	.963		13	1	.392	.180
						14	7	.454	.227
5	5	31	.077	.028		15	1	.522	.283
	6	34	.096	.035		16	1	.596	.347
	7	4	.120	.046		17	8	.676	.422
	8	1	.151	.063		18	1	.762	.506
	9	1	.188	.086		19	1	.855	.600
	10	5	.231	.116		20	9	.954	.706
	11	1	.281	.152					
	12	1	.336	.196	8	8	51	.198	.055
	13	6	.398	.248		9	1	.225	.061
	14	1	.466	.308		10	6	.259	.074
	15	1	.540	.377		11	1	.299	.092
	16	7	.620	.456		12	1	.346	.118
	17	1	.707	.545		13	7	.398	.150
	18	1	.799	.644		14	1	.457	.191
	19	8	.898	.755		15	1	.522	.239
	20	1	1.003	.877		16	8	.593	.297
						17	1	.670	.363
6	6	4	.111	.037		18	1	.753	.439
	7	1	.133	.044		19	9	.843	.526
	8	1	.160	.056		20	1	.938	.623
	9	5	.194	.074					
	10	1	.235	.098	9	9	6	.250	.062
	11	1	.281	.129		10	1	.281	.069
	12	6	.333	.167		11	1	.318	.081
	13	1	.392	.212		12	7	.361	.099

TABLE D.—Continued.

Table of Coefficients.—Three Lives. $x : x + m : x + n$.

Standard range of ages, 18 years.

m	n	Average age $= x +$	Coefficient of 1st correction	Coefficient of 2nd correction	m	n	Average age $= x +$	Coefficient of 1st correction	Coefficient of 2nd correction
9	13	7 $\frac{1}{3}$	·410	·125	13	13	8 $\frac{1}{3}$	·522	·073
	14	8	·466	·158		14	9	·565	·074
	15	8 $\frac{1}{3}$	·528	·199		15	9 $\frac{1}{3}$	·614	·082
	16	9	·596	·249		16	10	·670	·099
	17	9 $\frac{1}{3}$	·670	·307		17	10 $\frac{1}{3}$	·731	·123
	18	10	·750	·375		18	11	·799	·155
	19	10 $\frac{1}{3}$	·836	·453		19	11 $\frac{1}{3}$	·873	·196
	20	11	·929	·542		20	12	·954	·248
10	10	6 $\frac{2}{3}$	·309	·069	14	14	9 $\frac{1}{3}$	·605	·067
	11	7	·343	·074		15	10	·651	·067
	12	7 $\frac{1}{3}$	·383	·086		16	10 $\frac{1}{3}$	·704	·074
	13	8	·429	·104		17	11	·762	·089
	14	8 $\frac{1}{3}$	·481	·129		18	11 $\frac{1}{3}$	·827	·112
	15	9	·540	·163		19	12	·898	·143
	16	9 $\frac{1}{3}$	·605	·204		20	12 $\frac{1}{3}$	·975	·185
	17	10	·676	·254	15	15	10	·694	·058
	18	10 $\frac{1}{3}$	·753	·314		16	11	·744	·056
	19	11	·836	·383		17	11 $\frac{1}{3}$	·799	·061
	20	11 $\frac{1}{3}$	·926	·463		18	12	·861	·075
11	11	7 $\frac{1}{3}$	·373	·073		19	12 $\frac{1}{3}$	·929	·095
	12	8	·410	·077		20	13	1·003	·126
	13	8 $\frac{1}{3}$	·454	·088	16	16	10 $\frac{1}{3}$	·790	·044
	14	9	·503	·105		17	11	·843	·040
	15	9 $\frac{1}{3}$	·559	·131		18	11 $\frac{1}{3}$	·901	·042
	16	10	·620	·164		19	12	·966	·054
	17	10 $\frac{1}{3}$	·688	·206		20	12 $\frac{1}{3}$	1·037	·074
	18	11	·762	·256	17	17	11 $\frac{1}{3}$	·892	·025
	19	11 $\frac{1}{3}$	·843	·317		18	12	·948	·018
	20	12	·929	·387		19	12 $\frac{1}{3}$	1·009	·018
12	12	8	·444	·074		20	13	1·077	·027
	13	8 $\frac{1}{3}$	·485	·077	18	18	12	1·000	·000
	14	9	·531	·087		19	12 $\frac{1}{3}$	1·059	—·010
	15	9 $\frac{1}{3}$	·583	·103		20	13	1·123	—·012
	16	10	·642	·129	19	19	12 $\frac{1}{3}$	1·114	—·031
	17	10 $\frac{1}{3}$	·707	·162		20	13 $\frac{1}{3}$	1·176	—·056
	18	11	·778	·204	20	20	13 $\frac{1}{3}$	1·235	—·068
	19	11 $\frac{1}{3}$	·855	·255					
	20	12	·938	·315					

NOTES ON CALCULATION OF TABLE D.

Let the ages be $x, x+m, x+n$, i.e., differences of age are m and n .

$$\text{Mean} = x + \frac{m}{3} + \frac{n}{3}, \quad \text{Departures from mean} = -\frac{m+n}{3}, \quad \frac{2m-n}{3}, \quad \frac{-m+2n}{3}$$

Departure	Square	Cube
$-\frac{m+n}{3}$	$\frac{1}{9}(m^2 + 2mn + n^2)$	$\frac{1}{27}(-m^3 - 3m^2n - 3mn^2 - n^3)$
$\frac{2m-n}{3}$	$\frac{1}{9}(4m^2 - 4mn + n^2)$	$\frac{1}{27}(8m^3 - 12m^2n + 6mn^2 - n^3)$
$\frac{-m+2n}{3}$	$\frac{1}{9}(m^2 - 4mn + 4n^2)$	$\frac{1}{27}(-m^3 + 6m^2n - 12mn^2 + 8n^3)$
$\Sigma = 0$	$\Sigma = \frac{1}{9}(6m^2 - 6mn + 6n^2)$ $= \frac{2}{3}(m^2 - mn + n^2)$ $= \frac{2}{3} \frac{m^3 + n^3}{m+n}$	$\Sigma = \frac{1}{27}(6m^3 - 9m^2n - 9mn^2 + 6n^3)$ $= \frac{1}{9}(2m^3 - 3m^2n - 3mn^2 + 2n^3)$ $= \frac{1}{9}(m+n)(2m-n)(m-2n)$
Putting $m=0, \quad n=18$ or $m=18, \quad n=18$	Value = $\frac{2}{3} \times 324$ 1st coeff. = $\frac{\Sigma m^2}{\frac{2}{3} \times 324} = \frac{\frac{2}{3} \frac{m^3 + n^3}{m+n}}{\frac{2}{3} \times 324}$ $= \frac{m^3 + n^3}{324}$	Value = $\pm \frac{1}{9} \times 11664$ $\frac{\Sigma m^3}{11664} = \pm \frac{(m+n)(2m-n)(m-2n)}{11664}$
MODE OF CALCULATION	<p>To obtain numerators, calculate first two of each group (m constant and n varying) and remainder by 2nd differences.</p> <p>If m is constant, $\Delta_n(m^2 - mn + n^2)$ $= 2n + 1 - m$ $\Delta^2_n = 2$</p> <p>The numerator for m, n is the same as for $n-m, n$.</p>	<p>To obtain numerators form products directly, or calculate first three of each group (m constant and n varying) and remainder by 3rd differences.</p> <p>The numerator for m, n is numerically the same as for $n-m, m$, but with the reverse sign.</p> <p>2nd coefficient = one-half the algebraical sum of the 1st coefficient and the above fraction taken \pm.</p>

TABLE E.

Three Joint Lives. $0^M 2\frac{1}{2}$ per-cent. $0^M 2\frac{1}{2}$ per-cent.

x	$a_x : x : x$	1st correction	2nd correction	x	$a_x : x : x$	1st correction	2nd correction
20	18.779	-.648	-.186	37	13.460	-.731	-.322
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 20.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 37.333			
21	18.484	-.648	-.187	38	13.135	-.745	-.328
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 21.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 38.333			
22	18.185	-.647	-.189	39	12.808	-.760	-.331
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 22.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 39.333			
23	17.882	-.647	-.193	40	12.479	-.773	-.335
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 23.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 40.333			
24	17.577	-.647	-.200	41	12.148	-.785	-.337
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 24.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 41.333			
25	17.269	-.646	-.208	42	11.817	-.799	-.339
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 25.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 42.333			
26	16.958	-.646	-.217	43	11.484	-.811	-.338
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 26.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 43.333			
27	16.646	-.647	-.226	44	11.149	-.820	-.337
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 27.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 44.333			
28	16.333	-.649	-.237	45	10.812	-.829	-.336
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 28.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 45.333			
29	16.018	-.654	-.247	46	10.475	-.835	-.335
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 29.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 46.333			
30	15.702	-.658	-.259	47	10.137	-.841	-.332
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 30.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 47.333			
31	15.385	-.664	-.270	48	9.799	-.844	-.330
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 31.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 48.333			
32	15.067	-.671	-.280	49	9.461	-.846	-.326
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 32.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 49.333			
33	14.748	-.682	-.290	50	9.123	-.845	-.323
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 33.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 50.333			
34	14.428	-.692	-.300	51	8.785	-.842	-.319
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 34.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 51.333			
35	14.106	-.704	-.308	52	8.451	-.840	-.314
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 35.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 52.333			
36	13.783	-.716	-.316	53	8.117	-.835	-.308
$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 36.333				$\frac{1}{3} \frac{1}{3} \frac{1}{3}$ 53.333			

TABLE E—*continued.**Three Joint Lives.* $0^M 2\frac{1}{2}$ per-cent. $0^M 2\frac{1}{2}$ per-cent.

x	$a_x : x : x$	1st correction	2nd correction	x	$a_x : x : x$	1st correction	2nd correction
54 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	7.785	-.825	-.304	70 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	3.224	-.536	-.185
55 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	7.457	-.816	-.299	71 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	3.005	-.513	-.177
56 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	7.133	-.806	-.292	72 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	2.795	-.490	-.169
57 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	6.812	-.792	-.286	73 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	2.595	-.467	-.160
58 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	6.495	-.778	-.280	74 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	2.404	-.444	-.153
59 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	6.184	-.763	-.273	75 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	2.222	-.422	-.144
60 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	5.879	-.747	-.265	76 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	2.049	-.399	-.136
61 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	5.580	-.729	-.258	77 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	1.884	-.378	-.128
62 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	5.287	-.711	-.250	78 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	1.728	-.356	-.120
63 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	5.001	-.691	-.243	79 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	1.581	-.336	-.111
64 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	4.722	-.670	-.235	80 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	1.442	-.315	-.105
65 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	4.451	-.648	-.227	81 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	1.312	-.295	-.097
66 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	4.188	-.627	-.218	82 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	1.190	-.276	-.090
67 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	3.934	-.604	-.210	83 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	1.075	-.257	-.082
68 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	3.689	-.582	-.202	84 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$.968	-.238	-.075
69 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$	3.452	-.559	-.194	85 $\frac{1}{3} \frac{2}{3} \frac{3}{3}$.868	-.221	-.062

TABLE F.

Last Survivor of Three Lives. $0^M 2\frac{1}{2}$ per-cent. $0^M 2\frac{1}{2}$ per-cent.

x	$a_{x:\overline{x}:x}$	1st correction	2nd correction	x	$a_{x:\overline{x}:x}$	1st correction	2nd correction
20	29.84	.56	-.05	37	25.03	.86	-.09
21	29.60	.57	-.05	38	24.69	.88	-.10
22	29.36	.58	-.05	39	24.35	.91	-.10
23	29.11	.60	-.06	40	24.00	.93	-.10
24	28.86	.61	-.06	41	23.64	.96	-.11
25	28.59	.63	-.06	42	23.28	.98	-.11
26	28.33	.65	-.06	43	22.91	1.01	-.12
27	28.06	.67	-.07	44	22.53	1.04	-.12
28	27.78	.69	-.07	45	22.15	1.06	-.12
29	27.50	.70	-.07	46	21.76	1.09	-.13
30	27.21	.72	-.08	47	21.36	1.12	-.13
31	26.92	.74	-.08	48	20.96	1.15	-.14
32	26.62	.76	-.08	49	20.55	1.18	-.14
33	26.32	.78	-.08	50	20.13	1.21	-.15
34	26.00	.80	-.09	51	19.71	1.24	-.16
35	25.68	.82	-.09	52	19.28	1.27	-.16
36	25.36	.84	-.09	53	18.85	1.31	-.17

TABLE F—continued.

Last Survivor of Three Lives. $0^M 2\frac{1}{2}$ per-cent. $0^M 2\frac{1}{2}$ per-cent.

x	$a_{x:\overline{x}:x}$	1st correction	2nd correction	x	$a_{x:\overline{x}:x}$	1st correction	2nd correction
54	18.41	1.34	-.18	70	10.98	1.76	-.35
$\frac{1}{54\frac{1}{2}}$				$\frac{1}{70\frac{1}{2}}$			
55	17.97	1.37	-.19	71	10.52	1.78	-.36
$\frac{1}{55\frac{1}{2}}$				$\frac{1}{71\frac{1}{2}}$			
56	17.52	1.40	-.20	72	10.07	1.79	-.37
$\frac{1}{56\frac{1}{2}}$				$\frac{1}{72\frac{1}{2}}$			
57	17.06	1.44	-.21	73	9.62	1.79	-.38
$\frac{1}{57\frac{1}{2}}$				$\frac{1}{73\frac{1}{2}}$			
58	16.60	1.47	-.22	74	9.18	1.79	-.39
$\frac{1}{58\frac{1}{2}}$				$\frac{1}{74\frac{1}{2}}$			
59	16.14	1.50	-.23	75	8.74	1.79	-.40
$\frac{1}{59\frac{1}{2}}$				$\frac{1}{75\frac{1}{2}}$			
60	15.68	1.53	-.24	76	8.31	1.79	-.41
$\frac{1}{60\frac{1}{2}}$				$\frac{1}{76\frac{1}{2}}$			
61	15.21	1.57	-.25	77	7.89	1.78	-.41
$\frac{1}{61\frac{1}{2}}$				$\frac{1}{77\frac{1}{2}}$			
62	14.74	1.59	-.26	78	7.48	1.77	-.42
$\frac{1}{62\frac{1}{2}}$				$\frac{1}{78\frac{1}{2}}$			
63	14.27	1.62	-.28	79	7.08	1.76	-.42
$\frac{1}{63\frac{1}{2}}$				$\frac{1}{79\frac{1}{2}}$			
64	13.80	1.65	-.29	80	6.68	1.74	-.42
$\frac{1}{64\frac{1}{2}}$				$\frac{1}{80\frac{1}{2}}$			
65	13.32	1.68	-.30	81	6.31	1.72	-.43
$\frac{1}{65\frac{1}{2}}$				$\frac{1}{81\frac{1}{2}}$			
66	12.85	1.70	-.31	82	5.94	1.69	-.43
$\frac{1}{66\frac{1}{2}}$				$\frac{1}{82\frac{1}{2}}$			
67	12.38	1.72	-.32	83	5.58	1.67	-.43
$\frac{1}{67\frac{1}{2}}$				$\frac{1}{83\frac{1}{2}}$			
68	11.91	1.74	-.33	84	5.23	1.64	-.44
$\frac{1}{68\frac{1}{2}}$				$\frac{1}{84\frac{1}{2}}$			
69	11.44	1.75	-.34	85	4.90	1.60	-.43
$\frac{1}{69\frac{1}{2}}$				$\frac{1}{85\frac{1}{2}}$			

The Values in Table F are given to 2 decimal places only, as the errors of approximation for last survivor annuities usually affect the second decimal place.

TABLE G.

Abstract Table of Coefficients.—Four Lives. $x : x+m : x+n : x+p$.

Standard range of ages, 20 years.

m	n	p	Average age = $x +$	Coefficient of 1st correction	Coefficient of 2nd correction	m	n	p	Average age = $x +$	Coefficient of 1st correction	Coefficient of 2nd correction
0	0	4	1	·040	·024	4	12	12	7	·360	·160
		8	2	·160	·112			16	8	·533	·267
		12	3	·360	·288			20	9	·787	·477
		16	4	·640	·576						
		20	5	1·000	1·000	4	16	16	9	·680	·312
								20	10	·907	·453
0	4	4	2	·053	·027	4	20	20	11	1·107	·517
		8	3	·147	·085						
		12	4	·320	·224						
		16	5	·573	·467	8	8	8	6	·160	·048
		20	6	·907	·837			12	7	·253	·091
								16	8	·427	·213
0	8	8	4	·213	·107			20	9	·680	·440
		12	5	·360	·200						
		16	6	·587	·389	8	12	12	8	·320	·096
		20	7	·893	·699			16	9	·467	·173
								20	10	·693	·347
0	12	12	6	·480	·240						
		16	7	·680	·368	8	16	16	10	·587	·197
		20	8	·960	·608			20	11	·787	·309
0	16	16	8	·853	·427	8	20	20	12	·960	·352
		20	9	1·107	·589						
0	20	20	10	1·333	·667	12	12	12	9	·360	·072
								16	10	·480	·112
								20	11	·680	·240
4	4	4	3	·010	·016						
		8	4	·107	·053	12	16	16	11	·573	·107
		12	5	·253	·163			20	12	·747	·181
		16	6	·480	·368						
		20	7	·787	·693	12	20	20	13	·893	·195
4	8	8	5	·147	·061	16	16	16	12	·640	·064
		12	6	·267	·133			20	13	·787	·093
		16	7	·467	·293						
		20	8	·747	·565	16	20	20	14	·907	·069
						20	20	20	15	1·000	·000

NOTES ON CALCULATION OF TABLE G.

Let the ages be x , $x+m$, $x+n$, $x+p$, *i.e.*, differences of age are m , n and p .

Put $m+n+p=s$. Mean $= x + \frac{s}{4}$, departures from mean are $-\frac{s}{4}$, $m - \frac{s}{4}$, $n - \frac{s}{4}$ and $p - \frac{s}{4}$

Departure	Square	Cube
$-\frac{s}{4}$	$\frac{s^2}{16}$	$-\frac{s^3}{64}$
$m - \frac{s}{4}$	$m^2 - m\frac{s}{2} + \frac{s^2}{16}$	$m^3 - \frac{3}{4}m^2s + \frac{3}{16}ms^2 - \frac{s^3}{64}$
$n - \frac{s}{4}$	$n^2 - n\frac{s}{2} + \frac{s^2}{16}$	$n^3 - \frac{3}{4}n^2s + \frac{3}{16}ns^2 - \frac{s^3}{64}$
$p - \frac{s}{4}$	$p^2 - p\frac{s}{2} + \frac{s^2}{16}$	$p^3 - \frac{3}{4}p^2s + \frac{3}{16}ps^2 - \frac{s^3}{64}$
$\Sigma = 0$	$\Sigma = m^2 + n^2 + p^2$ $- (m+n+p)\frac{s}{2} + \frac{s^2}{4}$ $= m^2 + n^2 + p^2 - \frac{s^2}{2} + \frac{s^2}{4}$ $= \frac{1}{4}[4(m^2 + n^2 + p^2) - s^2]$	$\Sigma = \Sigma m^3 - \frac{3}{4}(\Sigma m^2)s + \frac{3}{16}(\Sigma m)s^2 - \frac{s^3}{16}$ $= \Sigma m^3 - \frac{3}{4}(\Sigma m^2)s + \frac{3}{16}s^3 - \frac{s^3}{16}$ $= \frac{1}{8}[8\Sigma m^3 - 6(\Sigma m^2)s + s^3]$
Putting $n = n = 0, p = 20$ or $m = n = p = 20$	Value $= \frac{1}{4} \times 3 \times 20^2$ $= \frac{1}{4} \times 1200$	Value $= \pm \frac{1}{8} [3 \times 20^3]$ $= \pm \frac{1}{8} \times 24000$
	1st coefficient $= \frac{4(m^2 + n^2 + p^2) - (m+n+p)^2}{1200}$	2nd coeff. $= \frac{1}{2} \{ \text{1st coeff.} \pm \frac{[\quad]}{24000} \}$
MODE OF CALCULATION	To obtain numerators, calculate first two of each group (m and n constant, p varying) and the remainder by second differences.	To obtain numerators, calculate first three of each group (m and n constant, p varying), and the remainder by third differences.

TABLE II.

Four Joint Lives. $0^M 2\frac{1}{2}$ per-cent. $0^M 2\frac{1}{2}$ per-cent.

x	$a_{x:x:x:x}$	1st correction	2nd correction	x	$a_{x:x:x:x}$	1st correction	2nd correction
20 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	16.965	To be estimated from succeeding values.	To be estimated from succeeding values.	32 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	13.336	-.593	-.457
21 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	16.673			33 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	13.030	-.597	-.476
22 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	16.378			34 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	12.724	-.604	-.491
23 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	16.079			35 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	12.416	-.610	-.506
24 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	15.779			36 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	12.110	-.620	-.518
25 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	15.477	-.600	-.337	37 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	11.803	-.631	-.528
26 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	15.172	-.595	-.352	38 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	11.496	-.642	-.538
27 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	14.867	-.593	-.367	39 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	11.187	-.654	-.543
28 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	14.562	-.590	-.386	40 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	10.877	-.665	-.548
29 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	14.255	-.588	-.403	41 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	10.566	-.675	-.551
30 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	13.949	-.589	-.421	42 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	10.255	-.686	-.552
31 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	13.643	-.591	-.439	43 $\begin{smallmatrix} 1 \\ 4 \\ 12 \\ 34 \end{smallmatrix}$	9.943	-.696	-.551

TABLE H—continued.

Four Joint Lives. $0^M 2\frac{1}{2}$ per-cent. $0^M 2\frac{1}{2}$ per-cent.

x	$a_{x:x:x:x}$	1st correction	2nd correction	x	$a_{x:x:x:x}$	1st correction	2nd correction
44 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	9.631	-.705	-.549	56 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	5.942	-.683	-.455
45 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	9.318	-.713	-.546	57 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	5.653	-.670	-.443
46 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	9.004	-.718	-.542	58 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	5.369	-.656	-.431
47 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	8.691	-.723	-.536	59 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	5.091	-.641	-.417
48 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	8.378	-.725	-.531	60 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	4.819	-.625	-.404
49 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	8.065	-.725	-.524	61 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	4.554	-.608	-.391
50 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	7.754	-.724	-.516	62 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	4.295	-.589	-.378
51 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	7.445	-.722	-.507	63 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	4.044	-.570	-.364
52 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	7.138	-.717	-.498	64 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	3.801	-.551	-.351
53 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	6.834	-.712	-.487	65 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	3.565	-.530	-.337
54 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	6.533	-.704	-.477	66 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	3.338	-.510	-.323
55 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	6.235	-.693	-.467	67 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	3.119	-.489	-.309

TABLE II —continued.

Four Joint Lives. $O^M 2\frac{1}{2}$ per-cent. $O^M 2\frac{1}{2}$ per-cent.

x	$a_x : x : x : x$	1st correction	2nd correction	x	$a_x : x : x : x$	1st correction	2nd correction
68 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	2·908	—·469	—·295	77 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	1·397	—·289	—·177
69 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	2·706	—·448	—·282	78 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	1·271	—·271	—·164
70 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	2·512	—·427	—·268	79 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	1·152	—·253	—·153
71 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	2·327	—·406	—·254	80 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	1·041	—·236	—·140
72 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	2·151	—·386	—·240	81 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	·937	—·220	—·130
73 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	1·984	—·366	—·227	82 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	·841	—·205	—·112
74 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	1·825	—·346	—·214	83 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	·751	—·189	—·094
75 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	1·674	—·326	—·202	84 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	·667	—·174	—·094
76 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$	1·532	—·308	—·189	85	·590	—·160	—·109

TABLE J.

Last Survivor of Four Lives. $0^M 2\frac{1}{2}$ per-cent. $0^M 2\frac{1}{2}$ per-cent.

x	$a_x : x : x : x$	1st correction	2nd correction	x	$a_x : x : x : x$	1st correction	2nd correction
20 1 4 1 2 2 3 3 4	30.18	To be estimated from succeeding values.	To be estimated from succeeding values.	32 1 4 1 2 2 3 3 4	27.39	.97	-.22
21 1 4 1 1 2 2 3 3 4	30.25			33 1 4 1 1 2 2 3 3 4	27.10	.99	-.23
22 1 4 1 1 2 2 3 3 4	30.02			34 1 4 1 1 2 2 3 3 4	26.80	1.02	-.24
23 1 4 1 1 2 2 3 3 4	29.78			35 1 4 1 1 2 2 3 3 4	26.49	1.05	-.25
24 1 4 1 1 2 2 3 3 4	29.53			36 1 4 1 1 2 2 3 3 4	26.17	1.08	-.26
25 1 4 1 1 2 2 3 3 4	29.28	.81	-.18	37 1 4 1 1 2 2 3 3 4	25.83	1.10	-.27
26 1 4 1 1 2 2 3 3 4	29.03	.83	-.19	38 1 4 1 1 2 2 3 3 4	25.52	1.13	-.27
27 1 4 1 1 2 2 3 3 4	28.77	.84	-.19	39 1 4 1 1 2 2 3 3 4	25.19	1.15	-.28
28 1 4 1 1 2 2 3 3 4	28.51	.87	-.20	40 1 4 1 1 2 2 3 3 4	24.85	1.18	-.29
29 1 4 1 1 2 2 3 3 4	28.24	.89	-.21	41 1 4 1 1 2 2 3 3 4	24.50	1.21	-.29
30 1 4 1 1 2 2 3 3 4	27.96	.92	-.21	42 1 4 1 1 2 2 3 3 4	24.15	1.25	-.31
31 1 4 1 1 2 2 3 3 4	27.68	.94	-.22	43 1 4 1 1 2 2 3 3 4	23.79	1.28	-.32

TABLE J—continued.

Last Survivor of Four Lives. $0^M 2\frac{1}{2}$ per-cent. $0^M 2\frac{1}{2}$ per-cent.

x	$a_{x:x:x:x}$	1st correction	2nd correction	x	$a_{x:x:x:x}$	1st correction	2nd correction
44 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	23.42	1.32	-.33	56 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	18.48	1.77	-.50
45 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	23.04	1.35	-.34	57 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	18.02	1.82	-.52
46 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	22.66	1.39	-.35	58 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	17.57	1.86	-.54
47 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	22.28	1.42	-.36	59 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	17.11	1.90	-.56
48 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	21.88	1.46	-.37	60 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	16.64	1.94	-.58
49 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	21.48	1.49	-.39	61 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	16.17	1.98	-.60
50 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	21.07	1.53	-.40	62 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	15.70	2.02	-.63
51 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	20.65	1.57	-.42	63 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	15.22	2.06	-.65
52 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	20.23	1.61	-.43	64 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	14.75	2.10	-.67
53 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	19.80	1.65	-.45	65 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	14.26	2.13	-.69
54 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	19.36	1.69	-.46	66 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	13.78	2.16	-.71
55 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	18.92	1.73	-.48	67 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	13.31	2.19	-.73

TABLE J—continued.

Last Survivor of Four Lives. $0^M 2\frac{1}{2}$ per-cent. $0^M 2\frac{1}{2}$ per-cent.

x	$a_{x:x:x:x}$	1st correction	2nd correction	x	$a_{x:x:x:x}$	1st correction	2nd correction
68 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	12.83	2.22	-.75	77 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	8.65	2.33	-.90
69 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	12.35	2.25	-.77	78 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	8.22	2.33	-.91
70 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	11.87	2.27	-.79	79 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	7.79	2.32	-.92
71 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	11.39	2.29	-.81	80 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	7.38	2.31	-.93
72 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	10.93	2.31	-.83	81 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	6.98	2.28	-.94
73 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	10.46	2.32	-.85	82 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	6.58	2.26	-.94
74 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	10.00	2.33	-.86	83 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	6.19	2.23	-.94
75 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	9.55	2.33	-.88	84 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	5.82	2.20	-.94
76 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	9.09	2.34	-.89	85 $\begin{smallmatrix} 1 \\ 4 \\ 1 \\ 2 \\ 3 \\ 4 \end{smallmatrix}$	5.47	2.17	-.95

The values in Table J are given to 2 decimal places only, as the errors of approximation for last survivor annuities usually affect the second decimal place.

ABSTRACT OF THE DISCUSSION.

Mr. D. C. FRASER said that the principle of the paper was a very simple one, and there was an easily-followed illustration of it in its application to the question of joint annuities on two lives. When Makeham's law held in a mortality table, actuaries were familiar with the fact that one table could be made to serve for all purposes—a table of joint annuities on lives of equal ages : and Mr. Lidstone had acted on the simple idea that in a mortality table where Makeham's law did not apply, if there were three tables instead of one, those three tables could be made to serve all purposes. The idea was obvious when it was so stated, but it was a stroke of genius to hit upon such a simple idea. The author gave an illustration in paragraph 14, where, to find a joint annuity on two lives, 38 and 62, he employed the values for ages 50 : 50, 60 : 40, and 65 : 35. The procedure was at once obvious ; but there was one point to be remarked—if a straightforward interpolation were made between the three values given a good result was not obtained. It might be noticed that the author's formula employed only even powers, but if one interpolated as between three values only, the whole of one's information was not used, because the function employed was a symmetrical function. The annuity-values given were on joint lives where the ages were of the form $x+t$ and $x-t$. If the value for any positive value of t were known, the value for a corresponding negative value of t was also known. In fact the left-hand branch of the curve was identical with the right-hand branch. The curve was absolutely symmetrical, so that, instead of interpolating between three values, one could interpolate between five values and when that was done Mr. Lidstone's formula was obtained. It would be noticed that the values given were not equidistant values, and Mr. Lidstone's result was at once obtained by Lagrange's formula. An important point was that exactly the same process could be used for last-survivor annuities as for joint-life annuities. The results came out with very great accuracy up to a disparity of 30 or 34 years. For example, if one took the mean ages 65 on page 9 the greatest error was .011. Comparing that with the value at the head of the table the error of 1 in the second decimal place was comparatively unimportant ; it was an error of about 2 per 1,000. One could use that value with absolute confidence for such a purpose as calculating an annual or single premium or the reserve in a valuation.

If one looked closely at the columns of errors in the Tables just referred to, it would be noticed that the errors were systematic and not accidental. For example, in the two pages dealing with joint lives, the errors were nearly all positive, and where negative errors did occur they occurred in the same place in every set of examples ; the fourth value in each set, for instance, was a negative error. If the errors in the last-survivor cases were compared with those in joint-life cases, it would be found that the errors in the

cases of two lives were reversed, and, taking the table for mean ages 50, the last-survivor error was within a unit, in the third decimal place, of the joint-life error. Since the errors appeared to follow a law, it gave some hope that they could be brought under control, and practically eliminated.

It would be noticed that the joint-life cases took the form of a symmetrical bell-shaped curve with a maximum value for the case of equal lives, and the last-survivor values took the form of a cup-shaped curve with a minimum value for the case of equal ages.

Before passing on to the case of more than two lives, which was more difficult, it might be worth while to make the following suggestion. Cases of several lives could be illustrated by taking simple functions of single lives. The difficulty in examining the cases of joint-life annuities and last-survivor annuities on three lives and more than three lives was that of obtaining examples. For a good many purposes functions of single lives could be used to illustrate the method: for example, the sum of annuities on three lives, the sum of annuities on four lives, and so on: and some of the features which Mr. Lidstone referred to in discussing three lives would be found to exist when functions of single lives were taken in the way he (the speaker) had suggested.

Passing on to the curves obtained in the case of three lives, they were not exactly symmetrical curves, but they were curves which had either a maximum or a minimum value for the case of equal ages. The term involving simply the sum of the variations from the average age vanished, as Mr. Lidstone pointed out. The consequence was there was either a minimum value or a maximum value when the lives were of equal age. The value $F(x+a, x+b, x+c)$ might be regarded as an ordinate of the curve $F(x+at, x+bt, x+ct)$, where t was the independent variable, and giving a, b, c different values, subject to the condition $a+b+c=0$, they would obtain, for a given value of x , a family of curves all in the same plane, and having a common maximum (or minimum) ordinate at the origin. The joint-life annuities constituted a series of bell-shaped curves, all having the same maximum value in the case of equal ages. There was a family of curves all having the same maximum value, and out of all these there was one curve which was symmetrical; that was the curve where the ages were equidistant. For the case of three lives, Mr. Lidstone picked out two curves, took one value from each, and used that along with the common maximum value for equal ages, and he obtained an approximation for every case of three lives within a given length. It was a very bold idea to suggest that such a simple process should give even a good approximation, but, as a matter of fact, the approximations were exceedingly good. The accuracy for the annuities on three lives was, he thought, remarkable. In the case of joint-life annuities, there were eight cases only in the examples given by Mr. Lidstone where the error exceeded two in the second decimal place. The last-survivor errors were larger than the joint-life errors, but relatively to the values of the last-survivor annuities, they were not on the

whole greater. Also, as he had pointed out in the case of two lives, the errors were systematic.

The approximations in the case of last-survivor annuities on three lives were probably the best that could be obtained by such simple means; and, as Mr. Lidstone pointed out, were sufficiently close for many practical purposes. But the question must inevitably arise, what modification or extension of Mr. Lidstone's processes was necessary if more accurate results were desired—and this was of great importance, not merely in connection with the values of last-survivor annuities, but in connection with the general question of interpolation for three variables. He thought that a reply to this question might also solve the difficulty mentioned by Mr. Lidstone in paragraph 23, where he remarked that in several cases the first approximation gave a better result than the second. He considered that a study of the curves which represented the last-survivor annuities in their relation to the curve which represented the first approximation would produce valuable results.

There was another point which had occurred to him. He had already referred to the relationship between the errors of joint-life annuities and last-survivor annuities. That suggested the possibility that the application of Mr. Lidstone's principle to some simple combination of joint-life and last-survivor annuities might have happy results. On making some rough tests it appeared that by simply taking the sum of the joint-life and last-survivor annuities they might perhaps secure greater accuracy than by taking the last-survivor alone. If the joint-life annuity could be otherwise correctly ascertained it might be a comparatively simple method of obtaining a good value of the last-survivor annuity.

Coming to the case of four lives, he thought he need only remark that, considering the simplicity of the means employed, the accuracy of the results was remarkable, though it could not be expected that such a close range of accuracy would be found as in the case of two and three lives. He did notice, however, that the great regularity of error which existed in the case of two and three lives did not exist to the same extent in the case of four lives.

In Part III. of the paper—"Further developments and special applications"—Mr. Lidstone made most interesting use of the principle of uniform seniority, and he thought that that was the gem of the paper. An illustration was given in paragraph 32 and the simplicity and beauty of the method would be appreciated by anybody who followed out the numerical working. The idea was a most simple one. In a table like the O^M Table, not following the law of uniform seniority, one might assume for a first approximation that uniform seniority did apply. The author used the O^{MC} Table of uniform seniority for this purpose. Three values were supposed to be known, one for equal ages, and two for ages not equal, and Mr. Lidstone also worked out the two given values for unequal ages on the assumption of uniform seniority, and used the two errors so ascertained to obtain the approximation for the error in the particular case in question. Applying the principles of

the paper to do that, he found a correction which practically cancelled the whole error, and gave results which might be considered exact. Mr. Lidstone suggested that the same principle might be used for three or four lives, and he thought there was no doubt that that would be the case, and that the method would prove to be equally successful. He would like to suggest a still further application of the principle of uniform seniority, for the cases of annuities on joint lives, where the disparity of age was wider than Mr. Lidstone treated of in the paper.

There was a property of the tables of uniform seniority which he thought might be applied usefully to cases of very wide disparity. If any value of uniform seniority was determined, it would be found that the deduction from the older age was practically constant after a range of 30 or 35 years was passed. The fact that tables were arranged by way of addition to the younger life concealed that rather important property. It applied also to uniform seniority tables for three and four lives, and there was a limit to the deduction from the oldest age in every case. For two lives in the case of the OMO Table the limit was about $7\frac{3}{4}$ years. At the disparity of 30 years the deduction from the oldest life was seven years; for a disparity of sixty years the deduction from the oldest life was $7\frac{3}{4}$ years. So that, for a range of 30 years or more, there was a difference of less than one year in the deduction to be made from the older life. For three lives the limit was 12 years, and the limit was reduced to $4\frac{1}{2}$ years, if the two older ages were equal. The tables of uniform seniority could be applied in succession. For example, in a case of three lives, two of the lives could be taken and a two-life table used to make the two older lives equal, and then the property to which he had referred came in. When the two older lives were equal there was a limit within which the deduction from the oldest age must fall.* It might be inferred that in any table, if equivalent ages were recorded for a few selected cases of wide disparity, a great number of cases could be dealt with by a simple interpolation. He would remark, in conclusion, that in everything Mr. Lidstone presented to the Institute there was evident his resolute determination to bring out results of practical utility. Mr. Lidstone's seniority as a Fellow was still under 20 years, and there was no person connected with the Institute of such a seniority who had brought before them such a volume and variety of work. In the paper he had presented that evening he had added a mighty stone to the pile of his achievements and to the weight of their indebtedness.

Mr. W. PALIN ELDERTON remarked that all actuaries would find some way of puzzling out a joint-life annuity, but last-survivor

* [It may easily be shewn that the maximum deduction from the oldest age where n lives are involved, is $\frac{\log_e n}{\log_e e}$; and that in the special case where the two

older of three lives are of equal age the limit is $\frac{\log_e 3}{\log_e e}$.—ED.]

annuities were far more troublesome, and what they required was, either some convenient way of tabulating them, or a simple method of calculating an isolated value for a particular purpose. Mr. Lidstone's paper was mainly concerned with the tabulation problem. There was one point which might be of use when one had to deal with cases outside the range given by the author. The last-survivor annuity on two lives, say 20 and 60, must lie between a last-survivor annuity on 20 and 20 and a single-life annuity on 20, because the single-life annuity on 20 must be equal to the last-survivor annuity on 20 and 100. Therefore the annuity for 20 and 60 lay between those two, and if the interpolated values were found by working on the square root of the age instead of on the age itself, he thought that in the majority of cases a fair result might be obtained. That method led one to think that possibly an alternative method of tabulation—less exact than the author's—would be to tabulate (1) last-survivor annuities for equal ages, x and x ; and (2) last-survivor annuities for ages x and $\frac{1}{2}(x+100)$. By means of these and the single-life annuities, an ordinary second difference formula could be used, and a certain amount of tabulation would be avoided.

There was one other method which might give a fairly good result, namely, to find the last-survivor annuity on two lives of equal age. He did not mean the arithmetical mean of the ages, but, instead of working on μ_x as in a joint-life case, one might use the reciprocal of μ_x as the function. By that means a result was obtained which would, he thought, give a quite reasonable approximation when the lives were old. With young ages he thought the same method could be used with an adjustment. The method would hold, he thought, to about the same extent as Gompertz's hypothesis, but he could not prove this.

Mr. R. TODHUNTER said that he would confine his remarks to two points relating to the two-life annuity-values. The first was the question of the range of Mr. Lidstone's method. He tabulated the annuity-values along three diagonals, whereas they were accustomed for interpolation purposes to tabulate in rows and columns. He suggested that the tabulation might be extended to the few remaining diagonals required to cover the whole of the possible age combinations. As the distance from the central diagonal increased, each parallel diagonal became shorter, so that the labour of completing the tabulation would be comparatively small. They would then have the material for interpolating any required value, but if it fell outside the range of Mr. Lidstone's standard formula, they would have to determine the constants from (say)

$$a_{x+15 \cdot x-15}, \quad a_{x+20 \cdot x-20}, \quad a_{x+25 \cdot x-25},$$

instead of

$$a_{xx}, \quad a_{x+10 \cdot x-10}, \quad a_{x+15 \cdot x-15}.$$

They would still preserve the fundamental advantages of interpolating along a line of symmetry, and of getting an interpolated value correct to fifth differences, while using only three annuity-values.

The other point which seemed to him to be of special interest was the question whether there was any other scheme which would compare favourably with the one which the author had proposed that evening. There were, of course, the ordinary methods of interpolation—either by a double first-difference interpolation, or by a double second central-difference interpolation, or by the simple formula for the two variables based on the values u_{00} , u_{01} , u_{0-1} , u_{10} , u_{-10} and u_{11} . But by none of these methods, so far as his experience went, did one get values so good as Mr. Lidstone's. A possible variation would be to tabulate the differences between the annuity-values by the table under discussion and the annuity-values by some standard table for which a complete set of joint-life annuity tables had been calculated, and then to interpolate the value of the difference for the particular combination of ages which was being considered, and to add that to the value by the standard table. That again, however, was not a very satisfactory process. Perhaps the only method giving a value which competed with the author's was a process which Mr. Lidstone virtually suggested in his "Further Development", namely, to tabulate the errors caused by calculating the annuity-value on the assumption of uniform seniority. Those differences were comparatively small, and if calculated it might be found that, at any rate throughout a considerable portion of the table, the error for many combinations of ages could be almost guessed by inspection. It, however, had not the completeness of the method proposed by Mr. Lidstone. The only ideally satisfactory method would be to obtain the equation to the joint-life annuity-value in a practicable form. Unfortunately that could not be done, but it did not seem impossible that some algebraical expression might be found which would fit the joint-life annuity-values with fair accuracy, just as it was possible to find a single algebraical expression which would fit the experience values of $\log p$. If that could be done, an expression would have been obtained from which any value could be calculated.

Mr. S. W. NEWLING said that he had investigated the mathematical portion of the paper from the standpoint of Analytical Geometry, and thought that the results might prove of interest. In the case of two lives, the geometrical interpretation was very simple, and might be studied rather as an introduction to the consideration of the three-life problem than as a matter of importance in itself.

In the case of three lives it was more difficult to picture the geometrical representation. If they took three rectangular axes for a , b , c , the locus of points, at which the joint-life annuities were equal, would be a surface symmetrical with respect to each of the three planes

$$a - b = 0, \quad b - c = 0, \quad \text{and} \quad c - a = 0.$$

The points which they were to consider would lie on the plane $a + b + c = 0$, and the two points $t, t, -2t$; $2t, -t, -t$; together with the four corresponding points found by changing the order, would lie on the circumference of a circle of radius $\sqrt{6}.t$, at intervals of

60 degrees. The locus of points in the plane at which the joint-life annuities were equal would be a curve symmetrical with respect to each of the three diameters found by joining three pairs of the six points t , $t - 2t$, &c.

If now they took rectangular axes in this plane, ξ along the line from the origin to the point t , $2 - t$, t , and η along the line from the origin to $-\sqrt{3}t$, 0 , $\sqrt{3}t$, and called the angle between the line from origin to ξ , η , and the axis of ξ , θ , then

$$r^2 = \xi^2 + \eta^2 = \frac{\Sigma a^2}{6t^2} : \xi = r \cdot \cos \theta, \quad \eta = r \cdot \sin \theta$$

Expanding the annuity in terms of ascending powers of ξ , η and substituting in terms of $r \cdot \cos \theta$ and $r \cdot \sin \theta$ they were able to determine the forms of the series by a consideration of the symmetry of the position of the points a , b , c ; b , c , a ; &c. The value of the annuity would be unaltered by the substitution of $(\pm \theta + 2n\pi/3)$ for θ , consequently the trigonometrical functions involved in the co-efficients of the powers of r must reduce to the forms

$$\text{powers of } (\sin^2 \theta + \cos^2 \theta) \text{ or } 1,$$

$$\text{or} \quad \text{powers of } \cos 3\theta,$$

and the expression took the form

$$A + r^2 \cdot B + r^3 \cos 3\theta \cdot C + r^4 D + r^5 \cos 3\theta \cdot E + r^6 (F + \cos^2 3\theta \cdot G) + \&c.,$$

$$\text{Now } \cos \theta \text{ was equal to } \frac{-3 \cdot b}{\sqrt{\Sigma a^2}} \cdot \sqrt{\frac{1}{6}}$$

$$\text{and } \cos 3\theta \text{ to } -\sqrt{\frac{1}{6}(\Sigma a^3)} \\ (\Sigma a^2)^{\frac{3}{2}}$$

$$\text{so that} \quad r^3 \cos 3\theta = -\frac{\Sigma a^3}{6t^3}.$$

Making the substitutions the series became

$$A + \frac{\Sigma a^2}{6t^2} B - \frac{\Sigma a^3}{6t^3} C + \frac{(\Sigma a^2)^2}{36t^4} D - \frac{\Sigma a^2 \Sigma a^3}{36t^5} E + \frac{(\Sigma a^2)^3}{216t^6} F + \frac{(\Sigma a^3)^2}{36t^6} G, \&c.$$

an expression which agreed with the formula in paragraph 44. after making allowance for the alteration in the scale of measurement, and noticing that in the case of three variables the terms in equation (3) involving D' and E' and also F' and G' coalesced.

Referring to paragraph 44, it was stated that "Terms of the 4th, 5th and higher orders may be similarly dealt with." He was afraid that the method of paragraph 43 would be exceedingly tedious when applied to higher orders, and for the benefit of students he

would point out a simpler method, explained in Chrystal's Algebra, Chapter XVIII., and elsewhere.

Let a, b, c be the roots of the equation

$$x^3 - p_2x + p_3 = 0,$$

then any symmetrical function of a, b, c can be expressed in terms of p_2, p_3 and their powers :

For example a symmetrical function of the eleventh order can be put equal to

$$m.(p_2)^4.p_3 + n.p_2.(p_3)^3$$

where the undetermined coefficients m, n , can be at once determined by giving two sets of values to a, b, c .

He would now like to offer a few observations on paragraph 23. The method was applied to three last-survivor annuities α, β, γ , and the effect was the same as when the method was applied to the three constituents separately and the results summed.

Taking as an example mean age 35, ages 30, 30, 45—The three-life value was .012 in excess, while the sum of the two-life values was .038 in excess or, taking it with the negative sign, .038 in defect. The two combined were therefore .026 in defect: this amount being reduced to .021 by the excess of .005 in the single life portion. This suggested that the errors were due to the insufficiency of the number of terms taken in finding the two-life portion. The method must be said to fail in the case of these last-survivor annuities, unless the series were to be extended to further terms, an extension which would render it troublesome in its practical application.

The method of approximation described in paragraph 9, *i.e.*, the successive calculation of $A, A + B\Sigma a^2, A + B\Sigma a^2 + C\Sigma a^3$ differed from that used in the tables of examples, where the first approximation (corresponding to the second in paragraph 9) was not equal to $A + B\Sigma a^2$ owing to the method adopted for facilitating the calculation of the co-efficients. If the tables were re-arranged so that the column headed "2-term approximation" were made equal to $A + B\Sigma a^2$ they would be in a better position to compare the results. In those cases in which the first approximation was then closer than the second it would not be allowable to take either the first or the second approximation, and it would be necessary to include further terms.

Mr. G. F. HARDY said that he desired to join, as he was sure all did, in the high praise which the paper had evoked from the speakers. Like all Mr. Lidstone's productions, it combined great simplicity and elegance with a remarkable amount of thoroughness. The principle underlying the paper was extremely simple, and he thought that was a great advantage to students, who would be able to follow with great ease the demonstrations Mr. Lidstone had given, and to follow with a great deal of facility the specimen applications of the method which the author had made in the paper. He was afraid that

some would think the amount of preliminary calculation required in the application of the method was extremely heavy, but that, he considered, was more apparent than real. There appeared to be a great mass of figures which had to be computed, but after all it only meant that certain specimen annuity-values had to be calculated at certain agreed intervals of age, and on the basis of those annuity-values the whole of the rest of the calculation rested. The coefficients which were used in applying the formula were constant, as Mr. Lidstone had pointed out, for all rates of interest and for all tables of mortality. Therefore, given the mortality tables and the specimen joint annuities, the work became very simple, and the preliminary calculations, however extensive they might be, could all be done at leisure once and for all.

Mr. Fraser and Mr. Todhunter had referred to the question of the particular age intervals which should be employed. That was rather an interesting point. Mr. Lidstone had given some reasons for the age intervals he had used, and perhaps in determining those regard should be had to the kind of intervals which were likely to arise in practice. For instance, with a three-life annuity one was likely perhaps to have two older lives, who might be life tenants, and a younger life who might be a reversioner, and in such cases the ages would fit in with Mr. Lidstone's selected ages $(x-2t)$, $(x+t)$ and $(x+t)$.

The great merit of the paper was that it produced an entirely simple and universal method which was applicable both to joint-life and survivorship statuses. There were a great many methods by which approximately good results could be arrived at in the case of joint lives. Even Simpson's Rule produced fairly approximate results in its time, and although actuaries' ideas of accuracy were a little more strict now than they were then, still, with a little alteration and modification in that rule, very good results could be obtained. But it was not by any means so easy to deal with the question of last-survivor annuities, and Mr. Elderton had made a suggestion as to the limits between which those last-survivor annuities would be found to fall. He would like to add another suggestion. Taking a case of a last-survivor annuity on a young life and an old life, certain limits which were tolerably close might be found between which the annuity would fall. Supposing there was a life of 20 and a life of 60, if for the life of 60 was substituted a perpetuity at such a high rate of interest that it was equal to an annuity at age 60, one approximation to the last-survivor annuity would be obtained. One would have $a_{20} + a_{60} - a'_{20}$, where the accented annuity-value was calculated at a rate of interest i' , where $\frac{1}{i'} = a_{60}$. Then, an alternative approximation to the true value would be obtained by the formula $a_{20} + a_{60} - a''_{20}$, where the accented value was calculated at a rate i'' , where $i'' = \mu_{60} + \delta$. It would be found that these two approximations would give rather narrow limits, between which the true value should be found.

It was an interesting point for students to notice that, while in

the case of joint annuities the appropriate equal age status in the joint lives gave an age always greater than the mean age, the reverse of that was always the case with last-survivor annuities. That, of course, was due to the fact that in the joint annuity the important life was the older one, whereas in the last-survivor annuity the important life was the younger. In the case of joint annuities, if the table approximately followed Makeham's law, an exact formula could be deduced which would hold good for all combinations of ages. Unfortunately that could not be done in the case of last-survivor annuities, even where there were only two lives. He had, himself, made some attempt to see whether some formula such as Mr. Elderton suggested—not necessarily using the reciprocal of the force of mortality but some function of that kind—could be found which would give the equivalent age in last-survivor cases, but he found that it was not practicable to obtain any formula which would give results with anything like the degree of accuracy at which Mr. Lidstone had arrived in his table. He had been very much interested in the author's further development of his method, in which it was shown that they were not bound to treat annuities as a function of the age of the lives, but could treat them as a function of any other function of the age that they pleased, and in the case of a table graduated by Makeham, if one adopted the function of the age as the Makeham constant c , raised to the power of the age, and treated that and not the age as the independent variable, then perfectly accurate results were obtained without any corrections whatever by Mr. Lidstone's method; and as had been pointed out by more than one speaker, that method could be employed to arrive at fairly approximate results when the table did not follow Makeham's law.

He had made some experiments in that direction, and found in the case of the O^M Table, for instance, a series of equivalent ages were obtained which exhibited the feature which was pointed out by Mr. Fraser, namely, that at those ages where the Makeham formula applied, certain constant differences between the oldest ages and the weighted mean age were obtained—he was speaking of only two lives—and, at those ages where the table departed from Makeham's law, one got a gradual but not very rapid departure from those mean ages as determined by the Table of Uniform Seniority, and by tabulating those departures from the normal mean age one was able very simply to interpolate and get close results. That, of course, was in principle what the author had done in the last part of his paper. In the case of the H^M or the O^M Table, there was already a complete set of joint-life annuities calculated and it might be sometimes convenient, if a three or four-life annuity was required, to employ those already calculated results, and perhaps he might be allowed to refer to a paper in the *Journal* (Vol. xxiii, p. 274) in which he had shown how annuities on three or four lives could be obtained with very considerable accuracy. Even in a table like the Carlisle, for instance, which was very badly graduated, errors of only about 2 in the second place of decimals were obtained. In

the annuity, the old principle of substituting one status for another might be to some extent carried further, and it would lead to the result that in lieu of a table such as the H^M or the O^M or the $O^{(af)}$ not following strictly Makeham's law, there would be a table adjusted as nearly as possible to represent that table, but following Makeham's law, and there would be substituted then for the annuities in the original table the annuities in the Makeham table which were as nearly equivalent as possible; that was to say, the annuities at the equivalent ages.

He desired to conclude by acknowledging that all those methods of dealing with the particular problem under discussion were practically superseded by Mr. Lidstone's paper, which had the great merit of dealing in a perfectly symmetrical manner with joint-life and last-survivor annuities, whether of two, three or four lives, and of dealing with them on a principle which was so simple that a student could easily understand it, and by a method which required such little calculation that, once the initial tables were computed, it might be applied in a very few minutes to any required problem.

The PRESIDENT said it became his pleasant duty to convey to Mr. Lidstone the members' thanks for the paper he had presented that evening. He was quite sure he could offer Mr. Lidstone the Institute's most hearty congratulations upon an excellent piece of work. His paper had elicited nothing but praise from authorities on the subject, and the discussion which had taken place had been so full and valuable that he was sure Mr. Lidstone would recognise in that fact alone the greatest compliment that could be paid to him.

A hearty vote of thanks was accorded to Mr. Lidstone for his paper.

Mr. LIDSTONE, in reply, after expressing his appreciation of the vote of thanks, said that his experience had been similar to that of Mr. Elderton: on the occasion of his last valuation he had had some very unpleasant quarters of an hour in grappling with last-survivor annuities, and it had occurred to him that the time had come to look into the subject, to see if something could not be done to place the calculation of such annuities on an easier basis. When he arrived at the method given in the paper, he found it was applicable also to joint-life functions, and he considered it was worthy of being developed. He thought if there was anything he was to be congratulated upon, it was that he had been privileged to be the first to stumble upon a method which really lay very near the surface of the subject, and which perhaps for that very reason had been most unaccountably missed by other workers in the same field. When he found that the method seemed to present some capacity for dealing in a systematic way with both last-survivor annuities and joint-life annuities, he thought, like a loyal son of the Institute, the best thing to do was to submit the method at once to the Institute, leaving it to others to push the subject further, and he much hoped that those who had contributed to the discussion would take up the matter and investigate it further on its theoretical side.

Mr. Fraser's remarks were of the greatest interest, but he would not attempt to deal with them at any length on that occasion, because he did not feel competent to do so without a great deal of additional study. He would, therefore, refer only to one or two points which lay on the surface. Mr. Fraser spoke of the errors revealed by his (Mr. Lidstone's) tables of examples as being systematic, but surely any method of interpolation which took in a certain number of orders of differences and excluded others would have a systematic error, because the error would be a systematic function of the neglected orders of differences. In fact if the function itself were a smooth or systematic one, then the part retained in the formula would be a systematic approximation, and the part rejected, which represented the error, would also be systematic. For instance, if a third difference function were taken, and an approximate result were got out by a second difference formula, a systematic error, being a function of the third difference, would naturally follow. The reason why there were errors in different directions in the last-survivor annuities and the joint lives was simply because the error involved in the approximation was much less for the sum of the single lives than it was for the joint lives. As the joint life occurred negatively in one case, and positively in the other, the error also changed its sign. He was not able to follow fully the explanation of Mr. Fraser with regard to the advantages of the different curves in the three-life cases, but he hoped to give that point the study it deserved when he had seen Mr. Fraser's remarks in print. In the meantime, he would only say that a number of other experiments, besides the particular curves selected in the paper for the interpolation, had been made, and he hoped that Mr. A. E. King, who had helped him so much with the present paper, would on a future occasion give some of the results. On the whole they had come provisionally to the conclusion that better results would not be obtained by the alternative curves than by those that had actually been used in the paper.

Various other methods had been suggested during the course of the discussion. He might say, frankly, that he held no particular brief for his own method, and he was not prepared to defend it against all comers. He hoped the result of the paper might be that other methods would be investigated and developed; and if these presented greater practical advantages, he, for one, should adopt them and not the one described in the paper. But, so far as he could see at present, the method he had developed had the great practical advantage that it was perfectly systematic. In form it was so simple that any intelligent clerk could apply it without any knowledge of anything else than that certain coefficients had to be multiplied by certain corrections taken from the table. It involved also comparatively little calculation or tabulation to arrive at the results. He had considered the question which Mr. Hardy had raised, and, he thought, Mr. Fraser also, as to whether he could not tabulate equivalent equal ages instead of annuity-values. That no doubt

could be done, but on the whole he had come to the conclusion that more advantages were derived from dealing with the annuity-values as presented in his paper, because then one had one set of coefficients and a very simple set of functions to calculate, whereas by the other plan one would have, after calculating annuity-values, to find a number of ages, which, he thought, would present a good deal of additional work, and would have to be determined separately for every table and every rate of interest. In conclusion, he again thanked the members for the very kind reception given to his paper.

[NOTE.—Mr. Fraser's discussion of certain points arising on the paper has been omitted at his request, as he proposes to develop his views in a later number of the *Journal*. The author of the paper also intends to contribute some more detailed comments on the discussion.—ED.]

NATIONAL INSURANCE ACT.

REPORT BY MR. G. F. HARDY, F.I.A., AND MR. F. B. WYATT, F.I.A.,
ON THE REVISION OR EXTENSION OF THE ORIGINAL ESTIMATES,
CAUSED BY CHANGES EFFECTED IN COMMITTEE IN PART I OF
THE BILL.

TO THE LORDS COMMISSIONERS OF HIS MAJESTY'S TREASURY.

MAY IT PLEASE YOUR LORDSHIPS,

28 November 1911.

1. We have at your Lordships' request revised certain tables and estimates contained in our Report of the 20 May last upon the case submitted to us in connection with the National Insurance Bill as introduced, and have made such corrections therein as are rendered necessary by the changes embodied in the Bill as it left the Committee on the 21st instant. The changes effected in the Bill, which render necessary a revision or extension of our original estimates, are :

- (1) Those relating to the population coming within the scope of the scheme.
- (2) Those involving separate estimates for England, Wales, Scotland and Ireland in lieu of the original estimates for the United Kingdom as a whole.
- (3) The alterations that have been made in the benefits payable to insured persons.

POPULATION COMING UNDER THE SCHEME.

2. As regards the population included in the Bill the principal change is the omission of young persons under 16 and the inclusion as voluntary insured persons, upon a reduced scale of contributions and benefits, of married women, not being "employed persons",

who have been contributors prior to marriage. As regards young persons under 16 these were separately dealt with in our Report of the 20 May and their exclusion will not affect the tables in that report showing the estimated number of compulsory and voluntary contributors as at 1 May 1912. It is now proposed that the Act should come into operation at a later date, and we have, for the purpose of estimating the income and expenditure under the Bill, assumed, in accordance with instructions, that this date will be the 1 July 1912. We have not, however, considered it necessary to re-estimate the number of contributors at the outset, as a postponement for a period of two months will not materially affect these. The estimated numbers in the case of England and Wales might be increased by about 2 per 1,000, in the case of Scotland by about 1 per 1,000, while in the case of Ireland, as we have assumed the population to be stationary, there would be no change.

3. Tables I to V appended give separately for England, Wales, Scotland, and Ireland, the estimated number of persons becoming members of approved societies at the commencement of the scheme, distinguishing between compulsory and voluntary contributors, and in the case of women between spinsters, married, and widows. The estimated number of members of approved societies (compulsory and voluntary) in future years are shown in Table VI. The figures in Tables I to VI do not include any estimate in respect of married women who may become voluntary contributors under clause 41, as there is no data by which the probable number of these can be calculated. The figures are also exclusive of the Army and Navy, as there are no means of estimating how many of those in the Services will become members of approved societies. The estimated number of contributors in the Services in various groups is stated in our Report of 26 July last (Table D.).

4. As regards the deposit contributors, we estimate the numbers at the commencement of the scheme, as follows:

	Men	Women	Both Sexes
England . . .	499,000	186,000	685,000
Wales . . .	32,000	11,000	43,000
Scotland . . .	72,000	26,000	98,000
Ireland . . .	35,000	21,000	56,000
United Kingdom . .	638,000	244,000	882,000

As pointed out in paragraph 66 of our Report of 20 May last, it is impossible to make an estimate of any value as to the probable number of deposit contributors in future years.

BENEFITS.

5. The "minimum benefits" provided by the Bill, in the case of fully insured persons, are set out in Schedule 4 of the Bill. They

differ from the benefits provided by the Bill in its original form in the following particulars.

Sickness Benefit.—The allowance of 10s. a week in the case of men and 7s. 6d. a week in the case of women during the first 13 weeks of sickness, reckoning from the fourth day, is now extended to the first 26 weeks of sickness, reckoning from the fourth day. We have not, for the purpose of these estimates, taken account of the options specified in the Schedule, as the value of the alternative benefits is, on the average, nearly the same as that of the benefits above mentioned.

This allowance is reduced in the case of unmarried persons under 21, in the case of men, to 6s. a week during the first 13 weeks and 5s. a week during the second 13 weeks, and, in the case of women, to 5s. a week and 4s. a week respectively for these periods.

It is also reduced in the case of persons who are between the ages of 50 and 60 at the time of becoming employed contributors, in the case of men to 7s. a week for the whole period of 26 weeks, and in the case of women to 6s. a week for the whole period, and for persons over 60 at the time of becoming employed contributors, in the case of both men and women to 6s. a week for the first 13 weeks and 5s. a week during the second 13 weeks.

Maternity Benefit.—This benefit is now extended to the wife of an insured person notwithstanding that she is herself an insured person.

The medical and sanatorium benefits remain unchanged, and according to our original instructions have been taken as equivalent to 6s. per head per annum, and 1s. 2d. per head per annum respectively throughout life.

CONTRIBUTIONS.

6. The rates of contribution necessary to provide the benefits for persons entering insurance at age 16, we now estimate as follows :

TABLE A.
Contributions at Age 16 for Minimum Benefit.

Benefit	POUNDS PER ANNUM		PENCE PER WEEK CORRESPONDING THERETO	
	Men	Women	Men	Women
	£	£	d.	d.
(a) Medical	·328	·328	1·51	1·51
(b) Sanatorium	·070	·070	·32	·32
(c) Sickness	·519	·378	2·39	1·74
(d) Disablement	·170	·175	·78	·81
(e) Maternity	·143	·039	·66	·17
Total benefits	1·230	·990	5·66	4·55
Cost of administration	·200	·200	·92	·92
Total	1·430	1·190	6·58	5·47

These contributions are to be payable up to age 70, and are computed on the assumption that they will not be payable during the sickness or unemployment of the contributor.

7. In estimating these contributions the same assumptions have been made as to the rates of mortality and sickness as in our previous Report. As stated in paragraph 45 of that Report we consider that any saving that may be effected by the sickness benefit not commencing until the fourth day of sickness must be kept as a margin, and without this margin we do not consider that the rates of sickness employed in our calculations are applicable to the conditions of a national scheme as set out in the Bill. As regards the estimated cost of the medical and sanatorium benefits we are of opinion that no larger sum than that assumed, namely, 7s. 2d. per annum for these combined benefits, can safely be set aside out of the proposed contributions, and that the whole margin between the estimated contributions and those actually payable under the Bill should be regarded as available to meet the heavier rates of sickness and disablements which must be expected in a certain number of societies. Unless this margin is retained it is probable that a considerable number of societies will show deficiencies upon valuation, and this might endanger the success of the scheme.

INITIAL RESERVE VALUES.

8. The aggregate initial reserves, computed in accordance with the principles explained in paragraphs 73 and 74 of our Report of 20 May last, we estimate as shown in the following Table B. These sums include the estimated capitalised liability in respect of women, married at the commencement of the scheme, who will eventually come into insurance during widowhood with title to full benefits.

TABLE B.

Estimated Aggregate Initial Reserve Values.

	Men	Women	Both Sexes
	£	£	£
England	36,700,000	13,678,900	50,378,900
Wales	2,301,000	780,200	3,081,200
Scotland	5,217,000	1,877,200	7,094,200
Ireland	3,288,000	1,594,600	4,882,600
	47,506,000	17,930,900	65,436,900
Navy and Army . .	1,206,000	...	1,206,000
Totals	48,712,000	17,930,900	66,642,900

9. The estimated number of years required to liquidate the initial deficiencies represented by the above aggregate reserve values is as follows:

	Men	Women	Both Sexes
England	18 $\frac{1}{2}$	17 $\frac{1}{2}$	18 $\frac{1}{4}$
Wales	18 $\frac{1}{4}$	17 $\frac{1}{2}$	18 $\frac{1}{2}$
Scotland	18 $\frac{1}{4}$	16 $\frac{1}{4}$	17 $\frac{3}{4}$
Ireland	18 $\frac{1}{2}$	19 $\frac{3}{4}$	19
United Kingdom .	18 $\frac{1}{2}$	17 $\frac{1}{2}$	18 $\frac{1}{4}$

In computing these terms of years the figures for the Navy and Army have been included in the figures for England.

INCOME FROM CONTRIBUTIONS.

10. The estimated aggregate contributions to be received from employers and insured persons who are members of approved societies in successive years is shown for each country in Table VII. The figures there given, however, include the special contributions of 1*d.* per week per head, to be paid by the State in the cases of employed persons whose wages are under 2*s.* a day, as provided in clause 6 of the Bill.

ANNUAL COST OF BENEFITS.

11. Tables VIII to XI show the estimated annual cost in respect of the "minimum benefits" for England, Wales, Scotland, and Ireland. These figures do not include the sums that will be payable in respect of the maternity benefit in the case of the Navy and Army, and which may be taken approximately at £28,000 per annum.

The estimated annual cost of the "maximum benefits", including cost of administration, is shown in Table XII. The principle upon which these "maximum benefits" are determined is explained in paragraph 82 in our Report of 20 May last. As in the case of the "minimum" benefits they do not include the figures for the Navy and Army.

MARRIED WOMEN VOLUNTARY INSURERS.

12. Neither the statement of contributions given in Table VII, nor the statements of the cost of benefits in Tables VIII to XII take any account of those women contributors who, upon marriage, may elect to continue insurance under clause 41, upon the reduced scale of contributions and benefits provided for this class. There are, obviously, no means of estimating what proportion of women will so elect to continue insured, but the number may be considerable. These insurances will not affect the amount of the initial deficiencies represented by the aggregate reserve values in Table B, nor the terms required to liquidate these deficiencies. They will, however,

affect the amount of the annual State subsidy, representing one-fourth of the annual payments for "maximum" benefits.

On the basis of the rates of marriage of spinsters and widows respectively, given in our Report of 20 May last, the annual number of marriages among contributors to the scheme, where the married woman is not an "employed person", will be approximately 150,000, and if in one-half of these cases the contributor elects to continue insured under clause 41 there will, in the course of 10 years, be nearly 700,000 such insurances subsisting, representing an annual income from contributions of about £450,000, and an expenditure on account of benefits of somewhat less than this sum.

AMOUNT OF STATE SUBSIDY.

13. The estimated annual charge in respect of the State proportion of the "maximum" benefits and cost of administration in the four countries is shown in Table XIII. The figures there given are exclusive of those having reference to the Navy and Army and to the additional State contributions of 1*d.* a week in the case of employed persons whose wages are under 2*s.* a day. These sums are, however, given in Table XIV showing the annual amounts of the State subsidy for the United Kingdom. The figures for the additional contributions in respect of low-paid labour have been deduced from figures furnished to us by the Local Government Board, but we have no means of distributing them among the four countries.

14. The amount of the State subsidy, as shown in Table XIV, will also be increased by the State proportion of the benefits paid out to deposit contributors and of the benefits paid to married women voluntarily insured. As regards the first of these items it is not practicable to determine what proportion of the contributions paid in to the Post Office by deposit contributors will be claimed in the form of benefits. In the early years of the scheme's operation the sums so paid out must necessarily be less, and may be considerably less, than the contributions paid in, but in later years the outgo may very well reach the level of the income, or possibly exceed it. As already stated, however, there are no means by which the number of deposit contributors in future years can be estimated. On the basis of the estimated numbers at the commencement of the scheme, as given on page 1 of the Report, it would seem probable that the annual cost to the State in respect of deposit contributors will not exceed £250,000 per annum, and is not likely to exceed about half that sum in the first financial year.

15. The cost of the State proportion of benefits in the case of married women voluntary insurers has not been included in Table XIV owing to the impossibility of making any trustworthy estimate. In the nature of the case the cost must be insignificant at the outset, but, on the assumption that one-half of those who upon marriage are entitled to remain assured upon the reduced scale of benefits will so elect, the cost to the State 10 years from the commencement of the scheme may reach about £150,000 per annum, the exact sum depending not only upon the numbers exercising their option to remain insured but also upon their average age at marriage.

16. If rough estimates of the cost made in the above manner of the State proportion of benefits in the case of deposit contributors and married women voluntary insurers may be taken as reasonable, the result will be to raise the estimated cost to the State, as shown in Table XIV, approximately to the following amounts:

Estimated Total Cost of State Subsidy.

						£
1912-13	1,600,000
1913-14	4,050,000
1922-23	5,780,000

17. In submitting the above further estimates we wish to point out that they must be taken in connection with the various qualifications described in our Report of 20 May last.

We have the honour to be,
Your Lordships' obedient Servants,

GEORGE F. HARDY,)
FRANK B. WYATT,) *Actuaries.*

TABLE I.

Estimated Number of Persons becoming Members of Approved Societies as at 1 May 1912.

ENGLAND.

Men.

Ages	Compulsory	Voluntary	Both Classes
16-20	917,000	11,300	928,300
20-25	1,034,000	27,300	1,061,300
25-30	944,000	44,200	988,200
30-35	868,000	58,300	926,300
35-40	769,000	69,600	838,600
40-45	659,000	75,300	734,300
45-50	545,000	35,800	580,800
50-55	425,000	25,400	450,400
55-60	310,000	16,900	326,900
60-65	211,000	10,400	221,400
	6,682,000	374,500	7,056,500

Women.

Ages	COMPULSORY				VOLUNTARY			Both Classes
	Spinsters	Married	Widows	Total	Spinsters	Widows	Total	
16-20	689,000	6,000	...	695,000	18,900	...	18,900	713,900
20-25	628,000	34,000	2,000	664,000	22,600	100	22,700	686,700
25-30	329,000	58,000	8,000	395,000	18,400	500	18,900	413,900
30-35	180,000	72,000	17,000	269,000	16,500	1,500	18,000	287,000
35-40	108,000	71,000	26,000	205,000	14,500	3,500	18,000	223,000
40-45	70,000	60,000	36,000	166,000	12,600	6,300	18,900	184,900
45-50	46,000	48,000	42,000	136,000	4,900	4,500	9,400	145,400
50-55	30,000	35,000	45,000	110,000	3,400	5,100	8,500	118,500
55-60	18,000	23,000	41,000	82,000	1,700	4,000	5,700	87,700
60-65	11,000	13,000	36,000	60,000	900	2,900	3,800	63,800
	2,109,000	420,000	253,000	2,782,000	114,400	28,400	142,800	2,924,800

TABLE II.

Estimated Number of Persons becoming Members of Approved Societies as at 1 May 1912.

WALES.

Men.

Ages	Compulsory	Voluntary	Both Classes
16-20	57,000	700	57,700
20-25	66,000	1,700	67,700
25-30	59,000	2,800	61,800
30-35	54,000	3,700	57,700
35-40	48,000	4,400	52,400
40-45	41,000	4,700	45,700
45-50	34,000	2,200	36,200
50-55	27,000	1,600	28,600
55-60	20,000	1,100	21,100
60-65	13,000	600	13,600
	419,000	23,500	442,500

Women.

Ages	COMPULSORY				VOLUNTARY			Both Classes
	Spinsters	Married	Widows	Total	Spinsters	Widows	Total	
16-20	39,000	39,000	1,100	...	1,100	40,100
20-25	36,000	2,000	...	38,000	1,300	...	1,300	39,300
25-30	19,000	3,000	...	22,000	1,100	...	1,100	23,100
30-35	10,000	4,000	1,000	15,000	900	100	1,000	16,000
35-40	6,000	4,000	2,000	12,000	800	200	1,000	13,000
40-45	4,000	4,000	2,000	10,000	700	400	1,100	11,100
45-50	3,000	3,000	2,000	8,000	300	300	600	8,600
50-55	2,000	2,000	2,000	6,000	200	300	500	6,500
55-60	1,000	1,000	2,000	4,000	100	200	300	4,300
60-65	1,000	1,000	2,000	4,000	...	200	200	4,200
	121,000	24,000	13,000	158,000	6,500	1,700	8,200	166,200

TABLE III.

Estimated Number of Persons becoming Members of Approved Societies as at 1 May 1912.

SCOTLAND.

Men.

Ages	Compulsory	Voluntary	Both Classes
16-20	148,000	2,000	150,000
20-25	163,000	4,000	167,000
25-30	143,000	5,000	148,000
30-35	125,000	6,000	131,000
35-40	110,000	7,000	117,000
40-45	93,000	8,000	101,000
45-50	75,000	4,000	79,000
50-55	59,000	3,000	62,000
55-60	44,000	2,000	46,000
60-65	29,000	1,000	30,000
	989,000	42,000	1,031,000

Women.

Ages	Compulsory				Voluntary			Both Classes
	Spinsters	Married	Widows	Total	Spinsters	Widows	Total	
16-20	109,000	1,000	...	110,000	4,000	...	4,000	114,000
20-25	95,000	5,000	...	100,000	5,000	...	5,000	105,000
25-30	50,000	9,000	1,000	60,000	3,900	100	4,000	64,000
30-35	26,000	10,000	2,000	38,000	2,700	300	3,000	41,000
35-40	14,000	10,000	3,000	27,000	2,400	600	3,000	30,000
40-45	9,000	8,000	4,000	21,000	2,000	1,000	3,000	24,000
45-50	6,000	6,000	5,000	17,000	500	500	1,000	18,000
50-55	4,000	4,000	6,000	14,000	400	600	1,000	15,000
55-60	2,000	3,000	6,000	11,000	200	500	700	11,700
60-65	1,000	2,000	5,000	8,000	100	200	300	8,300
	316,000	58,000	32,000	406,000	21,200	3,800	25,000	431,000

TABLE IV.

Estimated Number of Persons becoming Members of Approved Societies as at 1 May 1912.

IRELAND.

Men.

Ages	Compulsory	Voluntary	Both Classes
16-20	70,000	15,000	85,000
20-25	77,000	22,000	99,000
25-30	65,000	24,000	89,000
30-35	62,000	27,000	89,000
35-40	54,000	29,000	83,000
40-45	46,000	30,000	76,000
45-50	40,000	14,000	54,000
50-55	32,000	11,000	43,000
55-60	24,000	8,000	32,000
60-65	19,000	5,000	24,000
	489,000	185,000	674,000

Women.

Ages	Compulsory				Voluntary			Both Classes
	Spinsters	Married	Widows	Total	Spinsters	Widows	Total	
15-20	54,000	54,000	2,000	...	2,000	56,000
20-25	62,000	3,000	...	65,000	3,000	...	3,000	68,000
25-30	35,000	6,000	1,000	42,000	1,900	100	2,000	44,000
30-35	21,000	8,000	2,000	31,000	2,700	300	3,000	34,000
35-40	13,000	8,000	3,000	24,000	3,200	800	4,000	28,000
40-45	8,000	7,000	4,000	19,000	3,300	1,700	5,000	24,000
45-50	5,000	5,000	5,000	15,000	1,600	1,400	3,000	18,000
50-55	4,000	4,000	5,000	13,000	1,200	1,800	3,000	16,000
55-60	2,000	3,000	6,000	11,000	600	1,400	2,000	13,000
60-65	1,000	2,000	5,000	8,000	200	800	1,000	9,000
	205,000	46,000	31,000	282,000	19,700	8,300	28,000	310,000

TABLE V.

Estimated Number of Persons becoming Members of Approved Societies as at 1 May 1912.

UNITED KINGDOM.

Men.

Ages	Compulsory	Voluntary	Both Classes
16-20	1,192,000	29,000	1,221,000
20-25	1,340,000	55,000	1,395,000
25-30	1,211,000	76,000	1,287,000
30-35	1,109,000	95,000	1,204,000
35-40	981,000	110,000	1,091,000
40-45	839,000	118,000	957,000
45-50	694,000	56,000	750,000
50-55	543,000	41,000	584,000
55-60	398,000	28,000	426,000
60-65	272,000	17,000	289,000
	8,579,000	625,000	9,204,000

Women.

Ages	COMPULSORY				VOLUNTARY			Both Classes
	Spinsters	Married	Widows	Total	Spinsters	Widows	Total	
16-20	891,000	7,000	...	898,000	26,000	...	26,000	924,000
20-25	821,000	44,000	2,000	867,000	31,900	100	32,000	899,000
25-30	433,000	76,000	10,000	519,000	25,300	700	26,000	545,000
30-35	237,000	94,000	22,000	353,000	22,800	2,200	25,000	378,000
35-40	141,000	93,000	34,000	268,000	20,900	5,100	26,000	294,000
40-45	91,000	79,000	46,000	216,000	18,600	9,400	28,000	244,000
45-50	60,000	62,000	54,000	176,000	7,300	6,700	14,000	190,000
50-55	40,000	45,000	58,000	143,000	5,200	7,800	13,000	156,000
55-60	23,000	30,000	55,000	108,000	2,600	6,100	8,700	116,700
60-65	14,000	18,000	48,000	80,000	1,200	4,100	5,300	85,300
	2,751,000	548,000	329,000	3,628,000	161,800	42,200	204,000	3,832,000

TABLE VI.

Estimated Number of Members of Approved Societies (Contributors under age 70).

ENGLAND.

Year	COMPULSORY			VOLUNTARY		
	Men	Women	Both Sexes	Men	Women	Both Sexes
1912-13	6,682,000	2,782,000	9,464,000	375,000	143,000	518,000
1917-18	7,468,000	3,083,000	10,551,000	376,000	139,000	515,000
1922-23	8,118,000	3,347,000	11,465,000	362,000	137,000	499,000
1927-28	8,756,000	3,610,000	12,366,000	346,000	135,000	481,000
1932-33	9,373,000	3,873,000	13,246,000	328,000	134,000	462,000

WALES.

1912-13	419,000	158,000	577,000	23,000	8,000	31,000
1917-18	468,000	175,000	643,000	24,000	8,000	32,000
1922-23	509,000	192,000	701,000	23,000	7,000	30,000
1927-28	549,000	206,000	755,000	22,000	7,000	29,000
1932-33	588,000	220,000	808,000	21,000	7,000	28,000

SCOTLAND.

1912-13	989,000	406,000	1,395,000	42,000	25,000	67,000
1917-18	1,102,000	449,000	1,551,000	41,000	23,000	64,000
1922-23	1,197,000	482,000	1,679,000	40,000	23,000	63,000
1927-28	1,284,000	518,000	1,802,000	39,000	23,000	62,000
1932-33	1,372,000	553,000	1,925,000	34,000	23,000	57,000

IRELAND.

1912-13	489,000	282,000	771,000	185,000	28,000	213,000
1917-18	548,000	313,000	861,000	179,000	28,000	207,000
1922-23	588,000	334,000	922,000	174,000	29,000	203,000
1927-28	622,000	353,000	975,000	166,000	27,000	193,000
1932-33	651,000	369,000	1,020,000	157,000	26,000	183,000

UNITED KINGDOM.

1912-13	8,579,000	3,628,000	12,207,000	625,000	204,000	829,000
1917-18	9,586,000	4,020,000	13,606,000	620,000	198,000	818,000
1922-23	10,412,000	4,355,000	14,767,000	599,000	196,000	795,000
1927-28	11,211,000	4,687,000	15,898,000	573,000	192,000	765,000
1932-33	11,984,000	5,015,000	16,999,000	540,000	190,000	730,000

TABLE VII.

The Aggregate Contributions to be Received from Employers and Insured Persons, i.e., Members of Approved Societies, in the under-mentioned years.

ENGLAND.

Year	COMPULSORY			VOLUNTARY			Total, Both Classes
	Men	Women	Both Sexes	Men	Women	Both Sexes	
	£	£	£	£	£	£	£
1912-13	7,097,000	2,543,000	9,640,000	196,000	162,400	658,400	10,298,400
1913-14	9,672,000	3,460,000	13,132,000	657,000	214,300	871,300	14,003,300
1914-15	9,880,000	3,531,000	13,411,000	653,000	212,300	865,300	14,276,300
1915-16	10,000,000	3,600,000	13,690,000	649,000	210,400	859,400	14,549,400
1916-17	10,299,000	3,679,000	13,969,000	641,000	208,600	849,600	14,818,600
1917-18	10,494,000	3,738,000	14,232,000	633,000	206,900	839,900	15,071,900
1922-23	11,384,000	4,055,000	15,439,000	570,000	190,200	760,200	16,199,200
1927-28	12,254,000	4,363,000	16,617,000	514,000	175,400	689,400	17,306,400
1932-33	13,101,000	4,673,000	17,774,000	453,000	166,400	619,400	18,393,400

WALES.

1912-13	445,000	145,000	590,000	31,000	9,300	40,300	630,300
1913-14	606,000	198,000	804,000	41,000	12,300	53,300	857,300
1914-15	620,000	202,000	822,000	41,000	12,100	53,100	875,100
1915-16	632,000	206,000	838,000	40,000	12,000	52,000	890,000
1916-17	646,000	209,000	855,000	40,000	11,900	51,900	906,900
1917-18	658,000	213,000	871,000	40,000	11,800	51,800	922,800
1922-23	714,000	231,000	945,000	36,000	10,900	46,900	991,900
1927-28	768,000	249,000	1,017,000	32,000	10,000	42,000	1,059,000
1932-33	821,000	266,000	1,087,000	28,000	9,500	37,500	1,124,500

SCOTLAND.

1912-13	1,050,000	371,000	1,421,000	56,000	28,100	84,100	1,505,100
1913-14	1,431,000	506,000	1,937,000	74,000	37,400	111,400	2,048,400
1914-15	1,461,000	516,000	1,977,000	73,000	37,400	110,400	2,087,400
1915-16	1,492,000	526,000	2,018,000	72,000	37,500	109,500	2,127,500
1916-17	1,522,000	536,000	2,058,000	71,000	37,300	108,300	2,166,300
1917-18	1,551,000	546,000	2,097,000	69,000	37,400	106,400	2,203,400
1922-23	1,679,000	583,000	2,262,000	61,000	34,700	95,700	2,357,700
1927-28	1,799,000	628,000	2,427,000	56,000	32,300	88,300	2,515,300
1932-33	1,919,000	671,000	2,590,000	47,000	31,400	78,400	2,668,400

IRELAND.

1912-13	410,000	193,000	603,000	198,000	29,800	227,800	830,800
1913-14	557,000	263,000	820,000	261,000	39,400	300,400	1,120,400
1914-15	568,000	269,000	837,000	258,000	39,300	297,300	1,134,300
1915-16	581,000	275,000	856,000	255,000	39,000	294,000	1,150,000
1916-17	593,000	280,000	873,000	253,000	38,700	291,700	1,164,700
1917-18	604,000	284,000	888,000	251,000	38,500	289,500	1,177,500
1922-23	644,000	303,000	947,000	226,000	33,500	259,500	1,206,500
1927-28	680,000	318,000	998,000	199,000	28,700	227,700	1,225,700
1932-33	710,000	330,000	1,040,000	175,000	24,800	199,800	1,239,800

UNITED KINGDOM.

1912-13	9,002,000	3,252,000	12,254,000	781,000	229,600	1,010,600	13,264,600
1913-14	12,266,000	4,427,000	16,693,000	1,033,000	303,400	1,336,400	18,029,400
1914-15	12,529,000	4,518,000	17,047,000	1,025,000	301,100	1,326,100	18,373,100
1915-16	12,795,000	4,607,000	17,402,000	1,016,000	298,900	1,314,900	18,716,900
1916-17	13,060,000	4,635,000	17,695,000	1,005,000	296,500	1,301,500	19,056,500
1917-18	13,307,000	4,781,000	18,088,000	993,000	294,600	1,287,600	19,375,600
1922-23	14,421,000	5,172,000	19,593,000	893,000	269,300	1,162,300	20,755,300
1927-28	15,501,000	5,558,000	21,059,000	801,000	246,400	1,047,400	22,106,400
1932-33	16,551,000	5,940,000	22,491,000	703,000	232,100	935,100	23,426,100

TABLE VIII.

Total Amount required in each of the under-mentioned financial years to meet the Cost of the various "Minimum Benefits."

ENGLAND.

Men (Compulsory).

Year	BENEFITS					Total Minimum Benefits	Cost of Administration	Total Outgo
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>			
	£	£	£	£	£	£	£	
1912-13	507,000	317,000	751,000	...	231,000	1,806,000	933,000	2,739,000
1913-14	2,075,000	432,000	3,119,000	...	940,000	6,566,000	1,271,000	7,837,000
1914-15	2,123,000	442,000	3,233,000	852,000	954,000	7,604,000	1,299,000	8,903,000
1915-16	2,170,000	452,000	3,348,000	1,228,000	968,000	8,166,000	1,326,000	9,492,000
1916-17	2,217,000	462,000	3,463,000	1,320,000	983,000	8,445,000	1,353,000	9,798,000
1917-18	2,261,000	471,000	3,635,000	1,388,000	997,000	8,755,000	1,380,000	10,135,000
1922-23	2,192,000	519,000	1,285,000	1,601,000	1,076,000	9,973,000	1,496,000	11,469,000
1927-28	2,708,000	565,000	4,629,000	1,810,000	1,146,000	10,858,000	1,610,000	12,468,000
1932-33	2,913,000	607,000	5,001,000	2,007,000	1,222,000	11,750,000	1,721,000	13,471,000

Men (Voluntary).

1912-13	27,200	17,900	48,900	94,000	51,800	145,800
1913-14	112,900	23,500	201,400	...	42,300	380,100	68,700	448,800
1914-15	112,900	23,500	205,100	52,700	55,500	449,700	68,700	518,400
1915-16	112,900	23,500	208,900	76,300	53,600	475,200	68,700	543,900
1916-17	112,900	23,500	212,700	81,900	51,800	482,800	68,700	551,500
1917-18	112,900	23,500	215,500	85,700	50,800	488,400	67,800	556,200
1922-23	110,100	22,600	223,000	103,600	41,400	500,700	64,900	565,600
1927-28	106,300	21,700	228,700	121,400	33,900	512,000	62,100	574,100
1932-33	98,800	20,500	229,600	140,200	30,100	519,200	58,300	577,500

Women (Compulsory).

1912-13	211,000	133,000	222,000	...	30,000	596,000	390,000	986,000
1913-14	863,000	179,000	917,000	...	124,000	2,083,000	531,000	2,614,000
1914-15	881,000	183,000	946,000	239,000	127,000	2,376,000	542,000	2,918,000
1915-16	898,000	185,000	974,000	346,000	129,000	2,532,000	552,000	3,084,000
1916-17	916,000	190,000	998,000	370,000	131,000	2,605,000	563,000	3,168,000
1917-18	935,000	194,000	1,029,000	392,000	133,000	2,683,000	573,000	3,256,000
1922-23	1,024,000	213,000	1,167,000	448,000	140,000	2,992,000	622,000	3,614,000
1927-28	1,103,000	231,000	1,274,000	514,000	150,000	3,278,000	669,000	3,947,000
1932-33	1,195,000	249,000	1,393,000	579,000	161,000	3,577,000	717,000	4,294,000

Women (Voluntary).

1912-13	10,700	6,600	13,000	30,300	19,600	49,900
1913-14	42,800	8,900	52,100	103,800	26,000	129,800
1914-15	42,700	8,900	52,500	17,000	...	121,100	26,000	147,100
1915-16	42,600	8,900	53,000	24,500	...	129,000	25,900	154,900
1916-17	42,500	8,900	53,400	26,100	...	130,900	25,800	156,700
1917-18	42,400	8,900	53,800	27,700	...	132,800	25,700	158,500
1922-23	42,500	8,800	57,100	31,300	...	139,700	25,400	165,100
1927-28	42,400	8,800	58,400	34,200	...	143,800	25,000	168,000
1932-33	42,500	8,900	59,000	38,200	...	148,900	24,700	173,600

TABLE IX.

Total Amount required in each of the undermentioned financial years to meet the Cost of the various "Minimum Benefits."

WALES.
Men (Compulsory).

Year	BENEFITS					Total Minimum Benefits	Cost of Administration	Total Outgo
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>			
	£	£	£	£	£	£	£	£
1912-13	32,000	13,000	47,000	...	15,000	107,000	58,000	165,000
1913-14	130,000	27,000	195,000	...	59,000	411,000	80,000	491,000
1914-15	133,000	28,000	203,000	53,000	60,000	477,000	81,000	558,000
1915-16	136,000	28,000	210,000	77,000	61,000	512,000	83,000	595,000
1916-17	139,000	29,000	217,000	83,000	61,000	529,000	85,000	614,000
1917-18	142,000	30,000	228,000	87,000	62,000	549,000	86,000	635,000
1922-23	156,000	33,000	269,000	100,000	67,000	625,000	94,000	719,000
1927-28	170,000	35,000	290,000	113,000	72,000	680,000	101,000	781,000
1932-33	183,000	38,000	314,000	126,000	77,000	738,000	108,000	846,000

Men (Voluntary).

1912-13	1,800	1,100	3,100	6,000	3,200	9,200
1913-14	7,100	1,500	12,600	...	2,700	23,900	4,300	28,200
1914-15	7,100	1,500	12,900	3,300	3,500	28,300	4,300	32,600
1915-16	7,100	1,500	13,100	4,700	3,400	29,800	4,300	34,100
1916-17	7,100	1,500	13,300	5,100	3,200	30,200	4,300	34,500
1917-18	7,100	1,500	13,500	5,300	3,200	30,600	4,200	34,800
1922-23	6,900	1,400	14,000	6,100	2,600	31,300	4,100	35,400
1927-28	6,700	1,300	14,300	7,600	2,100	32,000	3,900	35,900
1932-33	6,200	1,200	14,400	8,800	1,900	32,500	3,700	36,200

Women (Compulsory).

1912-13	12,000	8,000	13,000	...	2,000	35,000	22,000	57,000
1913-14	49,000	10,000	52,000	...	7,000	118,000	30,000	148,000
1914-15	50,000	10,000	54,000	14,000	7,000	135,000	31,000	166,000
1915-16	51,000	11,000	56,000	20,000	7,000	145,000	32,000	177,000
1916-17	52,000	11,000	57,000	21,000	8,000	149,000	32,000	181,000
1917-18	52,000	11,000	59,000	22,000	8,000	152,000	33,000	185,000
1922-23	58,000	12,000	67,000	26,000	8,000	171,000	35,000	206,000
1927-28	63,000	13,000	73,000	29,000	9,000	187,000	38,000	225,000
1932-33	68,000	14,000	80,000	33,000	9,000	204,000	41,000	245,000

Women (Voluntary).

1912-13	600	400	700	1,700	1,100	2,800
1913-14	2,400	500	3,000	5,900	1,500	7,400
1914-15	2,400	500	3,000	1,000	...	6,900	1,500	8,400
1915-16	2,400	500	3,000	1,400	...	7,300	1,500	8,800
1916-17	2,400	500	3,100	1,500	...	7,500	1,500	9,000
1917-18	2,400	500	3,100	1,600	...	7,600	1,500	9,100
1922-23	2,400	500	3,300	1,800	...	8,000	1,400	9,400
1927-28	2,400	500	3,300	2,000	...	8,200	1,400	9,600
1932-33	2,400	500	3,400	2,200	...	8,500	1,400	9,900

TABLE X.

Total Amount required in each of the undermentioned financial years to meet the Cost of the various "Minimum Benefits."

SCOTLAND.

Men (Compulsory).

Year	BENEFITS					Total Minimum Benefits	Cost of Adminis- tration	Total Outgo
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>			
	£	£	£	£	£	£	£	£
1912-13	75,000	46,000	109,000	...	34,000	264,000	138,000	402,000
1913-14	398,000	64,000	454,000	...	138,000	964,000	186,000	1,150,000
1914-15	314,000	65,000	471,000	120,000	140,000	1,110,000	192,000	1,302,000
1915-16	321,000	67,000	488,000	173,000	141,000	1,190,000	196,000	1,386,000
1916-17	328,000	68,000	505,000	185,000	144,000	1,230,000	200,000	1,430,000
1917-18	334,000	70,000	524,000	194,000	147,000	1,269,000	204,000	1,473,000
1922-23	367,000	77,000	614,000	224,000	159,000	1,441,000	221,000	1,662,000
1927-28	396,000	83,000	666,000	251,000	170,000	1,566,000	236,000	1,802,000
1932-33	425,000	89,000	718,000	277,000	178,000	1,687,000	252,000	1,939,000

Men (Voluntary).

1912-13	3,000	2,000	5,000	10,000	6,000	16,000
1913-14	13,000	3,000	21,000	...	4,000	41,000	8,000	49,000
1914-15	13,000	3,000	21,000	5,000	6,000	48,000	8,000	56,000
1915-16	13,000	3,000	22,000	8,000	6,000	52,000	8,000	60,000
1916-17	12,000	3,000	23,000	9,000	6,000	53,000	8,000	61,000
1917-18	12,000	3,000	23,000	9,000	6,000	53,000	8,000	61,000
1922-23	12,000	2,000	23,000	10,000	5,000	52,000	7,000	59,000
1927-28	12,000	2,000	23,000	12,000	3,000	52,000	7,000	59,000
1932-33	11,000	2,000	23,000	12,000	3,000	51,000	6,000	57,000

Women (Compulsory).

1912-13	30,000	19,000	31,000	...	13,000	93,000	56,000	149,000
1913-14	124,000	26,000	132,000	...	18,000	300,000	78,000	378,000
1914-15	128,000	27,000	137,000	30,000	18,000	340,000	79,000	419,000
1915-16	129,000	28,000	141,000	44,000	19,000	361,000	82,000	443,000
1916-17	133,000	28,000	144,000	47,000	19,000	371,000	83,000	454,000
1917-18	136,000	28,000	147,000	50,000	19,000	380,000	84,000	464,000
1922-23	147,000	31,000	165,000	57,000	20,000	420,000	89,000	509,000
1927-28	159,000	33,000	180,000	67,000	24,000	463,000	96,000	559,000
1932-33	169,000	35,000	192,000	75,000	24,000	495,000	102,000	597,000

Women (Voluntary).

1912-13	1,500	1,200	2,200	5,200	3,400	8,600
1913-14	7,400	1,600	9,000	18,000	4,700	22,700
1914-15	7,400	1,600	9,200	3,300	...	21,500	4,600	26,100
1915-16	7,400	1,500	9,300	4,800	...	23,000	4,600	27,600
1916-17	7,300	1,500	9,500	5,200	...	23,500	4,500	28,000
1917-18	7,300	1,600	9,700	5,500	...	24,100	4,500	28,600
1922-23	7,500	1,500	9,900	6,100	...	25,000	4,400	29,400
1927-28	7,800	1,600	10,500	6,700	...	26,600	4,500	31,100
1932-33	8,100	1,700	11,500	8,500	...	29,500	4,600	34,400

TABLE XI.

Total Amount required in each of the under-mentioned financial years to meet the cost of the various "Minimum Benefits."

IRELAND.
Men (Compulsory).

Year	BENEFITS				Total Minimum Benefit	Cost of Adminis- tration	Total Outgo
	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>			
	£	£	£	£	£	£	£
1912-13	23,000	55,000	...	16,000	94,000	68,000	162,000
1913-14	31,000	229,000	...	65,000	325,000	93,000	418,000
1914-15	32,000	238,000	70,000	66,000	406,000	95,000	501,000
1915-16	33,000	247,000	102,000	68,000	448,000	97,000	545,000
1916-17	34,000	256,000	111,000	67,000	468,000	99,000	567,000
1917-18	35,000	266,000	117,000	69,000	487,000	101,000	588,000
1922-23	38,000	317,000	131,000	73,000	562,000	108,000	670,000
1927-28	41,000	342,000	151,000	78,000	615,000	114,000	729,000
1932-33	43,000	365,000	168,000	82,000	658,000	119,000	777,000

Men (Voluntary).

1912-13	9,000	23,000	32,000	28,000	60,000
1913-14	11,000	93,000	...	20,000	124,000	33,000	157,000
1914-15	11,000	94,000	25,000	26,000	156,000	33,000	189,000
1915-16	11,000	96,000	35,000	24,000	166,000	33,000	199,000
1916-17	11,000	98,000	37,000	23,000	169,000	33,000	202,000
1917-18	11,000	98,000	39,000	22,000	170,000	33,000	203,000
1922-23	11,000	104,000	49,000	18,000	182,000	32,000	214,000
1927-28	11,000	106,000	58,000	15,000	190,000	30,000	220,000
1932-33	10,000	108,000	66,000	14,000	198,000	28,000	226,000

Women (Compulsory).

1912-13	13,000	23,000	...	4,000	40,000	39,000	79,000
1913-14	18,000	97,000	...	13,000	128,000	53,000	181,000
1914-15	19,000	100,000	29,000	13,000	161,000	54,000	215,000
1915-16	19,000	103,000	43,000	13,000	178,000	57,000	235,000
1916-17	20,000	107,000	45,000	13,000	185,000	57,000	242,000
1917-18	20,000	110,000	48,000	13,000	191,000	58,000	249,000
1922-23	21,000	126,000	56,000	13,000	216,000	63,000	279,000
1927-28	23,000	134,000	64,000	13,000	234,000	65,000	299,000
1932-33	23,000	144,000	71,000	14,000	252,000	68,000	320,000

Women (Voluntary).

1912-13	1,300	2,900	4,200	3,800	8,000
1913-14	1,800	12,000	13,800	5,100	18,900
1914-15	1,800	12,400	4,900	...	19,100	5,200	24,300
1915-16	1,800	12,800	7,000	...	21,600	5,200	26,800
1916-17	1,900	13,200	7,500	...	22,600	5,300	27,900
1917-18	1,900	13,500	8,000	...	23,400	5,400	28,800
1922-23	1,900	14,200	8,900	...	25,000	5,100	30,100
1927-28	1,900	14,100	9,800	...	25,800	5,200	31,000
1932-33	1,800	13,900	11,000	...	26,700	4,800	31,500

TABLE XII.
Annual Expenditure for "Maximum" Benefits and Cost of Administration.

ENGLAND.

Year	MEN		WOMEN		Total
	Compulsory	Voluntary	Compulsory	Voluntary	
	£	£	£	£	£
1912-13	2,914,000	156,200	1,081,000	54,700	4,205,900
1913-14	8,336,000	477,500	2,865,000	142,700	11,821,200
1914-15	9,470,000	551,400	3,199,000	161,800	13,282,200
1915-16	10,097,000	578,500	3,381,000	170,300	14,226,800
1916-17	10,422,000	586,000	3,473,000	172,300	14,653,900
1917-18	10,781,000	591,600	3,569,000	174,300	15,115,900
1922-23	12,200,000	601,600	3,962,000	181,500	16,945,100
1927-28	13,262,400	610,700	4,327,000	185,600	18,385,300
1932-33	14,329,000	614,300	4,707,000	190,800	19,841,100

WALES.

1912-13	176,000	9,800	62,000	3,100	250,900
1913-14	522,000	30,000	162,000	8,100	722,100
1914-15	593,000	34,700	182,000	9,200	818,900
1915-16	633,000	36,300	194,000	9,600	872,900
1916-17	652,000	36,700	198,000	9,900	896,600
1917-18	675,000	37,000	203,000	10,000	925,000
1922-23	764,000	37,600	226,000	10,300	1,037,900
1927-28	830,000	38,100	247,000	10,500	1,125,600
1932-33	900,000	38,500	269,000	10,900	1,218,400

SCOTLAND.

1912-13	428,000	17,000	163,000	9,400	617,400
1913-14	1,223,000	52,000	414,000	25,000	1,714,000
1914-15	1,385,000	60,000	459,000	28,700	1,932,700
1915-16	1,474,000	64,000	486,000	30,400	2,054,400
1916-17	1,521,000	65,000	498,000	30,800	2,114,800
1917-18	1,567,000	65,000	509,000	31,500	2,172,500
1922-23	1,769,000	63,000	558,000	32,300	2,422,300
1927-28	1,917,000	63,000	613,000	34,200	2,627,200
1932-33	2,063,000	61,000	651,000	37,800	2,815,800

IRELAND.

1912-13	172,000	64,000	87,000	8,800	331,800
1913-14	445,600	167,000	198,000	20,800	830,800
1914-15	533,000	201,000	236,000	26,700	996,700
1915-16	578,000	212,000	258,000	29,500	1,077,500
1916-17	603,000	215,000	265,000	30,700	1,113,700
1917-18	625,000	216,000	273,000	31,700	1,145,700
1922-23	711,000	228,000	306,000	33,400	1,278,400
1927-28	776,000	234,000	308,000	34,100	1,372,000
1932-33	826,000	240,000	351,000	34,600	1,451,600

UNITED KINGDOM.

1912-13	3,690,000	247,000	1,393,000	76,000	5,106,000
1913-14	10,526,000	726,500	3,619,000	196,000	15,088,100
1914-15	11,981,000	847,100	4,076,000	226,400	17,130,500
1915-16	12,782,000	890,800	4,319,000	239,800	18,231,600
1916-17	13,198,000	903,300	4,434,000	243,700	18,779,000
1917-18	13,648,000	909,000	4,554,000	247,500	19,359,100
1922-23	15,444,000	930,200	5,052,000	257,500	21,683,700
1927-28	16,785,000	945,800	5,515,000	264,400	23,510,200
1932-33	18,118,000	953,800	5,981,000	274,100	25,326,900

TABLE XIII.

Annual Charge in respect of the State Proportion of "Maximum" Benefits and Cost of Administration.

ENGLAND.

Year	MEN		WOMEN		Sanatoria	Total
	Compulsory	Voluntary	Compulsory	Voluntary		
	£	£	£	£	£	£
1912-13	648,000	34,700	270,000	13,700	31,600	998,000
1913-14	1,852,000	106,100	716,000	35,700	43,000	2,752,800
1914-15	2,104,000	122,500	800,000	40,500	43,800	3,110,800
1915-16	2,243,000	128,600	845,000	42,600	44,800	3,304,000
1916-17	2,316,000	130,300	868,000	43,100	45,700	3,403,100
1917-18	2,396,000	131,500	892,000	43,600	46,600	3,509,700
1922-23	2,711,000	133,700	991,000	45,400	51,600	3,932,100
1927-28	2,947,000	135,700	1,082,000	46,400	55,100	4,266,200
1932-33	3,185,000	136,500	1,177,000	47,700	59,100	4,605,300

WALES.

1912-13	39,000	2,200	16,000	800	1,900	59,900
1913-14	116,000	6,700	41,000	2,000	2,600	168,300
1914-15	132,000	7,700	46,000	2,300	2,700	190,700
1915-16	141,000	8,100	49,000	2,400	2,700	203,200
1916-17	145,000	8,200	50,000	2,500	2,800	208,500
1917-18	150,000	8,200	51,000	2,500	2,800	214,500
1922-23	170,000	8,400	57,000	2,600	3,100	241,100
1927-28	185,000	8,500	62,000	2,600	3,400	261,500
1932-33	200,000	8,600	67,000	2,700	3,600	281,900

SCOTLAND.

1912-13	95,000	4,000	41,000	2,400	4,700	147,100
1913-14	272,000	12,000	101,000	6,300	6,300	409,600
1914-15	308,000	13,000	115,000	7,200	6,400	449,600
1915-16	327,000	14,000	122,000	7,600	6,500	477,100
1916-17	338,000	14,000	125,000	7,700	6,700	491,400
1917-18	348,000	14,000	127,000	7,900	6,800	503,700
1922-23	393,000	14,000	140,000	8,100	7,400	562,500
1927-28	426,000	14,000	153,000	8,600	8,000	609,600
1932-33	458,000	13,000	164,000	9,500	8,500	653,000

IRELAND.

1912-13	51,000	18,000	27,000	2,800	3,100	101,900
1913-14	135,000	50,000	73,000	7,600	4,200	269,800
1914-15	155,000	58,000	83,000	9,000	4,300	309,300
1915-16	166,000	60,000	90,000	9,800	4,400	330,200
1916-17	172,000	60,000	92,000	10,100	4,400	338,500
1917-18	178,000	61,000	94,000	10,400	4,500	347,900
1922-23	201,000	63,000	104,000	10,900	4,800	383,700
1927-28	218,000	64,000	112,000	11,000	5,100	410,100
1932-33	233,000	65,000	119,000	11,000	5,300	433,300

TABLE XIV

Annual Charge in respect of the State Proportion of "Maximum" Benefits and Cost of Administration, including Contributions to the Navy and Army Special Fund and on account of low-paid labour.

UNITED KINGDOM

Year	MEN		WOMEN		Sanatoria	Both Sexes	Navy and Army	LOW-PAID LABOUR		Total
	Compulsory	Voluntary	Compulsory	Voluntary				Men	Women	
1912-13	833,000	58,900	351,000	19,700	41,300	1,306,900	32,000	15,500	115,800	1,470,200
1913-14	2,375,000	171,800	931,000	51,600	56,100	3,591,500	86,000	20,600	154,100	3,852,500
1914-15	2,699,000	201,200	1,044,000	59,000	57,200	4,060,100	88,000	20,600	154,100	4,323,100
1915-16	2,877,000	210,700	1,105,000	62,100	58,400	4,314,500	89,000	20,600	154,100	4,578,500
1916-17	2,971,000	212,500	1,135,000	63,100	59,600	4,441,500	89,000	20,600	154,100	4,705,500
1917-18	3,072,000	214,700	1,164,000	64,400	60,700	4,575,800	89,000	20,600	154,100	4,839,800
1922-23	3,475,000	219,100	1,292,000	67,000	66,300	5,119,400	89,000	20,600	154,100	5,383,100
1927-28	3,776,000	222,200	1,109,000	68,600	71,600	5,547,100	89,000	20,600	154,100	5,811,400
1932-33	4,076,000	223,100	1,527,000	70,900	76,500	5,973,500	89,000	20,600	154,100	6,237,500

REPORT BY MR. T. G. ACKLAND, F.I.A., ON THE ESTIMATED
FINANCIAL EFFECT OF AMENDMENTS INTRODUCED IN COMMITTEE
IN PART II. OF THE BILL.

To Sir H. Llewellyn Smith, K.C.B.,
Permanent Secretary,
Board of Trade.

DEAR SIR,

(1) I now beg to submit, as desired by you, a Report as to the estimated financial effects of the amendments introduced in Committee in Part II. of the National Insurance Bill, so far as the same are ascertainable, or calculable.

In describing the Amendments I have given references, in the first place, to the clauses of the Bill as introduced, and have added, in brackets, the corresponding references to the Bill as amended in Committee.

(2) *Clause 71 ; page 60 ; lines 11 and 12* [Clause 89 ; page 98].—Provision for the allowance of the return, granted to workmen at age 60, to the representatives of those who die after that age.

The calculations upon which the estimated value of the return at age 60 have been based, and from which the estimated figures in my Report of May last have been deduced, are founded on the assumption that the benefit will be paid, provided the workman reaches age 60, and fulfils the other conditions of the Clause. The case of his subsequent death does not therefore affect the financial estimates then made.

(3) *Clause 71 ; page 60* [Clause 89 ; page 99].—New subsection (2), providing for the continuance, as an insured member, of a workman who has attained the age of 60, and has received the return of contributions under subsection (1) of the Clause, and varying the conditions under which he shall receive benefit, in respect of contributions paid before age 60.

The data upon which the estimates made in my Report of May last were based, may be considered as including workmen over 60 years of age, and up to the usual age of superannuation, say 65 years of age. The higher rate of unemployment presumably experienced by members at these advanced ages may therefore be deemed to have been allowed for, proportionately, in my previous estimates, and the finances of the Scheme would, therefore, not be affected by the inclusion of such members. The amending provision that the benefit to be received after age 60, in the case where a refund has been made at that age, is, so far as the contributions paid before that age are concerned, to be in the proportion of one week in eight of the contributions so paid, instead of one week in five, as provided in the Seventh Schedule of the Bill, is not a necessary one, from the actuarial point of view, as the refund at age 60 has in my estimates been separately provided for by a specific portion of the members' contributions. It is, however, as I understand, desired on other grounds to reduce the benefit in respect of the contributions paid before age 60, and, so far as the effect of this reduction is calculable

I find that it would affect the finances of the Scheme to a relatively small extent, and it is probable that the reduction in the benefit in such cases would probably in practice be rarely operative.

The amount required to secure the refund at age 60, on the conditions of Clause 71 [89] was estimated, in the calculations upon which my Report of May last was based, at about £10,000 per annum in respect of the total insured members, or approximately a penny per member per annum.

(4) *Clause 79 ; page 64 ; line 20* [Clause 100 : page 105 ; line 1].—The words “two-thirds” deleted, and the words “three-fourths” inserted in lieu thereof.

I do not think that the substitution of the proposed words will have any material effect on the finances of the scheme. It may in practice be reasonably assumed that Associations or Trade Unions would not apply for an arrangement under Clause 79 [100] if this meant a financial loss to them and their members, *i.e.* if their own benefits were so low that a refund under the arrangement, limited to two-thirds of these benefits, would amount to less than the statutory benefits which their members would have drawn apart from the arrangement. Any association, in other words, on applying to make an arrangement, would presumably raise its benefits, if necessary, to a point at which the arrangement could be made without loss. The change from two-thirds to three-quarters therefore affects the administration of the associations rather than the Unemployment Fund, on which its influence can only be indirect.

(5) *Clause 81 ; page 65 ; line 19*.—The word “eighteen” deleted, and the word “sixteen” inserted in lieu thereof.

Line 21.—The words “or apprenticeship” deleted.

Line 25.—At end insert the words, “but does not include an indentured apprentice.” [Clause 102 ; page 106.]

It seems reasonable to assume, from the somewhat scanty data available, that there would be about 159,000 workmen between 16 and 18 years of age in the insured trades, and that of these about 26,000 would be indentured apprentices. This would bring in 133,000 fresh contributors to the Scheme. The final effect of the introduction of this estimated number of insured members between the ages of 16 and 18 can be most conveniently considered later on in the present Report, when the amendments to Schedules 7 and 8, providing for reduced benefits and contributions in respect of such workmen, are under consideration.

I estimate that the indentured apprentices over 18 years of age included in the original Scheme of the Bill were about 42,500 in number, in respect of whom little or no unemployment benefit would be payable ; and that their contributions, together with those of the employer (with full allowance for refund under Clause 70 [88] as amended), and after allowance for expenses, were about £45,000 per annum, in respect of which an amount of £43,651 was available to increase the margin of profit in the Fund, as estimated in my Report of May last. Their subsequent exclusion therefore involves a loss

to the Fund of this margin of profit, available in my Report of May last, but then treated as an uncalculated margin.

(6) *New Clause* [88], providing for the refunding of part of the contributions paid by the employer, in the case of workmen continuously employed, inserted in lieu of Clause 70.

I have made a careful estimate as to the probable effect upon the finances of the scheme of the change made by providing that, instead of a payment in advance by the employer of 15s. per annum, in respect of selected workmen, a refund shall be made to the employer, the end of the year, of one-third of the contributions paid to him on his own behalf during the year, in respect of any workman who has been continuously in his service through the year, and who has paid not less than 45 contributions. Making assumptions which in the circumstances appear to be reasonable, as to the proportion of the men in respect of whom the employer would receive this benefit, I estimate that the contributions of the workman, the employer, and the State, would be as under, as compared with the figures given in paragraph 16 of my Report of May last :

	<i>s.</i>	<i>d.</i>	
Average contribution from Workman . .	9	2	per annum
.. .. . Employer ..	7	10	..
.. .. . State ..	6	11 $\frac{1}{2}$..
	23	11 $\frac{1}{2}$	
Expenses of administration	2	3 $\frac{2}{3}$..
Average contribution available for benefits	20	9 $\frac{2}{3}$..

The total average contribution available for benefits would thus be increased by 9 $\frac{2}{3}$ d. per insured member per annum, or by about £97,000 in respect of the total insured membership.

(7) *Schedule 7 ; page 75 ; leave out lines 20-23* [Schedule 7 ; page 121 ; lines 3-5] and insert "seven shillings."

The effect of this amended provision, granting an equalised benefit of 7s. weekly in respect of all branches of the insured trades, is estimated to increase the average benefit per insured member per annum, after making due allowance for the proportionate numbers of insured members in each trade, from 18s. 3d., deduced in paragraph 14 of my Report of May last, to 20s. 2d., thus requiring an additional sum of 1s. 11d. per member per annum, or about £232,000, in respect of the total number of insured members.

(8) *Schedule 7 ; page 75 ; line 25* [Schedule 7 ; page 121 ; lines 6-11].—Provision inserted that no unemployment benefit shall be paid whilst the workman is below the age of 17, and one-half benefit whilst he is between 17 and 18.

As stated in paragraph (5) above, the estimated effect of bringing in, under the Scheme, workmen between 16 and 18 years of age, is to introduce 133,000 fresh contributors. There seems to be good reason for assuming that the rate of unemployment, in respect of members at these young ages, would be less, and perhaps materially

less, than the normal rate at ages 18 and upwards : and, making reasonable assumptions in this respect, I estimate that the amount required to secure the allowance of 3s. 6d. weekly during unemployment between 17 and 18 years of age would be about £23,000.

(9) *Schedule 8 ; page 77 ; line 10* [Schedule 8, page 122 ; lines 16-20].—The words added “ provided that in the case of a workman below the age of 18, a penny shall be substituted for 2½d., as the contribution from the workman and from the employer, but for the purposes of reckoning the number of contributions in respect of such a workman, the penny shall be treated as two-fifths of a contribution.”

I estimate, on the assumption of a low rate of unemployment to the workman between 16 and 18 years of age, and therefore a relatively high rate of refund to the employer, under the amended Clause 70, that the contributions of 133,000 insured members between these ages, together with the contributions of the employers and of the State, and after allowance for expenses, would amount to about £56,000. As the total amount of the reduced benefit to these members was estimated above at £23,000, there would be an estimated nett gain to the Fund of about £33,000, which may be considered as some set-off against the estimated loss of £45,000 arising from the exclusion of indentured apprentices.

The variation of the “ one-in-five ” rule, carried out by the above amendment in respect of members between 16 and 18 years of age, does not seem to be justified from any actuarial considerations, and I understand that this portion of the amendment is to be reconsidered on Report. It appears to me that an equitable manner of dealing with the question would be to provide that, in respect of unemployment before age 18, a workman shall be entitled to one week's benefit, at the reduced rate of 3s. 6d., in respect of every five weeks' contributions : but that, in respect of unemployment after age 18, he shall be entitled to one week, at the full rate of 7s., in respect of every 12½ weeks' contribution paid prior to age 18, and of every five weeks' contribution paid after that age.

(10) *Schedule 8 ; page 77 ; line 13* [Schedule 8 ; page 122 ; lines 21-30].—Provision for contributions at the rate of a penny, where the period of employment does not exceed one day, and at the rate of twopence, where such period exceeds one day, but is less than a week, with consequential provisions as to reckoning such contributions for the “ one-in-five ” rule, under the 7th Schedule.

In my estimates of May last, 44 weeks' contributions were assumed as payable on the average in each year, and, so far as these weeks of employment would in practice be split up into portions of a week, further integral weeks' contributions would have been received at the rate of 2½d., resulting in uncalculated margins, referred to in paragraph (19) of my Report, over and above the estimates made in that Report.

The proposed reduction of contributions in respect of portions of a week will materially diminish these uncalculated margins, which will also be somewhat reduced in respect of the amendment to Clause 72, dealing with “ short time.” It is, however, impossible to

make any present estimate as to the financial effect of these amendments, but, so far as I can judge, the amounts in question cannot be at all considerable.

(11) The following amendments (and possibly one or two others) may probably have some small financial effect, either favourable or unfavourable, upon the Scheme of the Bill, but the effect would appear to be relatively quite negligible in amount, and there appear to be no data available upon which any estimate could be based as to the financial effect of such alterations:

Clause 62 ; page 53 ; line 37 [Clause 80 ; page 92 ; lines 5–6].—After the word “weeks,” the words inserted “in the preceding five years.”

Clause 67 ; page 58 ; line 3 [Clause 85 ; page 96 ; lines 22–27].—Additional subsection (e), providing for the continued payments of contributions and benefits during any period intervening between an application for the decision of any question or any claim for benefit and the final determination of the question or claim.

Clause 69 ; page 58 ; lines 35 and 36 [Clause 87 : page 97 ; line 28 at end].—The words “or such larger sum as the Treasury may fix” deleted.

Clause 69 ; page 59 ; line 8.—After the word “week,” the words inserted “or increase those rates unequally as between employers and workmen.” [Clause 87 : page 97 ; line 40.]

Clause 72 ; page 60 ; line 36 [Clause 90 ; page 99 ; lines 24–27].—Provision for the employer receiving the benefit of the refund of contributions, in respect of workmen working “short-time,” and under the conditions stated in the Clause, in the case where the short-time is effected by stopping the work for some day in the week, usually recognised as a working day of at least four hours.

Clause 76 ; page 63 ; line 19 [Clause 96 : page 103 ; lines 12–14].—At the end of the Clause, the words added, “above the rates specified in the Eighth Schedule to this Act, or shall vary such rates unequally as between employers and workmen.”

New Clause (91) exempting from compulsory insurance workmen in agricultural districts, usually following an occupation other than an insured trade, and employed in an insured trade occasionally only, and providing for the payment of their contributions by agreement between the employer and the workman.

(12) The following Summary statement shows the combined estimated effect on the finances of the Fund, so far as calculable, of the several amendments discussed in the preceding paragraphs, and also the estimated variations in the number of insured members brought under the Scheme :

SUMMARY OF ESTIMATED MARGINS (so far as calculable).

	£
Estimated margins as brought out in Report of May 1911	211,837
Estimated loss from equalization of benefit at 7s. in all insured trades	£232,000
Estimated profit from varied provision as to refund to employers instead of reduced yearly contributions	97,000
	<hr/> 135,000
	<hr/> 76,837
Addition to margin (available after giving effect to the modification as shown above, but then uncalculated) in respect of the inclusion of 42,500 indentured apprentices in the 2,421,000 insured members, originally reckoned as normal insured members	43,651
	<hr/> 120,488
Estimated loss from exclusion of 42,500 indentured apprentices over 18 years of age	£45,000
Estimated profit from introduction of 133,000 workmen between 16 and 18 years of age	33,000
	<hr/> 12,000
Nett estimated margin of profit	<hr/> £108,488

INSURED MEMBERS.

Total number, as estimated in Report of May, 1911	2,421,000
Since added :	
Workmen between 16 and 18 years of age	133,000
	<hr/> 2,554,000
Since excluded	
Indentured apprentices over 18 years of age	42,500
	<hr/> 2,511,500
Total number of insured members, as varied by amendments to Bill	2,511,500
Estimated margin per insured member per annum	<hr/> 10d.

(13) It will be seen that the estimated effect of the amendments as a whole is to reduce the available margin of profit from £211,837, as estimated in paragraph (17) of my Report of May last, to £108,488, or to reduce the margin from 1s. 9d. per member per annum (on 2,421,000 insured members) to about 10d. per member per annum

(on 2,511,500 insured members). The estimated available margin of £108,488 is equal to 4.3 per-cent on the nett cost of the benefit, as compared with 9.6 per-cent, deduced in my Report of May last.

(14) The above figures do not, of course, bring into account the uncalculated margins, referred to in paragraphs (19), (20) and (21) of my Report of May last. Those referred to under the headings (i), (iii) and (iv), in those paragraphs respectively, appear to be unaffected by the amendments to the Bill, but the uncalculated margins under (ii), in respect of contributions for portions of a week, must be regarded as materially reduced by the amendment to the Eighth Schedule. A general investigation into the scope and probable operation of the uncalculated margins still available indicates that they may, in practical operation, be expected to have some considerable effect, in the provision of income additional to that estimated, or in the reduction of the estimated outlay on benefits, so that, on the whole, I should anticipate some increase from these sources in the estimated calculable margins referred to in the preceding paragraphs.

(15) Whilst, therefore, I should have preferred, in view of the inadequacy of the available data, and the consequent measure of uncertainty as to the results deduced therefrom, to have seen a larger margin for possible departures from the estimated rates, I consider that there is reasonable ground for the expectation that the margins available—calculable and incalculable—will in practical operation be sufficient to preserve the financial stability and solvency of the Scheme: subject, however, in the event of their not so proving sufficient, to the powers taken, under the Clauses and Schedules of the Bill, for the revision, either generally or in respect of particular trades, of the relation between benefit and contribution, on such revision proving to be necessary for the restoration of financial solvency.

I am, dear Sir,

Your obedient Servant,

THOMAS G. ACKLAND,

Fellow of the Institute of Actuaries,

Hon. Fellow of the Faculty of Actuaries.

24 November 1911.

THE foregoing Reports, taken in conjunction with the Reports reprinted on pp. 406–475 of the current volume, afford a substantially complete account of the actuarial basis of the National Insurance Act, but for the convenience of readers of the *Journal* who may wish to study the subject more fully, we print the following list of the subsidiary actuarial reports on special questions, which

were issued as Parliamentary Papers during the progress of the Bill through the House :—

	Reference Number.
Actuarial Report on the position of Persons in the Naval and Military Service of the Crown.	
Report of Messrs. Hardy and Wyatt.	Cd. 5809
Copy of Memorandum Explanatory of the Principal Amendments which it is proposed to incorporate in the National Insurance Bill.	
Report (Sickness Claims—First three days waiting periods), by Messrs. Hardy and Wyatt.	
Ditto by Mr. A. W. Watson.	
(Proposed Reduced Insurance in certain classes of employment).	
Report by Messrs. Hardy and Wyatt.	
Report by Mr. A. W. Watson.	
(Married Women as Voluntary Contributors).	
Report by Messrs. Hardy and Wyatt.	
Report by Mr. A. W. Watson.	Cd. 5930
Actuarial Report on proposed Government amendments to Clause 36. (Special provision with regard to persons in the Navy and Military Service with Crown).	
Report by Messrs. Hardy and Wyatt.	Cd. 5943
Copy of Memorandum explanatory of the amendments which it is proposed to incorporate in the National Insurance Bill in regard to Seamen in the Mercantile Marine.	
Report by Messrs. Hardy and Wyatt.	
Report by Mr. Duncan C. Fraser.	Cd. 5942
Rate of sickness prevailing in the Agricultural Districts of Scotland.	
Report by Messrs. Hardy and Wyatt.	
Report by Mr. A. W. Watson.	Cd. 5966
Report of the Actuaries in relation to the National Insurance Bill as amended in Committee.	
Report by Messrs. Hardy and Wyatt.	Cd. 5983

Copies of any of these Reports may be obtained from Wyman and Sons, Ltd., Fetter Lane, E.C., and 32, Abingdon Street, S.W.; or Oliver & Boyd, Tweeddale Court, Edinburgh; or E. Ponsonby, Ltd., 116, Grafton Street, Dublin.*

* A further Report by Mr. A. W. Watson dealing with the National Insurance Bill and the Buntingford Union Association was laid on the table of the House of Commons on 14 December, and will, it is understood, be issued shortly.

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., *Barrister-at-Law*.

Rights of
co-sureties.

THE recent case of *Stirling v. Burdett* [1911] 2 Ch. 418, is of some interest to those concerned with advances secured by personal covenants and policies of assurance, and may, therefore, be noted here. The case in question deals with the position of co-sureties under a joint and several guarantee for the repayment of a loan and the payment of premiums on two policies of assurance included in the security. The following are the material facts: By a deed dated 22 May 1907, entered into between the S. & K. Syndicate Limited (called the mortgagors) of the first part, the plaintiffs and the defendants (called the sureties) of the second part, and the Norwich Union Life Assurance Society (called the society) of the third part, the mortgagors and sureties became jointly and severally liable for the payment to the society of the sum of £15,000, with interest at six per-cent. The deed contained an assignment to the society of two policies of assurance, and a joint and several covenant on the part of the mortgagors and the sureties for the payment of the premiums. There was a proviso, limiting the liability of the sureties, in the following form: "Provided also that notwithstanding anything in these presents contained, the sureties respectively shall not respectively be liable to pay in respect of the said principal sum of £15,000 hereby secured, and the interest thereon, and the said premiums, or otherwise by virtue of these presents, any larger sums respectively than the sums set opposite their respective names in the second schedule hereunder written." In the second schedule there was a list of the sureties, twelve in number, and against the name of each was placed the maximum sum for which they were respectively to be liable. These sums varied considerably, the largest being £5,000 and the smallest £1,000, the total amount of the maxima thus guaranteed being £30,000. The mortgage money was not to be called in for ten years, provided certain conditions were fulfilled.

The plaintiffs, and two of the defendants, paid various amounts which, together, were sufficient to cover the interest on the £15,000 and the premiums on the policies down to a certain date, and the sums paid by the individual plaintiffs

were more than their due proportion of the total of such interest and premiums, but did not, in any case, reach their respective maximum limits. The period of ten years, during which the mortgage money was not to be called in, had not expired, nor had there been any breach of the conditions relating thereto.

In these circumstances the plaintiffs applied for a declaration that the defendants, as co-sureties with them, were jointly and severally liable to contribute to, and make good, a proper proportion of the calls made and all further calls, if any, according to the respective limits provided for in the deed.

The case came before Warrington, J., who found in favour of the defendants. In delivering judgment to this effect, he said: "This is an action by some of a number of co-sureties, as plaintiffs, against the rest of the co-sureties, as defendants, claiming contribution by the defendants to money paid by the plaintiffs under the contract of suretyship. The defendants say, and it is the fact, that the plaintiffs have not paid, or been called upon to pay, anything in excess of their proper proportion of the moneys which are the subject of the suretyship; and they insist that under those circumstances the plaintiffs are not in law entitled to the relief they ask. The plaintiffs reply that though they have not paid, or been called upon to pay, more than their proper proportion of the whole of the moneys covered by the contract, they have between them paid more than their proportion of what was actually due at the time of payment; and they insist that this fact entitles them to recover. . . . In my opinion the contract is indivisible, and the limitation of liability would make it impossible, with justice, to deal with individual payments in the way contended for by the plaintiffs, and I think the defendants are right on the point of law raised by them. The result is that as regards the defendants who have specially raised the point, the action must be dismissed with costs."

Transfer of life
assurance
business under
Assurance
Companies Act,
1909.

The case of *Empire Guarantee and Insurance Corporation, Limited*, Petitioners, 1911, XLVIII. S.L.R. 1038, relates to the transfer of the life assurance business of that corporation to the Royal Exchange

Assurance Corporation. The Empire Corporation was registered

on 31 December 1900, and was empowered to carry on most branches of insurance, including life assurance. It made the necessary deposit under the Life Assurance Companies Act, 1870, and transacted a certain amount of such business. This branch did not, however, make much progress, and the directors came to the conclusion that it would be in the interest, both of the shareholders and of the life policyholders, that the business should be transferred to another company. Accordingly, on 14 October 1910, they entered into an agreement with the Royal Exchange Assurance Corporation for the transfer of the life assurance business to that body.

The Empire Corporation had power under its memorandum and articles of association to sell or dispose of its business or any part thereof in consideration of payment in cash or in shares or securities of any other company. Under the proposed arrangement the policies issued by the Empire Corporation were to be cancelled, the policyholders receiving in lieu thereof policies of the same value in the Royal Exchange Corporation; but only non-participating policies were to be given in exchange for participating ones. Evidence was forthcoming to show that the transaction was an advantageous one for the policyholders in the Empire Corporation, all of whom were in favour of the proposed transfer.

On 28 January 1911 the Empire Corporation applied to the Court to sanction the proposed transfer, under the Assurance Companies Act, 1909, and the Court remitted the application for report to the Hon. J. M. Balfour, W.S., who, in his report, stated, after reciting the facts, that "Your reporter . . . has grave doubts as to (1) Whether the transaction set forth in the agreement is a transfer of the nature which falls to be submitted to the Court for its sanction in terms of section 13 of the Assurance Companies Act, 1909, and (2) If it is, whether the procedure adopted to carry out the transaction has been regular. . . . It appears to your reporter that the arrangement embodied in the agreement cannot be considered as a transfer of the assurance business of a class from one company to the other company, and that there is nothing in the section of the Act above quoted which would compel a policyholder to release his policy and accept that of another company. . . . The terms of section 13 of the Assurance Companies Act, 1909, before quoted, appear to your reporter to require that the petition should be presented by the

“ directors. . . . Should your Lordship be of opinion that
 “ the petition has been properly presented in name of the
 “ company, your reporter would respectfully submit that a
 “ meeting of the company should have been held to authorise
 “ the presentation of the petition in the name of the
 “ company.”

On the case coming before the First Division of the Court of Session, a majority of the Court decided in favour of sanctioning the proposed transfer, overruling the objections raised by the reporter. The judgment was to the following effect: “ Find
 “ that the requirements of section 13 of the Assurance Companies
 “ Act, 1909, have been duly complied with, and that it is within
 “ the power of the Empire Guarantee and Insurance Corporation,
 “ Limited, to carry out the proposed transfer: Sanction said
 “ transfer of the life assurance business of the said Empire
 “ Guarantee and Insurance Corporation, Limited, to the Royal
 “ Exchange Assurance Corporation in terms of the agreement
 “ mentioned in the petition, and decern.”

REVIEW.

Insurance Companies' Accounts. Second Edition. Revised by A. E. SPRAGUE, D.SC., M.A.

[LONDON: C. & E. LAYTON.]

MORE than thirty-seven years have elapsed since Dr. T. B. SPRAGUE published his well-known treatise on Life Insurance Accounts, and throughout this period his book has proved of very considerable assistance to both the student and the practical man. The passing of the Assurance Companies Act, 1909, and the extension of legislation to forms of insurance other than life, have naturally rendered this treatise somewhat out of date. It is interesting, however, to note that certain of the amendments suggested by Dr. Sprague in 1874 have found a place in the new Act.

During the course of a recent discussion at the Institute the hope was expressed that a new work on the lines of Dr. Sprague's would be undertaken, perhaps after the results of one year's working under the Act of 1909 were available, and this book, which is described as a second edition of the original treatise by Dr. Sprague, will therefore be welcomed. In this edition the original text has to a large extent been retained, but the valuable analysis

of accounts deposited with the Board of Trade and published in the Blue Book of 1872 has been omitted, and no similar analysis of accounts deposited under the new Act has been inserted. Possibly it was thought that the material available at the time of going to press was not sufficient to afford a reasonable basis for analysis, but, however this may be, we think that the omission of the analysis is a slight defect in an otherwise admirable revision.

The original treatise is so well known that there is little in the present edition which calls for particular reference. Beyond the introduction of fresh chapters dealing briefly with forms B., C., D., and E. of Schedule I. and with Schedule II., and the omission of the Chapters in the first edition dealing with the Third and Fourth Schedules of the 1870 Act, the alterations are more or less of a verbal nature. The following points, however, which have presented themselves on a perusal of the book may be worthy of notice. With reference to the item "Less Income Tax thereon," in Schedule I., it is stated that this item should be reduced by the amount of tax deducted by the Company from annuities. It seems that this statement may prove a little misleading and should be modified by a foot-note to the effect that a Company is only entitled to retain income tax so deducted from annuities in so far as the annuities are charged on a tax-bearing fund amply sufficient to pay them in full, and then only when it is assessed on its income from investments and not on its profits. It was further stated in the original treatise—and the statement is repeated in the revised edition—that any sum spent in establishing the business or in preliminary expenses may be entered as a deduction from the paid-up capital, and examples were given of accounts presented in this way. Schedule III. of the new Act does not appear to contemplate any such deduction, and it is apparently open to doubt whether such a course would be admissible to a company registered under the Companies Acts, as it would tend to suggest in the minds of the public that a reduction of capital, a matter which can only be effected with the sanction of the Court, had actually taken place. Note 4 to Schedule I., seems further to strengthen the contrary view, and it is quite possible, moreover, to conceive circumstances in which such a course might be adopted to conceal extravagant management.

It is suggested that such bonds and stocks as "City of London Corporation Bonds" and "Metropolitan Consolidated Stock" should properly be included amongst Loans on Rates, but with this suggestion it is probable that many authorities would disagree. The main point of difference appears to be that these securities can be dealt with freely on the Stock Exchange and are liable to fluctuation, while, as it is pointed out, loans on mortgages of rates are *usually re-payable by a fixed or varying annuity*, and the balance of the loan cannot be called in. If, therefore, it becomes necessary to realise a security of the latter description, a purchaser would have to be found for the remaining term of the annuity, a far more difficult matter than selling stocks or bonds on the market. Apparently the argument for the inclusion of these

investments under the heading of Loans on Rates supplies at least one very good reason for their separation.

There is something to be said for the inclusion in the Returns of a form of account embracing any capital or other transaction which is not specifically covered by the present schedules, particularly as there is apparently no obligation to supply the amount of subscribed capital. The amount of the subscribed and uncalled capital would seem to be a far more important matter, from the standpoint of security to policyholders, than the amount paid up, which may have been dissipated in expenses, and be only represented by paper assets. As regards the suggested amendments of Schedules I, II and III of the 1909 Act, and the further information which might be required from Companies, it is probable that the adoption of some of these recommendations would meet with strenuous opposition on the part of the Companies. For example, the minute returns regarding the business transacted in the year and remaining in force at the end of the year, however desirable they might be as a statistical record of the progress of life assurance amongst the community, would, we are afraid, be considered as information not very material to the policy-holders' interests, and as a more or less unnecessary repetition of the troublesome statement to be rendered under Schedule V, which even now has to be deposited at least once every five years. That a return of this description is recognised as imposing a somewhat heavy burden on life assurance companies is evidenced by the provision as to the deposit of returns under Schedule V in the case of companies making an annual return under Section IV.

A cursory examination of the accounts of a few offices for the year 1910, rendered, or purporting to be rendered, in accordance with the provisions of the Assurance Companies Act, 1909, has revealed one or two facts which may be worthy of remark.

First, with regard to the item 'Deposit with the High Court (Securities to be specified)', the following are a few examples of the information given under this heading :

- (1) Preference Stocks of railways in Great Britain.
- (2) British Government Securities.
- (3) British Railway Preference Stock.

The other accounts examined give sufficient details to enable the security to be identified, and also, in some cases, the nominal amount thereof. The wording of the Schedule is a little ambiguous, but, assuming all these accounts are accepted by the Board of Trade as they stand, it is evident that the want of uniformity experienced under the 1870 Act will be perpetuated, though this particular matter may be of small importance.

Secondly, under the heading "Other sums owing by the Company (to be separately stated under each class of business)", the following items are found :

- Amounts owing to other Insurance Companies.
- Current Accounts owing by Company.

As these items are not allocated to their respective accounts, it can hardly be claimed that they are in accordance with the Schedule.

The results of even this brief analysis are perhaps sufficient to emphasize the difficulties of effectuating that uniformity in accounts which is so greatly to be desired.

R. G. M.

INSTITUTE OF ACTUARIES STUDENTS' SOCIETY.

THE Annual General Meeting of the above Society was held at Staple Inn Hall on Monday, 16 October 1911, Mr. W. Palin Elderton presiding.

The Chairman, in moving the adoption of the Report and Accounts, said he thought that, although the Society had not yet realized all its aims, a very successful start had been made. From the senior members of the Institute of Actuaries who had taken the Chair at meetings much help had been received. In the earlier stages, especially, the President, Sir Gerald Ryan, had greatly assisted and encouraged the Society. The discussions held had been very useful for the examinations, and the periodical meetings had served a highly important purpose in providing members with an opportunity of meeting actuarial men connected with other offices and learning their views. A journal would shortly be published and it was hoped that in future the social side of the Society could be more developed. A considerable increase of membership was desirable and he urged all Institute men who were eligible to join the Society.

The following gentlemen were elected to serve on the Committee for the next twelve months: Mr. W. Palin Elderton (Hon. Member), Chairman; Messrs. H. T. Clarke, R. C. Fippard, A. Henry, J. A. Humphreys, S. E. Maenaghten (Hon. Member), L. A. Mouat Jones, R. J. Ledger, A. E. King, J. A. McCulloch, H. E. Melville, W. A. Osborne (Hon. Secretary, 11, Lombard Street, E.C.), J. R. Pickup, J. Stocks, and E. A. Woodall (Hon. Treasurer, 39, King Street, E.C.).

Before the proceedings terminated a vote of thanks was unanimously passed to the retiring Honorary Secretary, Mr. J. A. Humphreys, whose resignation, due to pressure of other duties, was accepted with very great regret.

In accordance with the rules of the Society the Report and Accounts have been submitted to the Council of the Institute.

Since the above meeting was held the Council have nominated Mr. H. M. Trouneer to be a member of the Committee of the Society.

During October, November and December six ordinary meetings were held. The following is a brief note of the proceedings—

23 *October* 1911.—Paper on “The Law of Bankruptcy”, by R. C. Simmonds. Mr. S. G. Warner in the Chair.

The following gentlemen joined in the discussion: Messrs. H. E. Melville, E. W. Townley, G. M. Reeve, R. Hammond, J. G. Kyd, W. Mouat Jones, K. O. Nash, W. Penman, Junr.

30 *October* 1911.—Paper on “Surrender-Values and Paid-up Policies”, by R. Hammond. Mr. A. Levine in the Chair.

The following gentlemen joined in the discussion: Messrs. A. E. King, R. D. Anderson, W. J. Fulford, A. Henry, R. C. Fippard, E. B. Nathan, V. P. A. Derrick, J. Stocks, R. C. Simmonds, C. F. Peters, E. W. Phillips, E. W. R. Spiegel.

6 *November* 1911.—Paper on “Whether it is ever permissible in the theory of life contingencies to assume an even distribution of deaths over the year of age”, by R. J. Ledger. Mr. G. J. Lidstone in the Chair.

The following gentlemen joined in the discussion: Messrs. H. L. Trachtenburg, E. W. Phillips, G. M. Reeve, W. P. Elderton (Hon. Member), A. Henry, E. F. Spurgeon (Hon. Member).

13 *November* 1911.—Paper on “Bimetallism”, by G. M. Reeve. Mr. G. E. May in the Chair.

The following gentlemen joined in the discussion: Messrs. R. C. Simmonds, R. C. Fippard, C. H. Maltby, S. J. Gunningham, A. Henry, H. E. Melville.

20 *November* 1911.—Paper on “Graduation: Its Objects and Principles”, by A. Henry. Mr. G. Green, M.A., in the Chair.

The following gentlemen joined in the discussion: Messrs. J. H. Duffell, C. C. H. Drake, J. Stocks, R. C. Simmonds, R. D. Anderson, E. W. Phillips, S. J. Perry, A. E. King.

4 *December* 1911.—Discussion on “Office Premiums,” opened by J. Stocks. Mr. S. E. Macnaghten in the Chair.

The following gentlemen took part in the discussion: Messrs. E. C. Farmer, R. Hammond, A. Henry, G. M. Reeve, W. C. Fielder, R. C. Simmonds and H. G. Sharp.

THE INSTITUTE OF ACTUARIES.

At the first Ordinary General Meeting for the Session 1911-12, held on 27 November 1911, the President (Sir GERALD H. RYAN) said that before passing on to the ordinary business of the meeting there were two matters of considerable importance to the profession which he might be justified in referring to that evening.

INDIAN LIFE ASSURANCE LEGISLATION.

In the first place, since their last meeting in June, a Bill had been introduced in the Viceroy's Council regulating the business of life assurance in India, and on that point it would be sufficient for him to say that the representations of the British companies operating in India had received very satisfactory consideration at the hands of the Indian Government, so that companies complying with the Assurance Companies Act, 1909, would be excused from making a new deposit under the Indian Act, and from certain other formalities and restrictions. Probably some member of the Institute would contribute a Note concerning the new Indian Bill to the *Journal*, in order that the full purport of the measure might be on record.

THE NATIONAL INSURANCE BILL.

The next matter to which he wished to devote a few minutes was the National Insurance Bill, which had absorbed so large an amount of public interest and excited so much public criticism within the last few months. As he had mentioned when addressing the members in June last, a Special Committee of the Council was appointed to watch the Bill, in its passage through Parliament, in the interests of the profession, and a great deal of time had been given by the members of that Committee to the work which thus devolved upon them. Quite recently the Institute had been consulted by the Treasury on matters of financial importance, and it had placed the result of its deliberations before the Chancellor of the Exchequer. But it seemed desirable that he should take the present opportunity—perhaps the last which would be available—to call attention to two features of the Bill which could not be regarded with satisfaction. He referred to the position first of Actuaries, and secondly of Insurance Clerks, under the Bill as it at present stood. The Clauses of the Bill dealing with Accounts and Valuations provided that the special and periodic valuations of approved Friendly Societies were to be undertaken by valuers to be appointed by the Treasury. There were no rules as to what constituted the qualification of a "valuer." Apparently the selection was left to the absolute discretion of the Treasury. It would be quite unnecessary for him to remind the members of the technical knowledge required in the performance of the duty which attached to the position of valuer under the Bill, nor need he press the point that unsound advice had been admitted

to be a frequent reason for the failure and insolvency of friendly societies in the past. But, looking at the undoubted fact that the success of the Insurance Bill would depend not a little on the skill and competency of the expert advisers consulted, it would be highly regrettable if the Bill did not prescribe the qualifications of those valuers. In the interests of the profession the Institute had made representations to the Government that the words "Qualified Actuary" should take the place of the word "Valuer" in the Bill, but in order to meet certain objections raised it had been suggested that the precedent of the Assurance Companies Act, 1909, should be followed, which provided that in addition to the Fellows of the Institute and of the Faculty, there might be appointed such other persons having actuarial knowledge as the Treasury might approve. The Council held that that recommendation would have a three-fold effect; first, by fixing the standard of qualification on a high level it would protect members of approved societies from the lamentable results of bad advice; secondly, it would give to the profession of actuary that public recognition which had already been accorded to it by the Assurance Companies Act and many private Acts; and thirdly, it would enable the Treasury to have a due regard to legitimate vested interests and to the need for a certain elasticity to start with. On the last-mentioned point he might say that according to information he had received there were 27,000 registered Friendly Societies, including branches of the affiliated orders. Most of these Societies would need to have special schemes and special valuations under the Bill, so that it might be necessary at the outset to have a little elasticity in the appointment of valuers. The Council would have been failing in its duty to the younger members of the profession if it had not taken up the matter in an earnest spirit, and he trusted that their recommendations, which were cordially supported by the Faculty of Actuaries, would not fail to have an effect in producing that reasonable and much-needed amendment in the terms of the Bill.*

Dealing with the second point, namely, the position of Insurance Clerks under the Bill, he did not know whether sufficient attention had been drawn to the fact that a large and estimable body of men and women would probably suffer substantial

* In the Act as finally passed a "valuer" is defined as "a person possessing actuarial qualifications as may be approved by the Treasury."—ED.

damage under the provisions of the Bill. It was not too much to say that the prosperous Insurance Companies, Banks and other leading Joint-Stock Companies, granted benefits and privileges to the members of their staffs which were far more liberal than the Government scale of benefits, and that without specific contribution. It was true in most instances that that humane treatment proceeded from custom rather than contract or legal terms of service, but that difficulty could be, and he had no doubt would be, easily remedied. He could not tell the number of mercantile clerks throughout the country, as he believed no trustworthy statistics had been obtained, nor did he know the number of clerks earning less than £160 a year employed by Insurance Companies, Banks, and other great financial institutions. There must be a very large number of persons indeed who were at present, and who had been for many years, recipients of much greater benefits than the Government benefits, whose position would probably be altered for the worse by the Government Bill. He did not think any doubt could reasonably be entertained that employers and employed alike would come under the operations of the Bill with extreme reluctance and with a feeling that they would be much better off under the old régime; nor could he say why the line should be drawn so strictly in their case so as to include them against their will. The Fire Offices' Committee and the Life Offices' Association were jointly interesting themselves in the matter, and a recommendation had been sent to the proper quarter that the Insurance Commissioners to be appointed under the Bill should have the power of excluding from its operations the employés of any Company or public body which had proved to their satisfaction that they had entered into legal binding agreements with their staffs to supply higher benefits than those provided by the Bill. It was recognised that that might mean the guarantee of adequate reserve values on the withdrawal of any member of the staff, but that was a point with which the Insurance Commissioners would be perfectly competent to deal. All the insurance companies asked was that power should be granted to the Insurance Commissioners to allow them to make, or continue, in the interests of their employés more favourable schemes than that of the Government. The Bill gave the Commissioners power to exempt Railway Superannuation Funds established by Act of Parliament, and all that was being asked was that they should have similar power in respect of Insurance Offices and other influential bodies.

It was to be hoped that those friendly representations to the Government would not fail in their objects, and that that second blemish would be removed from the Bill.

LECTURES ON FRIENDLY SOCIETIES.

At the invitation of the Council, Mr. A. W. Watson (who has since been appointed to the position of chief Actuary to the Joint Committee of Commissioners under the National Insurance Act) is delivering at Staple Inn Hall a course of six lectures on Friendly Societies. We are indebted to Mr. Watson for the following abstract of the subjects dealt with in the Lectures :

Types of Friendly Society—Affiliated Orders—Centralized and Partially Centralized Societies—County Societies—Local Societies—Deposit and Dividing Societies. The constitutional features of each group, the methods of re-insurance practised in certain cases, and special provisions of the rules having reference to the sickness claims. Sickness, accident, mortality, secession and maternity claim experiences. Methods of tabulating and analysing the data ; notes on Standard and other important experiences. *e.g.*, M.U. 1866–70, 1893–97 ; A.O.F. 1871–75 ; Registered Friendly Societies, 1876–80. Valuation. Adjustments of results based on standard tables ; the “Ages-passed-through” method of valuation. Comparisons with former results. The “profit and loss” statement. Treatment of Deficiencies ; suggestions for reducing excessive claims for sick pay. Railway Funds, Orphan Benefit Insurances. Friendly Societies and the National Insurance Bill.

It is understood that after the conclusion of the course, the Lectures will be published by the Institute in book-form.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

*Some recent Statistical Results—A review by W. PALIN ELDERTON.
F.I.A., Actuary of The Star Assurance Society.*

[Read before the Institute, 18 December 1911.]

I.—INTRODUCTORY.

PROBABLY everyone who has been responsible in any way for the underwriting of life assurance has paused on some occasions to consider whether an extra premium which it is proposed to charge for personal or family history is really justifiable. It is comparatively easy to express this hesitancy as a problem that requires solution by asking whether the mortality among those persons having a certain family history or personal defect is heavier than that among other assured lives, and if so, to what extent. At present there is, however, little to help towards a direct solution and the amount of extra premium is based on an "educated guess." It is, of course, only statistical evidence that can finally prove the necessity for an extra or justify its amount, but although insurance offices have been doing business and charging extra premiums for many years no complete attempt has been made to solve the problem.

It would not be by any means impossible to set out on cards for every case for the past twenty years some particulars of family and personal history, but I fear that many would look upon the process as somewhat too laborious. A shorter alternative would be to take out the necessary particulars for all the new business in, say, the years 1890-1895 and investigate the

resulting mortality. In America an attempt has been made to study special risks, but even apart from the possible criticism of some of the methods and results one would no more feel safe in relying absolutely on such information for English business than one would in using a mortality table based on American lives for English premium rates or valuations.

The matter is perhaps specially worth mentioning at the present time owing to the growing tendency of insurance offices to grant assurances without medical examination, while the recent growth in the business under children's deferred assurance tables must inevitably cause one to consider whether the absence of definite evidence as to the connection between family history and the duration of life may not imply that much that we are doing is a little in the dark. A special point of criticism will indicate a possible danger. A deferred assurance is required for the first child (recently born) of a marriage; the parents, aged 26 and 23, being alive and in good health. Many proposal forms would ask for no further information, though they contain a general question as to the existence of certain diseases in near relatives. I am doubtful if the offices would care, even if they are able, to go back on the contract, if it afterwards turned out that both the grandmothers died of phthisis and there were other defects in the family history.

In schemes in which there is no medical examination the office protects itself against fraud and very early claims, and satisfies itself that the proposer's habits are satisfactory, but it would seem probable that a fuller knowledge of the effect of family or personal history on the duration of life would render such business far safer than many people consider it to be at the present time.

I am aware that many lay little stress on the doubts I have raised, that it is impracticable to ask for everything on the proposal form, and that it is our business to run risks. It is also open to anyone to assert that the premiums charged have been loaded to cover these contingencies, that the advice of medical officers on these questions is sufficient or that the expense of the necessary investigation would be out of proportion to the resulting gain. It is, however, easy to reply that if one does not know the contingency one cannot load for it, and that the doctor is in no better position than the actuary if he has no facts on which to base his view; while I am a little inclined to think that it is easy to overestimate the cost of the work by supposing an unnecessary amount of detail in it.

But even if the actuary finds that it is impracticable to take steps to utilize such information as may be available in the books and papers of insurance offices he can at any rate examine the evidence on heredity, disease, and environment that is being collected from other sources in order to see how far it can assist him in practical work. I think, however, he sometimes hesitates to undertake the study because he feels that much of the recent work on heredity is expressed in an unfamiliar language which must be mastered before the statistical results can be appreciated. This is a very natural point of view, but there are in reality few new terms to be learnt and the meaning of the most important one can be grasped easily by bearing in mind that its use is to give us a measure of the resemblance between two things. We want in fact to be able to say to what extent offspring resemble their parents in various characteristics or to what extent vaccination protects against smallpox and to enable us to do this a scale of resemblance or correlation has to be adopted similar to the probability scale. The latter of course runs from zero representing the certainty of an event not happening to unity representing the certainty of its happening. The correlation scale runs from -1 to $+1$; unity represents perfect correlation, and zero absence of correlation, while the negative sign implies that the two objects discussed vary in opposite directions. An example will explain this. Let us assume that we are considering the correlation between the height of a father and the height of his son. We know that, generally speaking, a tall father has a tall son; the correlation is therefore positive: if in some race in an out of the way part of the world tall fathers generally had short sons and short fathers tall sons, the correlation would be negative; while if in a third race one were unable to form the slightest idea of the height of a son when the height of a father is known, the correlation would be zero. The following table, drawn mainly from a series of measurements made on various parts of the same person, will give an additional idea of the scale. The table includes a few measurements which may be of special help to actuaries.

TABLE I. *Scale of Resemblance.*

Things measured	Coefficient of Correlation
Right and left femur98
Age of husband and age of wife96
Left cubit and left middle finger85
Height and left cubit80
Height and left foot74
Height and left middle finger66
Vaccination and recovery in smallpox64
Capacity of skull and greatest horizontal breadth (Naqada skulls, male)... ..	.43
Bank reserve and discount rate37
Severity of attack of smallpox and years since vaccination	.32
Anti-typhoid inoculation and immunity24
Age of maturity and unexpired term of endowment assurances19
Weight of healthy heart and age of person14
Weight of heart and spleen08

It is sometimes suggested that the use of such a scale is unnecessary because we can tell that two things are closely related or are only slightly related by general reasoning, but the position of the statistician in this respect will be clear to the actuary, as it is only necessary to remind the latter that he knows that a man aged 60 is more likely to die in a year than one aged 30, but a scale of probability is necessary to show to what extent the two probabilities differ.

A difficulty of a different nature will, however, occur to most actuaries. It is easy to obtain several thousand cases from which the chances of death can be calculated accurately, but there is a difficulty in obtaining an equally large amount of material bearing on the heredity of stature for instance. We are accustomed to feel that rates of mortality are reliable because they are based on large numbers, but if there are comparatively only a few cases from which the position in the correlation scale (coefficient of correlation) is judged we might easily be far out in our estimate. This is true to some extent: one has to be content with smaller numbers; but checks can be placed on the result so that we can be satisfied that we have not assumed a correlation that does not exist nor made any serious error in our numerical values owing to paucity of data. A larger body of facts might show slightly larger or smaller coefficients than those given, but there is no reason to

think that the error is sufficient to affect the conclusions we shall draw from our statistical results.

II.—HEREDITY.

With these preliminary explanations the scope of the present review may be indicated. We shall deal only with man and give the numerical results that have been reached for measurable and non-measurable characteristics and for the inheritance of liability to disease. With the exception of the experimental investigations of the biologists on plants and animals most of the work on the statistical aspect of heredity has been done by a few workers of whom Francis Galton, Karl Pearson, and W. F. R. Weldon are the best known. Recently through Francis Galton's generosity a laboratory has been established for studying such subjects, and within the last few years a large amount of valuable work has been done. It seems likely that this output will be maintained as Sir Francis Galton left the residue of his estate for the establishment of a laboratory and professorship, but the amount available will have to be supplemented by other gifts at the present time if the many necessities for the conduct of such work are to be adequately provided for.

However this may be, it will be readily admitted that the past ten years, in which most of the results given in Tables II and III have been produced, have given us a considerable amount of information about the resemblance between relatives.

TABLE II.—*Parental Resemblance.*

Characteristic	Source of Data	Computer	Locus	Measure of resemblance between characteristic in parent and child (coefficient of correlation)
<i>Physical.</i>				
Stature ...	K. Pearson, Family records.	K. Pearson & A. Lee.	Biometrika, Vol. II, pp. 357, <i>et seq.</i>	·51
Span	·45
Forearm	·42
Eye colour ...	F. Galton, Family records.	..	Phil. Trans. A. Vol. 195, pp. 102 <i>et seq.</i>	·49
<i>Psychical.</i>				
Ability... ..	Oxford class lists ...	E. H. J. Schuster & E.M.Elderton	Eugenics Lab. Memoir No. I.	·49
Intelligence	K. Pearson, Family records.	K. Pearson ...	Eugenics Lab. Lecture No. VI.	·58
<i>Diseases, etc.</i>				
Tuberculosis	Crossley Sanatorium (Dr. W. C. Rivers).	K. Pearson ...	Studies in National Deterioration, No. II.	·50
..	Family histories of 1,500 criminals.	C. Goring ...	Studies in National Deterioration, No. V.	·53
Insanity	·47
..	Dr. O. Diem's data	K. Pearson ...	British Medical Jl., 1905, p. 1176.	over ·30
..	James Murray's Royal Asylum, Perth (Dr. A. R. Urquhart).	D. Heron ...	Eugenics Lab. Memoirs, No. II.	·57
Deafness ...	Dr. E. A. Fay's American Data.	E. H. J. Schuster	Biometrika, Vol. IV, pp. 465 <i>et seq.</i>	·54
Defects in vision	Dr. Steiger's German Data.	A. Barrington & K. Pearson.	Eugenics Lab. Memoirs, No. V.	·53
Duration of Life	Landed Gentry and Peerage.	M. Beeton and K. Pearson.	Proc. Roy. Soc., 1899 or <i>J. I. A.</i> , xxxv, pp. 112 <i>et seq.</i>	·12
..	Society of Friends...	..	Biometrika, Vol. I, pp. 50 <i>et seq.</i>	·13

*For convenience of reference it may be mentioned that the Eugenics Laboratory Memoirs and Lectures, and the Studies in National Deterioration are published by Dulau & Co.

TABLE III.—*Resemblance between Brothers and Sisters.*

Characteristic	Source of Data	Computer	Locus	Measure of resemblance between characteristic in brothers, sisters, or brother and sister (coefficient of correlation)
<i>Physical.</i>				
Stature ...	K. Pearson, Family records.	K. Pearson and A. Lee	Biometrika, Vol II, pp. 357 <i>et seq.</i>	.53
Span ...	"	"	"	.54
Forearm ...	"	"	"	.48
Cephalic Index	K. Pearson, School Children.	K. Pearson ...	Biometrika, Vol. III, pp. 131 <i>et seq.</i>	.49
Head length	"	"	"	.46
Head breadth	"	"	"	.58
Auricular height.	"	"	"	.52
Eye Colour	"	"	"	.53
"	F. Galton, Family records.	K. Pearson and A. Lee.	Phil. Trans. A., Vol. 195, pp. 102 <i>et seq.</i>	.48
Hair Colour	K. Pearson, School Children.	K. Pearson ...	Biometrika, Vol. III, pp. 131 <i>et seq.</i>	.57
Straightness of hair.	"	"	"	.51
Athleticism...	"	"	"	.65*
<i>Psychical.</i>				
Vivacity ...	K. Pearson, School Children.	K. Pearson ...	Biometrika, Vol. III, pp. 131 <i>et seq.</i>	.46
Assertiveness	"	"	"	.50
Introspection	"	"	"	.56
Popularity ...	"	"	"	.52
Conscientiousness.	"	"	"	.62
Temper ...	"	"	"	.50
Handwriting	"	"	"	.52
Ability ...	"	"	"	.46
" ...	Oxford Class Lists...	E. H. J. Schuster and E. M. Elderton.	Eugenics Lab. Memoir, No. 1.	.56
" ...	Harrow and Charterhouse Lists.	"	"	.56
" ...	K. Pearson, Family records.	K. Pearson ...	Eugenics Lab. Lecture, No. 1.	.54
<i>Diseases, etc.</i>				
Tuberculosis	Crossley Sanatorium (Dr. W. C. Rivers).	K. Pearson ...	Studies in National Deterioration, No. II.	.43
Insanity ...	James Murray's Royal Asylum, Perth (Dr. A. R. Urquhart).	D. Heron ...	Eugenics Lab. Memoir, No. II.	say .50
Deafness ...	Dr. E. A. Fay's American Data.	E. H. J. Schuster	Biometrika, Vol. IV, pp. 465 <i>et seq.</i>	say .74
General Health	K. Pearson, School Children.	K. Pearson ...	Biometrika, Vol. III, pp. 131 <i>et seq.</i>	.53
Duration of Life.	Society of Friends...	M. Beeton and K. Pearson.	Biometrika, Vol. I, pp. 50 <i>et seq.</i>	.27

* Probably an overestimate because certain schools to which both brothers went encouraged athletics and certain other schools did not.

The first three items of Table II and the first seven of Table III clearly lend themselves most readily to statistical work, but satisfactory methods have also been devised for finding the coefficient of correlation even when the characters considered cannot be measured quantitatively.

We have headed Tables II and III as showing "resemblance" instead of "heredity", because, although the coefficients found may be taken as significant, more than one interpretation of the resemblance might be given. It is open to anyone to say that parent and child have like stature because their surroundings are the same, and a similar remark is made even more often about psychical characteristics. The obvious reply from Tables II and III is that it is hard to account for the resemblance of eye colour on the basis of environment, and if this resemblance is due to heredity one would naturally conclude that other resemblances showing approximately the same coefficient should be attributed to the same cause. There is, however, another reply: attempts have been made to measure the effects of environment, and the resulting coefficients have been very small; in fact the average of nearly fifty coefficients was only $\cdot03^*$ and we may therefore conclude that the resemblance is mainly the result of heredity.

The problem of the heredity of disease is, however, of especial interest to actuaries and may be discussed somewhat more fully. The most satisfactory way of estimating the correlation between a disease in parent and offspring is by working on a very large random sample of family histories of the general population. This is a slow process as it takes many years to collect sufficient family histories. For the present, therefore, we have to content ourselves with less satisfactory methods. We have to start from people that we know suffer from the disease and ascertain how many of their relatives (*i.e.*, parents if we are estimating parental resemblance, brothers and sisters if we are estimating fraternal resemblance, &c.) also suffer from it and how many have not suffered. In order to complete the information necessary to enable us to measure the correlation we have to estimate the corresponding number of parents without the disease having offspring who are also without it. This information cannot be obtained from the statistics and has to be estimated from con-

*See Eugenics Laboratory Lecture Series, VI, "Nature and Nurture", by Karl Pearson, p. 26. More coefficients have been calculated since and these lead to the same conclusion.

siderations based on the general population, but it is possible in most cases to get an upper and lower limit between which our estimate should fall, and in consequence an upper and lower limit can be found for the coefficient of correlation. In making our estimate we have to bear in mind that the age element enters into our calculations, because if we are dealing with living people they may still exhibit signs of the disease we are studying. The values given in Tables II and III were found after taking all these points into consideration and are midway between the maximum and minimum values.

It has been the custom of most insurance offices to charge an extra premium for those whose parents died of "consumption", and although in recent years this may have been relaxed it still continues. There is, I understand, no doubt that in the strict sense the disease is not inherited, but I think it probable that, apart altogether from the difficulty of explaining the statistical results we have given in our tables, it would be readily admitted that some are constitutionally inclined to certain kinds of disease and others are not. In other words the soil is important. If heredity is responsible for the soil the extra charge for a family history of consumption is probably as justifiable to-day as it was twenty years ago. But apart from this practical point it is worth going further into this question as there are one or two interesting ways of testing the part that infection plays.

One would expect if infection were of great importance that those whose parents or near relatives were consumptive would, owing to early direct contact, exhibit signs of the disease at an earlier age than those whose near relations had no such taint. The average age at onset leaves us, however, undecided—there is, practically speaking, no difference.* Another test of infection is the prevalence of the disease in husband and wife, and this has been studied by the late E. G. Pope (an American connected with the Adirondack Cottage Sanitarium) whose unfinished work was completed recently by K. Pearson.† The coefficient of correlation between pulmonary tuberculosis in husband and wife was between .1 and .3. It has, however, been shown there is a distinct tendency of like to marry like, and if this were true of the type of constitution as it is true of stature, eye colour, psychological characteristics and insanity, most of the resemblance in tuber-

* See Studies in National Deterioration, No. II. Dulau, 1907.

† See Studies in National Deterioration, No. III. Dulau, 1908.

culosis could be accounted for without having to resort to infection for an explanation.

Do not, however, misunderstand me. I do not say for one moment that I doubt the conclusion of the medical profession that people cannot get the disease without the germ; all I say is that the statistical evidence, so far as it goes, tends to show that the constitution of the person is of considerable importance and is inherited.

Before leaving the subject of heredity it will be well to refer to one or two difficulties which are more particularly likely to be felt by actuaries. The first is: "If heredity is more important than environment, why is the mortality of licensed victuallers or gold miners so heavy? Does not occupation outweigh heredity?" The second is: "Insurance Offices have those who propose to them examined and medical examination is so valuable as showing that the life is sound that everything else may be neglected. For example, if a man has a consumptive family history and is examined when he is thirty and found absolutely sound is it safe to neglect family history?" The third is: "If various diseases are inherited in an intensity represented by about $\cdot 5$, why is the inheritance of duration of life represented by only $\cdot 13$?"

The answer to the first question is, I think, that in studying such problems one must discuss like things. The real test is to compare the mortality of licensed victuallers, or of goldminers who have a good family history with the mortality among people of the same occupation with a bad family history. To put the matter in another way one must not in studying heredity compare the mortality of, say, goldminers who are the offspring of healthy people with the mortality of, say, clergymen who are selected as being examples of the offspring of tuberculous parents. Of course this does not mean that occupation may be neglected in fixing the rate of premium to be charged, but it may mean that a goldminer or a licensed victualler as a parent is no reason for an extra premium.

The answer to the second question is that while a medical examination is a great protection no medical examiner would dare to say that a man passed by him on examination alone would remain a better life than his fellows for even four years. The family history may help when the result of the personal examination has worn off.

The third question reminds us that people do not always die

of the disease from which they suffer, and death acts apparently as a marksman whose shooting is in some respects at random. Accidental causes of death, in which pneumonia can generally be included, probably account for the low correlation, and the values were found in order to test this point.

III. FERTILITY.

This concludes our review as far as it relates to heredity, and from some points of view it might perhaps naturally end here. but actuaries are now called into consultation over matters of wider significance, and the recent work which members of the Institute of Actuaries have done in connection with the Insurance Bill reminds one that there is a tendency for responsible Government officials to make use of the actuarial profession. Legislation such as that of the new Insurance Bill and the Acts relating to Old Age Pensions, Workmen's Compensation, Sanitary and Housing Measures, &c., have perhaps an almost immediate effect on mortality and sickness; they may also have a slower but no less definite effect on fertility, &c. If in combination with other factors the reforms—if that word can be used when the ultimate effects are unknown—affect fertility so as to cause the birth rate to fall among the better, more thrifty, harder working people in any social grade as compared with the birth rate in the less worthy sections of the same grade, the last state of the community may be far worse than the first, for it would mean that we are making our next generation out of the weaker stocks in the community. May it not be necessary for the actuary to consider such problems if he is called in by the State to advise on certain proposed legislation? May it not happen in the long run that all the temporary good even to mortality and sickness rates may be counterbalanced by an evil produced through another channel?

At any rate, it may not be amiss to call attention to what has already been done on this subject; although some of the work was necessarily based on scanty and imperfect material, it indicates the possibility of proper investigation and the lines on which such investigation should proceed.

The problem with which we are concerned is whether there has been a gradual change in the incidence of the birth rate: if it could be shown that there had been a decrease in the birth rate among the better sections of the population and an increase, or a smaller decrease, among the worse sections we should feel

that we had reached a state of affairs that was far from satisfactory. If we also found that this decrease was due to the smaller size of the family it would be worth while investigating whether the elder members of a family were less valuable than the younger, whether for instance, they were more likely to develop mental disorders or certain diseases.

Unfortunately the evidence, so far as it goes, does not bring one much comfort. Statistics from London boroughs seem to show that where there is the lowest type of labour, where there is more general pauperism, where there are more pauper lunatics, families are largest ; where there is presumably more culture and education, the birth rate is least. Nor does the heavier infantile mortality among the undesirable part of the community compensate for the higher birth rate ; the net fertility remains higher.

A comparison of the conditions in 1851 and 1901 showed that the relationship between inferior status and higher birth rate had practically doubled during that period.* These results relate only to one city : they require to be confirmed or refuted by information obtained from other cities and from rural districts.

Apart, however, from such attempts to find a direct solution of our problem we can with advantage consider the relative size of family among normal stock and among such stock as the tuberculous, the insane and the criminal. One would naturally wish to see a lower fertility among degenerate stocks, but the figures given in Table IV will hardly come as a surprise when we remember that no adequate steps are taken to provide for the permanent care of the feeble minded, that the habitual criminal returns to his family and propagates his kind, and that there is little social feeling against the marriage of or into degenerate stocks.

* Heron. *Studies in National Deterioration*, No. I.

TABLE IV.

Class	Source of Data	Computer	Locus	Nature of Marriage	Size of Family
<i>Normal.</i>					
English middle class.	—	K. Pearson	—	15 years at least; began before 35	6.4
Family records	K. Pearson.	K. Pearson	—	Completed	5.3
English intellectual class.	—	K. Pearson	—	„	4.7
English peerage.	Foster's peerage, etc.	K. Pearson	—	15 years at least	5.8
Edinburgh normal artizan.	Charity Organization Society's Report.	K. Pearson	Eugenics Lab. Lecture No. I.	Incomplete	5.9
London normal artizan.	Not stated ...	K. Pearson	Eugenics Lab. Lecture No. I.	„	5.1
Danish professional class.	Copenhagen statistics.	Rubin and Westergaard	<i>Statistik der Ehen.</i> Jena, 1890.	15 years at least	5.2
Danish working class.	Copenhagen statistics.	Rubin and Westergaard	<i>Statistik der Ehen.</i> Jena, 1890.	25 years at least	5.3
N.S.W. professional, etc., classes	Vital statistics, N.S.W., 1895-1902.	A. O. Powys	Biometrika, Vol. IV, pp. 284, 285	Completed	4.6
N.S.W. industrial class.	Vital statistics, N.S.W., 1895-1902.	A. O. Powys	Biometrika, Vol. IV, pp. 284, 285	Completed	5.3
N.S.W. agricultural class	Vital statistics, N.S.W., 1895-1902	A. O. Powys	Biometrika, Vol. IV, pp. 284, 285	Completed	6.2
<i>Undesirable.</i>					
Deaf mutes, England.	National Association in aid of deaf and dumb.	E. H. J. Schuster	Biometrika, Vol. IV, p. 477	Probably completed	6.2
Deaf mutes, America.	Dr. E. A. Fay's data.	E. H. J. Schuster	Biometrika, Vol. IV, p. 477	Probably completed	6.1
Tuberculous stock.	Crossley Sanatorium (W. C. Rivers).	K. Pearson	Studies in National Deterioration, No. II.	Probably completed	5.7
Alburotic stock.	Collected by K. Pearson and colleagues.	K. Pearson	Eugenics Lab. Lecture, Series No. I.	Probably completed	5.9
Insane stock.	James Murray's Royal Asylum Perth (Dr. A. R. Urquhart).	D. Heron	Eugenics Lab. Memoir, No. II.	Probably completed	6.0
Edinburgh degenerates	Not stated ...	K. Pearson	Eugenics Lab. Lecture No. I.	Incomplete	6.1
London mentally defectives.	„ ...	K. Pearson	Eugenics Lab., Lecture No. I.	„	7.0
Manchester mentally defectives.	„ ...	K. Pearson	Eugenics Lab. Lecture No. I.	„	6.3
Criminals ...	Family histories of 1,500 criminals	C. Goring	Eugenics Lab. Lecture No. I.	„	6.6

NOTE.—Childless marriages are excluded.

This Table sufficiently explains itself ; it affords evidence that the less desirable stocks have larger families than those of normal stock, and one naturally turns from this to consider whether the earlier born members of a family are weaker than those born later. If they are there is a double danger in the smaller families of normal stocks.

Table V deals with this problem, and the explanation of the Table is that if the actually insane are compared with the insane stocks (*i.e.*, the population found by taking into consideration all the members of a family of which one or more members are insane) or if the actually insane are compared with the normal classes they contain an undue proportion of first-born. In the same way one finds proportionately more first-born who are tuberculous than one would expect from the number of first-born in the general population.

TABLE V.

Showing number of First-born and Second-born in various classes for a sample population of 1,000.

Class	Source of Data	Computer	Locus	No. of First-born	No. of Second-born
Peerage ...	Foster's Peerage, &c.	D. Heron	Eugenics Lab. Memoirs No. II.	172	163
Family records	K. Pearson, family records.	D. Heron	Eugenics Lab. Memoirs No. II.	152	147
Danish professional class.	Rubin and Westergaard, Copenhagen statistics.	—	—	194	178
Insane stocks	James Murray's Royal Asylum, Perth (Dr. A. R. Urquhart).	D. Heron	Eugenics Lab. Memoir No. II.	168	159
Actually Insane	James Murray's Royal Asylum, Perth (Dr. A. R. Urquhart).	D. Heron	Eugenics Lab. Memoirs No. II.	231	171
Tuberculous Stocks.	Crossley Sanatorium (W. C. Rivers).	—	From particulars given in Studies in National Deterioration, No. II.	177	170
Actually tuberculous	Crossley Sanatorium (W. C. Rivers).	—	From particulars given in Studies in National Deterioration, No. II.	297	217

So far, therefore, one may summarize our information by saying that characteristics are inherited and at the present time there is a tendency for those having undesirable family traits, or, in the widest sense, a weak family history, to provide more than a proper proportion of the next generation.

IV.—CONCLUSION.

I began my review with the suggestion that the information given to insurance offices would probably be of use in the study of the influence of heredity on the duration of life. Might it not also be the case that the same source would help to give us information about those problems of fertility and disease to which we have just been referring ?

The occupation of the life assured is known to us, and this with the size of the policy gives some indication of his position. The number of members of his family is ascertained, and in view of the age at which assurances are effected it is unlikely that the families are incomplete.

As an example of a possible study I may suggest that a comparison might be made between the sizes of the families of (*a*) professional men, (*b*) shopkeepers, and (*c*) labourers who proposed several years ago, and the sizes of families among those with similar occupations who are effecting assurances to-day.* This would give valuable information as to changes in fertility as regards class during a period of years. The results would be of great use even although offices obtain assurances from practically none of those who are in the lowest grades of work and so far as possible avoid the most undesirable.

There are also one or two other points to which we may make reference. Would it not be of some service to insurance offices to know whether members of a large family had greater longevity than those of a small family : might it not be of some commercial value to know whether a family history in certain diseases is more likely to appear in earlier offspring than in later : might not more thorough statistical work on our own data enable us in the long run to take many risks we cannot now assess and find sources of profit which we now neglect ?

These are suggestions only. I know that insurance offices are not philanthropic institutions for the study of interesting social problems, but surely it is true that a great deal is still to be

* The assumption would be made that the grade of the parent could be judged by the grade of the insured offspring.

found out about our own work, and that these social problems may have a closer connection than some of us care to think with the mortality of those people of this country who will be our lives assured in the future.

ABSTRACT OF THE DISCUSSION.

Mr. C. C. MONKHOUSE thought that it would be possible to collect the data of all offices in the manner Mr. Elderton had suggested in such a way that it would be of great use to the companies themselves and would also help to solve the problems of heredity, &c., which seemed to bear such close relation to the prosperity of the country. The fact that American offices, although they had already made one endeavour to study special risks, had considered the matter so important that they were at present engaged on another investigation of the same character, seemed to indicate that they had a very different idea of the value of such statistics from what English offices appeared to have. Until the data were actually collected their true value could not be estimated, and if any such data were collected he thought that offices might also supply other details, which, although of no actual use from an assurance point of view, would be of great value to those who were studying social problems by this new statistical method.

Without in any way wishing to under-estimate the value of statistical enquiry—which must be the only way in which the power of heredity and other like forces could be truly measured—he thought that they could not accept the results without question. The calculus of correlation depended to a certain extent on their being able to graduate the statistical data of the two qualities they were comparing, and he looked with suspicion on any table, however well graduated, if the data used were not extensive. All the results shown in the author's tables depended on comparatively slender data.

The tables given in the paper were those on which conclusions had been based as to the cause of the supposed degeneracy of the race, and if the results were taken as they stood it would certainly appear as if the country were in a very bad way indeed. It appeared that the feeble-minded and the unsound stock are more prolific than the sound stock; that the birth rate is decreasing, and that the decrease is greater among the higher classes than the lower; if the truth of the heredity tables given at the beginning of the paper were assumed, and if the upper classes have practically the monopoly of intelligence and character, one could only come to the conclusion that intellectually England must be very much on the down grade. Were these conclusions justified by the facts? No amount of mathematical theory would extract true facts from inadequate data. Huxley

said, "What comes out of the Mathematical Mill depends entirely on the grist you put into it", and the material in the present instance, being so limited, did not justify the grave conclusions drawn from the results obtained.

With regard to the heredity of physical qualities, the slenderness of the data was perhaps the only adequate reason for doubting the results, but with regard to intelligence, health, &c., he thought there was a much stronger reason for doubt, because it was assumed that since offspring reproduce mental characters in the same degree as their physical characters, therefore, if the latter are inborn, the former must be inborn also. In other words it was assumed that one kind of sameness involves another and different kind of sameness. This seemed at first sight a very logical assumption, but when one considered the great issues at stake and the complexity of mental characteristics as compared with physical, the assumption seemed too drastic. Supposing the results to be comparatively correct, the only conclusion that could really be based upon them was, he thought, that offspring resemble their parents as regards physical and mental characteristics in an equal degree. In nearly every case the environment of the child is the same as that of its parents. That was a very different conclusion from the one that both mental and physical characteristics are inherited with equal intensity.

One of the objects of the calculation of the tables under consideration was to prove what a great force heredity is in determining the future welfare of the nation, and by similar comparisons with regard to environment to show that heredity is practically the only force that matters. He had before him certain published tables giving the coefficients in respect of environment, designed to prove that the force of environment on future generations is almost negligible. These tables compared the health and weight of children with the employment of their mothers, the wages of the father with the intelligence of the child. The data for the investigation were provided to a certain extent by the Charity Organization Society, and so included children of the lower class. This, he thought, did not treat environment fairly. If environment of the parents did influence their children surely such influence would not appear till later in life, so that to study children in this relation only was almost equivalent to trying to prove that the drink habit is not hereditary because children between the ages of 10 and 12 are not addicted to it.

He did not think the results warranted the conclusions based on them, viz., that heredity is practically the only force that matters, and that, consequently, in improving the environment of the people they were only wasting time so far as the improvement of the race was concerned. All outside evidence went to show that the force of environment is very powerful indeed, and he himself thought that environment was the only method available for influencing or modifying heredity. Could one really believe that in about a hundred years we had changed from an agricultural nation and become adapted to City life solely through heredity? This might be

explained by a selective death rate, but considering that modern legislation tends to take more care of the unfit than the fit the argument did not seem to hold good.

It might be said that if environment had as powerful an influence as heredity, how was it that, in spite of the great social reforms and the improvement of conditions, evidence seemed to show that the race was degenerating? A great deal was heard about the improved conditions of the people: but the conditions of the people could not be said to have improved unless the conditions in the city to-day were better than the conditions in the country a hundred years ago. With regard to the decrease in the birth rate, he doubted whether that really was an evil, if it did not get beyond bounds; the fact that the upper classes were not doing their duty in this direction, as well as the lower, was immaterial, unless the former had the monopoly of the desired character, and this he questioned. If they took a thousand children at random from the upper and lower classes and could measure their intelligence, he doubted whether they would find the intelligence of the street arab a whit less inferior: the very fact that, in spite of every disadvantage, there were men who rose from the very lowest class to be leaders of men was a proof positive what a mine of intelligence there must be if only opportunity were given for that intelligence to show itself.

Mr. H. J. REITSCHEL asked whether statistics proved that the transference of children of tuberculous parents from badly ventilated, insanitary and over-crowded homes into more favourable surroundings would not reduce tuberculosis amongst such offspring. Tuberculosis and unfavourable environment were closely connected so far as the general population was concerned, and it would therefore be interesting to know how far the author had considered this question of environment in connection with, say, the tuberculous family history of the 1,500 criminals in Table II, whose criminality may itself have been partly due to surroundings detrimental not only to morals but also to physical health.

The author stated that in order to estimate the correlation between a disease in parent and offspring it was necessary to start from people known to suffer from it and ascertain how many of their relatives also suffered from it and how many have not suffered from it. In order to complete the information it would also be necessary to estimate the corresponding number of parents without the disease having offspring also without it, and this should be estimated from considerations based on the general population. Unless environment had no effect on, say, the spread of tuberculosis, should not the comparison be made with healthy parents and their offspring, living in the same kind of environment as the tuberculous parents and their offspring, in view of the fact that disease and poverty frequently follow each other? This comparison, however, was probably impossible of attainment. As to physical characteristics, the Poor Law Commission said, referring to Poor Law children, "Few such children in after life fall back into pauperism; and it is probable that the children in some at least of the present Poor Law Schools

are being better fitted for earning their living than those outside." If these remarks were based on facts, then environment would seem to be of importance.

With reference to the applicability of these investigations to life assurance he felt some doubt because no two cases requiring consideration were identical. The lives having a tuberculous family history would require subdivision according to whether one or both parents had had the disease; according to the physical development of the assured; according to habits; according to pecuniary circumstances; according to occupation; and so on. The number of subdivisions would be so great as to render the final results of doubtful value on account of the paucity of the data in each section. Then there was the influence upon the statistics of the selection exercised by the offices, probably the most important factor of all, of varying intensity, and therefore quite incapable of assessment. If their methods of selection were altered as a result of these investigations, would not a new body of lives be admitted, bearing no sort of resemblance to those upon which the recent investigations had been based?

Mr. MAJOR GREENWOOD said that he would confine his remarks entirely to the special side of the subject which appealed to a medical statistician. The point which would probably strike the members, as professional mathematicians, was the extraordinary badness of medical statistics. For instance, if such a simple question as the following was asked—"What is, on the average, the case mortality of any of the common diseases, such, for instance, as acute pneumonia?" it would be impossible to give any adequate answer. What a physician would probably say was that the prognosis in a case of disease depended so much on the individual circumstances of the case that a general answer was very likely to be fallacious. That was quite evidently true, but it was impossible to find any adequate data upon which one could judge the average prognosis in any particular class of society for the common diseases. It seemed to him that on the question of the general mortality from such diseases as pneumonia, some collaboration should be possible between the mathematician and the medical adviser of life insurance companies. For instance, quite recently he was at a meeting of a Medical Society when a paper was read on Tuberculosis by the medical officer of one of the large companies, and he published in that paper a table in which he compared the difference in the incidence of tuberculosis upon those persons who had tuberculosis in the family and those who had not, but no effort was made to distinguish between direct inheritance and collateral inheritance. It was impossible to ascertain from the author's figures or from the discussion that followed whether the parents, or the consins, or the granduncles, or what other members of the family had had tuberculosis, the ages at which the various persons had died, and other matters that would naturally occur to an actuary. The impression he derived from contemporary medical literature was that the medical officers of life insurance companies ought, if he might say so, to be put more under

the control of the actuary than they were. They made general statements on the subject of the inheritance or otherwise of disease from their experience, and those statements were not supported by any evidence.

With regard to the question of environment, he suggested that a good deal of the discussion had been due to the fact that those persons who held, as he thought Prof. Pearson and Mr. Elderton held, that environment was a relatively unimportant matter in racial evolution, were generally supposed to believe that it would be a desirable thing to give up improving the environment. That seemed to him to be an erroneous deduction. To take an analogy which he thought Prof. Pearson had used, the best grindstone in the world would not make a good cutting-edge on a piece of bad steel; but it did not follow, if that position were taken up, that a better result would not be obtained with the best grindstone possible and a poor steel than if nothing at all were done. Consequently, so far as the question of improving the general environment of the slums and so on was concerned, the real point which Prof. Pearson and his associates seemed to be raising was that too much must not be expected in that direction. There was, he thought, far too great a tendency on the part of medical officers of health to proclaim the coming of a new heaven and a new earth if they were allowed to open people's windows and so forth. When it was found that many of those assertions were unjustifiable, the general public would rush to the conclusion that they had better stop doing all those things, and therefore from that point of view Prof. Pearson was really a better environist than the environists themselves.

Lastly, he desired to give an illustration from purely medical statistics of the kind of difficulty which occurred in connection with environment, namely, the question of infant mortality, which had been touched upon by Mr. Elderton. If the statements of any medical man on the subject were read, it would probably be found that two facts were put forward as being the chief causes of high infant mortality. One was the practice of artificial feeding and the other was poverty. It was very easy to construct quite a satisfactory explanation of infant mortality on that hypothesis. The children were in dirty homes and not properly looked after; they were fed from dirty feeding bottles; and septic contamination, diarrhoea, and so forth resulted. As everyone knew, about one hundred years ago Malthus, in a work which appeared to be less read than it should be, argued the old-fashioned view that a high infant mortality went with a high birth rate, and that the chief factor in infant mortality was the birth rate. Was that so? Dr. Newsholme had recently argued that there was no necessary connection between the birth rate and infant mortality. From his own figures a correlation of .36 was established, but he spoke of that as being a relatively low fraction. Of course, it could not be larger than 1. The difficulty was once more to get data; and about the best, or the least bad, data that he knew were those collected in Bavaria two or three years ago by Drs. Groth and Halm. They gave for all the districts

of Bavaria, the birth rate, the infant death rate, particulars as to the prevalence of artificial feeding, and—as a measure of poverty—the proportion of persons who received some form of public assistance. If all those data were treated by the method described by Mr. Elderton it would be found that the correlation between poverty and infant death rate, correcting, of course, for constant population and keeping the artificial feeding constant, was hardly larger than its probable error. At any rate, the correlation between the birth rate and the infant death rate, keeping poverty constant and keeping artificial feeding constant, was very large indeed, which led one to the suspicion that those views of the older writers should now be entirely put on one side.

Mr. O. T. FALK expressed the hope that some competent authority on the subject would, later in the evening, say something on what he thought was the important point for the members of the Institute to bear in mind, namely, the business side of the problem, rather than the purely statistical. He understood that, in a paper read a fortnight ago, Mr. L. P. Orr accused his professional brethren of not really making use of data at hand, and the present paper seemed to him to make the same reproach. He felt himself that that was a charge they should not admit, on the following grounds. He thought the primary question for insurance companies was not whether heredity or environment counted most, but whether either heredity or environment counted sufficiently for them to make an accurate measurement of the effects in the way which had been suggested. There were so many factors that affected the problem that it would be impossible to measure them all accurately, and to calculate rates for life assurance giving effect to them all. His reason for drawing attention to the point was that he was one of those people whom someone had called “lumpers”, i.e., he felt that insurance companies might take practically all their risks at the same rate of premium and have no rated-up lives. In saying that he knew he was very much opposed to the modern tendency of life insurance offices, although one large company acted on that principle, either declining a risk or accepting it at its ordinary rate of premium.

Mr. GEOFFREY MARKS referred to Mr. Orr's paper read before the Faculty. He thought the discussion, so far as it had gone, illustrated very well the necessity for some such arrangement as that which Mr. Orr proposed. It was obvious that the fact which emerged from the discussion was that all sorts of different opinions, based on considerations which could not be reduced to exact dimensions, arose, when the foundations which were necessary for an accurate discussion were wanting. He did not wish to follow Mr. Orr so far as to suggest the immediate establishment of a permanent Bureau of Research. He would like to mention in that connection that a similar proposal was made at a meeting of the Institute some years ago by Mr. C. R. V. Coutts. He thought if the Institute were to apply itself to collecting data on some specific points, possibly on cards which would admit of other details being inserted which might be of use in a more extended investigation, that

was as far as they could be expected to go at present. He wished to point out that the experience which had been collected in America during the last few years would provide the Institute with a very large body of data for comparison, and would also enable the members to judge what points were essential in such an investigation and what might be neglected to some extent. Even on such a detail as the form of the cards, which was of great importance, they would have the accumulated experience of two investigations in America, one past and one in progress now, to guide them. There were some minor matters arising out of the paper upon which he would have liked to offer some observations, but he felt that the large scope of the question which the author had raised was far more important than any question of detail. He confessed he was to some extent in sympathy with Mr. Faik's views, but he thought it would not be safe at present to put them into practice, on account of the absence of those statistical data which it was necessary to have if a solid foundation on which to build was to be laid down. He concluded by expressing the hope that the Council would see its way to taking up the very important point Mr. Elderton and Mr. Orr had raised, not only on account of its value to the members in their professional duties, but because he could conceive many aspects of social and economic work in which such investigations as were suggested would be of immense service, particularly in those researches with which Prof. Karl Pearson's name was specially identified.

PROF. KARL PEARSON said that however bad the statistics bearing on any statistical question might be, they could only be met by better statistics. He thought the Institute of Actuaries should, if possible, amass statistics. He admitted that relatively to the 20,000 cases with which actuaries were accustomed to deal, the thousands of cases dealt with in the Eugenics Laboratory might seem small, but he was not quite sure that they warranted the word "slender." For instance, each one of the entries in the tables in relation to physical characteristics was based upon four tables of one thousand each: the only question of doubt was the probable error of the result, and it was quite certain that in one thousand cases the probable errors were extremely small. He therefore did not think the slenderness of the data would explain the results. Coming to a further point, there was a very general agreement in Tables II and III, but there were two results in each table which were markedly different from the rest, those, namely, for longevity or the duration of life. The results were arrived at with a view to finding how many deaths were selected and how many were purely chance. If all deaths were purely chance—if they were simply shot at roughly by marksmen or killed by chance—there should be a zero relationship. The fact that it was not a zero relationship but that there was not the full strength of heredity in those cases enabled them at once to see what was the proportion of selected to non-selected deaths, and from those data it was found that roughly from 50 to 75 per-cent of deaths were selected, *i.e.*, they must depend upon the physical constitution of the individuals who died.

He thought some weight should be laid upon the divergence in those cases; because they were the very cases *à priori* in which investigators would expect a deviation from the full intensity of heredity.

It had been suggested that the agreement between the coefficients found for psychical and physical qualities was possibly due to a simple chance relation between the two causes, environment and inheritance. It was very strange, however, that on the full range from 0 to 1 they should come out practically equal. Why should not the environment have given .7 or .3 for the psychical qualities? One of the things that was worth remembering from mediæval logic and philosophy was "*Occam's razor*"—that it was desirable, when one explanation would do, to take one and not provide two. The use of "*Occam's razor*" was, he thought, a general principle of science. But he would be very sorry indeed to leave the matter there. In the cases of psychical and physical characters of children they were dealing practically with school children of the same age, or reduced by growth curves to the same age. Absolutely the same children, or similar children, had been treated from the environment standpoint; the same as regards ability, health, weight, and so on. The enquiry was perfectly unprejudiced, and although they expected in the case of alcohol to find a very high correlation indeed between the badness of the parents' environment, the badness of the wages, the badness of the home conditions, and the health and fitness of the children, it came out surprisingly little. In an investigation of the kind to which he was referring, until the numbers were actually worked out there were no means of saying what the result of the investigation would be. Quite recently his laboratory had received an enormous amount of material concerning infantile mortality and matters of that kind. They had tried every conceivable object, and of all of them it had been found that baby pacifiers were probably the worst thing for infantile mortality. They were a long way worse than artificial feeding. They were a long way worse than the employment of the mother. The employment of the mother was about as bad as the occupation of the father, and neither of them was very material. That probably arose from the fact that a feeble man, physically and mentally, received a poor wage, therefore his wife had to go into employment, and the result was that it was not so much the employment of the mother that mattered. If the results were corrected for breast-feeding it would be found there was very little relation between the employment of the mother and the mortality of the child, but if there was a feeble man, as measured by his wage, then it would be found that the child was bad. He took it that that was heredity and not environment, but it would be put down to environment by those who were very anxious to improve environment.

It was precisely the same with the question of back-to-back houses. The Local Government Board had just issued a Report proving how very bad back-to-back houses were. The Eugenics Laboratory happened to have particulars of something like six thousand cases of back-to-back houses and non-back-to-back houses, and they at

once set to work to investigate the point ; and it was found that when allowance was made for the type of people who lived in back-to-back houses, their wages, habits and so on, back-to-back houses made absolutely no difference to the results. He was only able to refer very superficially to the question of Dr. Goring's criminals, which had been mentioned in the paper. Dr. Goring and two other officials from the Prison Commission had recently been working in his Laboratory, and had drawn up a Report on 3,000 criminals, of which the first 1,500 were referred to in the paper. The question of the home environment of those criminals, their family histories and so forth, had been investigated. The Report was at present confidential. It was to be presented to the Prison Commissioners and would, no doubt, in due course appear. An answer would then be given as to how far environment affected criminals. He had not the least doubt in his own mind that most of the statements in the Government Report on Degeneration were entirely fallacious. They consisted chiefly of personal impressions, and not of things reduced to numbers. The Laboratory possessed at present dozens and dozens of pedigrees of pauper children and pauper parents. In all those cases it was found that pauperism ran from generation to generation : and if any school medical officer were questioned he would state that the greatest difficulty he had was with the over-fed pauper children, because, notwithstanding their over-feeding, they were so frightfully stupid ; they could not work with the other children, who might be half starving. They were the lower portion of the population. He did not use the word " lower " in the sense of a poorer population or a richer population, but in the sense of a mentally and physically lower section of the particular class of the population that was being dealt with, and from that point of view it was a perfectly right word. He knew nothing about upper or lower classes, but he did know there was a section of the population which was mentally and physically inferior, and naturally it gravitated to the low wage-earning portion of the community. But he had never made any such statement as that the whole of the ability, or the bulk of the ability, came from the upper classes. As a matter of fact, in one of his papers he showed that about ten times as much of the marked ability in the country came from undistinguished stock as came from distinguished stock, but that was only one point to note. Another point to note was that distinguished stock produced approximately ten times more distinguished men than it did undistinguished men. It was because the undistinguished stocks were about 100 times as large as the distinguished stocks in number that the bulk of the ability came from the undistinguished stocks.

With regard to the birth rate, he did not think that the table cited by Mr. Elderton fully brought out what was now known. Taking the purely intellectual classes, he was afraid the figure of 4.7 per-cent which he gave about 1885 was very exaggerated ; it would be found that 2 to 3 per-cent was a more correct figure, when allowance was made for light mortality. It would be found that the intellectual classes had an extraordinarily small death-rate among their children,

and that 2 or 3 per-cent would cover their reproductive power or, at any rate, their reproductive design.

Mr. JAMES PULLAR thought that valuable results could be arrived at by a comparison on very broad lines of the influence of heredity and environment—by comparing, for example, the records of the same race under different climatic conditions and different influences. He had been for 25 years in Australia, and he was convinced that the rate of vitality in Australia was very much more favourable than select vitality in this country. People lived there under different conditions and in a different environment perhaps, and it must be borne in mind that the question of environment was the controlling factor in obtaining better results. Australian societies had distinctly more favourable rates of mortality than those which prevailed in this country with similar lives, and he thought probably some of the author's conclusions might be affected by that fact. He was quite certain that the collection of data from life assurance practice would be most valuable. He was very much surprised to hear the opinion of one speaker, that he would like to insure all lives at ordinary rates. He thought all practical men knew that that was quite an impossibility, and that the whole difficulty which presented itself to life assurance managers was to assess lives in such a way that they were all equally handicapped.

Mr. S. G. WARNER, in closing the discussion, said that the subject as presented had three aspects: one which was intimately connected with the practical business of life assurance; a second relating to larger applications of the insurance principle which might in the future come under their survey; and a third, of a wider and more general nature. In the comments he had to make he would deal with those points in order.

The first aspect brought before them the question of the "rated-up life", who was always a source of difficulty. When, after much exertion, a proposer had been secured, he was medically examined, and then perhaps had to be told either that the company did not want him at all or that it would not take him on its normal terms. That was a disappointment to him, and an awkward situation to deal with. It was suggested that the companies had not been scientific in their treatment of the problem, and perhaps they must plead guilty to that indictment. They had had their hands pretty full of other work. They had, in little more than a century, so developed actuarial science on its practical side, as to build up the fabric of British life assurance to its present dimensions on a sound and solid foundation. Possibly then they had not hitherto been able to deal with the difficulty now under consideration so scientifically as might have been desired. A "rough and ready" method had been adopted, of adding years to the age. It was found, however, on the whole that the plan worked well in practice, although there again the facts had not in any systematic manner been scientifically ascertained. It was now suggested that actuaries should do with records of family history what they had already done with general

mortality experience; collect the data relative thereto which must exist in abundance, and endeavour, from the information so revealed, to arrive at some more systematic and scientific method of assessing under-average risks. It was difficult to see any objection to such a suggestion: it seemed to be an admirable one, and he hoped something might be done in that direction; but it must be remembered that the data would be exceedingly complex. They would fall into a large number of divisions and would depend a good deal upon unverified statements. Nevertheless, he admitted that the difficulty and complexity of such a task ought not to discourage or deter them, and he simply raised such points as cautionary considerations which should be in their minds when entering upon it. If it could be accomplished with any approach to accuracy its beneficial nature could hardly be questioned.

The second aspect of the subject related to questions which might in the future come before actuaries more than they had done in the past. The tendency of modern legislation was such as to bring under consideration more closely than hitherto questions regarding the health and mortality of the general population. That brought him to the question which had evidently interested most of the speakers that evening. It was the old question of the relative strength, as forces affecting humanity, of heredity and environment. It was not to be wondered at that most of the speakers devoted a good deal of attention to that subject, because it was one of great interest. Its bearing on the actuarial treatment of questions affecting national mortality was of vital importance. It was asserted—and not disputed—that the birth rate in this country was steadily declining. But a further important statement was made, and professedly justified by the statistics placed before them. It was pointed out not only that the decline existed, but where it existed—the relative incidence of that decline. It was shown that two forces were at work, one of them the tendency, as the social ambitions of modern life increased, to limit the families of the comparatively well-to-do, careful and intelligent classes; the other the tendency, admirable in many ways, of modern philanthropy to do all that possibly could be done to make the conditions of existence more tolerable for the classes who were not very intelligent or well educated, and who rested at the base of the social fabric, a tendency which might lead to the preservation and multiplication of the comparatively unfit. It was shown that there was a double danger here. The birth rate was declining in the wrong place, so far as the future prosperity and stability of the nation and its intellectual strength were concerned. That was a statement which the actuary would have to take very carefully into account when advising upon questions which affected national vitality. To the business that most of those present were engaged in, that of ordinary commercial life assurance, this was, if important, not so important, because they had the safeguard of medical selection, and they drew their lives on the whole from classes with which they were fairly well acquainted. But when national questions had to be dealt with the

contention assumed great importance, and what had to be considered was the evidence brought in support of that contention and the inferences to be drawn therefrom.

Thus they were led to the third or general aspect of the subject, stretching beyond merely actuarial questions, and appealing to all who were interested in the future welfare of their country. He desired to say a few words about this, because the discussion had naturally shown a tendency to include that aspect and not confine itself exclusively to technical ones. It was primarily a disappointment when one was confronted with any statement which would exalt heredity as a force at the expense of environment. It was a disappointment to all wellwishers of their fellow-men, for one great reason. Heredity was a thing with which it was very difficult to deal. Any attempts that were made to do so must be fitful, tentative and cautious, and under present social conditions it was not possible to do much, so that the process would be a slow one. But everyone was apt to think that environment was something which it was within his power to improve, something upon which he could set to work, contributing his little quota to leave the social state rather better than he found it. We had been accustomed, perhaps too much, to consider this the more important force of the two, and to take a comparatively hopeful view of the national future because of what it was felt could be done with regard to environment: and so we had been disposed to attach less weight to the other factor. Those who tried to do their work in a scientific spirit knew, however, that they had to look facts in the face: to get rid, as far as possible, of preconceptions, social or political, and to deal with things as they stood; to recognize, in the words of a great thinker of a bygone century, that "Things are what they are, and will be what they will be. Why, then, should we seek to be deceived?" He had been very much impressed by the arguments from heredity as presented in the light of the methods explained and figures adduced in the paper. As had been said by one of the speakers, it was entirely illogical to suppose that those who assumed the attitude he had been describing deprecated attempts to improve human environment or human society. By all means let these things be done. The unfortunate thing was that one felt, if faith in environment as a force which could appreciably advance the world's welfare were lost, the mainspring of a good deal of the best work being done in the world would be broken. At the same time one was compelled to admit the great force of the argument that had been put forward. It had been said, for instance, that boys rose from the lowest rank of society to be distinguished men. They did, and one was glad that they did. But, then, how many boys did not? How many boys were there in that class? If the progress of the average boy therein was compared with the progress of the average boy of a different social class, what was the result? That was where the real force of the heredity argument came in.

After all, was it not open to us to take courage and some comfort from the thought that possibly efforts, earnest enough, well enough

directed, skilful and persistent enough, to improve environment, might by slow degrees so lift up the social condition generally that, in turn, something might thereby be done to improve heredity itself? Might they not be able to form, even out of unpromising surroundings, the beginnings of a race of men and women who would do something in the future to cast out the heredity taint with which they had been born, and to climb by slow degrees to something better? It would be a pity for anyone possessed with the desire to work on such lines to feel disappointed or cast down by the force of the argument presented that evening. He felt, however, as had already been pointed out, that the risk of leaving heredity out of account was a subtle and a dangerous one. It was also one apt to beset enthusiasm for social regeneration, and therefore it could not but be a salutary thing to have the subject brought as it had been before the meeting. In any case, the members of the Institute could not thank Mr. Elderton too warmly for having brought under its notice questions of such absorbing interest; questions fraught with issues of such immense importance: questions so intimately interwoven with the well-being of the nation to which they belonged.

The PRESIDENT said that the discussion in all its bearings had been so admirably summed up by Mr. Warner that he would not re-open it; but he desired to say in regard to the practical part of the paper, dealing with extra premiums, that he looked forward to the time when some member of the Institute would take into consideration the American system of tabulating and classifying under-average lives in connection with the investigation that was now being conducted by the New York Life Office under Mr. Arthur Hunter's charge, and discuss the merits of that scheme of classification. In the criticisms passed that evening no suggestion had been made as to the manner in which the data available for dealing with extra risks or under-average lives could be turned to the best account. It would be remembered that in the American investigation the actuary and the doctor first started together to decide upon certain deviations from the normal to represent various classifications and categories of risks of a special character. Starting from 100 as the normal index they would assess at, say, 115 a risk of some special character. For instance, a man who had one parent and one collateral who had died of consumption, might be considered to fall in the class represented by 120. Thus the doctor and the actuary, working together, assessed a weighted number or index for almost all the main classes of extra risks which had to be dealt with, and in that way subsequent dealings with vast masses of material seemed to be immensely facilitated, as separate investigations could be made of the observations falling under each classification. He hoped one of the members of the Institute would take in hand a close investigation of the American system, for the purpose of explaining its merits and, possibly, of indicating its defects. He thought it was a plan of dealing with the matter of which actuaries had not taken sufficient account in this country. Americans claimed for it that it was the best means available for making the rate fit the

risk. In conclusion, he wished to say that he was afraid he did not perceive in the practical working of life assurance any greater desire to deal with the business on scientific lines than existed twenty-five years ago. He thought rough-and-ready, general, and easily-understood methods of conducting business were being adopted more and more, but that might be because the actuaries had not yet brought their scientific analysis up to a point at which the necessity of greater refinements had been clearly established.

Mr. W. PALIN ELDERTON, in reply, said he felt he ought, in the first place, to express his appreciation of the kindly criticism the paper had received. Much of the criticism on a statistical paper was given after a very rapid perusal of it, and results that had taken a long time to work out were very frequently dismissed with a reference to "practical experience." The fact, however, was overlooked that the statistics themselves were really practical experience which had been properly collected, and expressed all the facts instead of only the facts dealing with one side of the problem. He supposed it was inevitable that as the result of his paper the old talk about heredity and environment should occur. He was almost inclined to believe that Mr. Monkhouse was an agnostic on the subject, and that he expressed the views he did simply for the sake of debate. But if he was wrong in that conjecture he suggested that Mr. Monkhouse should think of the subject in the following manner. People were born with their heredity; it was with them all their lives; it did not vary and one did not take much note of it. But environment changed; its changes were noticed, and much was therefore attributed to its influence. Might it not be overlooked that environment would affect individuals in different ways, according to different natures received from their ancestors? Environment might affect an individual or a generation; it could, he thought, only affect a race if it so acted as to render one class more fertile than another. So little did actuaries think of it from the point of view of its connection with successive generations that they never troubled to ask a man who wished to be assured what the occupation of his parent was. The majority of Mr. Rietschel's points were answered by Professor Pearson: but he thought Mr. Rietschel made a mistake when he implied that the different practice of offices would affect the results obtained from the statistics. Surely if two offices received a proposal in which both parents, for instance, died of consumption, it would not matter to the statistician, who ultimately worked the figures out, if one of them took the case with five years' extra and the other with ten years' extra, or even if one of them took it at the ordinary rates. He did not think that it would matter even if many of the cases were declined altogether; if up to, say, five years ago no office ever accepted a certain type of case he did not see how the statistical work would be damaged if certain offices began to accept such cases.

He suggested, in reply to Mr. Pullar's remarks, that the possible selection of emigrants might be worth considering in connection with the data. He thought the answer to Mr. Falk's defence of the

actuaries of the past, was that they were not professedly "lumpers", therefore they ought to have worked out their statistics to see what they were doing. If Mr. Falk said that in the future he wanted to be a "lumper" that was no real defence for the people who had been working in the past. He was inclined to think, however, that it was impossible to lump until one knew what to lump. One would, of course, hope that any investigation made would prove that a large amount of "lumping" was permissible. He thought the answer to some of Mr. Warner's doubts was that he must not over-divide. It was unnecessary to split up everything into the smallest possible groups. Mr. Orr had made a splendid appeal to the Faculty for research. He did not know when he wrote his paper that Mr. Orr had written one. Perhaps in some respects that was as well, because if he had done so he might not have had the pleasure of listening to the discussion that evening. Personally he wished that some such Bureau as Mr. Orr suggested could be set up: it would put much that was being done on a better basis; it might encourage them to be a little more scientific, and it might enable actuaries to be of more service to the companies that employed them and to the community at large.

The Investment of Life Assurance Funds. By G. E. MAY, F.I.A.,
of the Prudential Assurance Company, Limited.

[Read before the Institute, 29 January 1912.]

AT first sight one might perhaps be loth to admit that a very close analogy can be drawn between the insurance of under-average lives and the investment of life assurance funds. It seems to me, however, that in some important respects the two operations are comparable.

I think it must be admitted that practically any life, however bad, would be insurable provided the risk to be run could be accurately assessed and the requisite premium charged. Similarly, almost any investment would be worth buying provided that the risk of the loss of capital could be accurately gauged and the rate of interest was sufficiently large to pay what may be called an investment rate and to provide a sufficient margin to cover the risk of loss of capital.

In practice it is, of course, impossible to judge the risks that will have to be run with the degree of accuracy required to enable, in the one case, any life to be accepted, or in the other, any investment to be purchased at a price, but it is, I believe, quite possible to go a long way in both directions, and it is mainly with a view to obtaining the opinions of the Members of the

Institute on this point in regard to investments that this paper has been written.

Practically all life assurance offices accept slightly damaged lives, but I understand that it is the practice of some offices to go still further and to accept proposals on lives which have been or would be refused by other companies. Similarly, higher rates of annuity are granted by some offices on those lives which, after medical examination, are found to be impaired. A somewhat higher rate of premium, however, is charged, or in the case of annuities a somewhat lower annual amount is granted, than it is estimated would really be required; in other words what may be called a safe rate in both cases is charged. I believe it to be a fact that on both these classes of business a larger percentage of profit is made than from ordinary risks, and this is quite in accordance with what one would have anticipated.

In the same way it will, I think, be found, that by careful selection relatively high yielding investments can be obtained which, after deducting from the interest what I have termed an "investment rate", will show a margin considerably greater than is required to cover the risk run.

It is obvious that both in the case of impaired lives and also in that of investments the success or failure of any new course of action can only be ascertained at some future date by the results obtained. It is true that in the case of normal lives the results can be foretold with great precision, and in the case of slightly impaired lives with a moderate degree of accuracy by means of past experience, but in the case of more seriously impaired lives we have but little, if any, past experience to serve as a guide, and it is only by an intelligent anticipation of the future that this business can be successfully undertaken. In the case of an extension of the field of investments for insurance companies it is, however, often possible to obtain a fairly good idea as to the future by means of past history, and this, coupled with an intelligent anticipation, should enable one to form a reasonably accurate forecast.

It is, of course, possible to investigate the past history of groups of investments, but so much depends upon the personal element in regard to selection of investments and the relative amounts invested in them, that I do not propose to introduce any such investigation into this paper. I would, however, add that the researches I have made in this direction have gone far towards convincing me that the investment of funds in relatively

high yielding securities carefully selected, not only gives a more satisfactory return than that shown by the lower yielding stocks, but on the average gives greater security for the capital.

I have used the words "on the average" advisedly, since it is obvious that some of the higher yielding investments do not turn out so satisfactorily as others; this is probably equally true of the lower yielding securities, though perhaps to a less degree, both in regard to appreciation and depreciation in the market value. In the case of high yielding investments, however, you have a margin of interest, over and above a fair investment rate, upon which you can rely, and we all know well how even a small margin will, at compound interest, rapidly amount to a considerable figure.

I have been speaking of low yielding and high yielding securities, and in this connection I think it is desirable to say, though it may possibly appear to be a truism.

(1) That because a security gives a low yield it by no means follows that it is a bad investment; on the other hand, because the yield is low the capital is not necessarily secure;

(2) Similarly, because an investment shows a high yield the capital is not necessarily insecure; nor does it follow that because the yield is high the investment will prove remunerative.

The placing of a certain portion of an insurance company's funds in relatively high yielding securities gives the further advantage of a considerably increased field for investment, thereby affording greater opportunities for averaging. It cannot, I think, be emphasized too strongly how important a factor this is in contributing to the safety of the capital as a whole. Since all of our assurance business is based on the law of averages one would naturally have thought that this principle would have been given full weight when considering the question of the investment of funds. I am afraid, however, that in the past very little attention was paid to this point; the ruling consideration appears to have been to endeavour to satisfy oneself as to the safety of the capital in each investment considered by itself. I do not at all wish to argue that this is not one of the most important factors for the safety of the capital, but I do wish to protest most strongly that it is not the only one. To what extent this practice was pursued in the earlier days is shown by the lists of assets of four leading insurance companies set out in the paper read before this Institute by Mr. Samuel Brown in January 1858. I reproduce these lists for reference.

Eagle Life, 30th June 1857.

Investments	Amount	Proportion per-cent	Bearing Interest per-cent
	£		
Government Securities ...	79,731	6·13	3·365
Mortgages	931,831	71·69	4·750
Decreasing Mortgages ...	76,483	5·88	3·963
Reversions	42,780	3·29	4·
On Policies	43,963	3·38	5·
Temporary Investments ...	97,000	7·46	5·
Unproductive	28,212	2·17	
Total capital	1,300,000	100·	4·500

London Life, 30th June 1857.

Investments	Amount	Per-cent of Total	Bearing Interest per-cent
	£		
3 per Cent stock	111,269	4·20	3·478
Exchequer Bonds	175,000	6·60	3·500
Canada Debentures	100,000	3·77	4·
Government Annuities ...	20,899	·79	4·938
	407,168	15·36	
Bank Stock	96,480	3·64	4·478
Mortgages, and on Policies ...	2,092,524	78·95	4·439
House, &c., unproductive ...	54,259	2·05	
Total capital	2,650,431	100·	4·234

Rock Life, 31st December 1856.

Investments	Amount	Per-cent of Total	Bearing Interest per-cent
£3,725 Government Annuities, 1880	£		
2,200 do. do. 1885			
7,002 Government Life Annuities			
4,755 Terminable Annuities...			
£17,682, say, 15 years' purchase	265,230	13·53	Say 4·
Canada Debentures ...	200,000	10·20	4·
British Guiana Debentures ...	65,000	3·32	4·
	530,230	27·05	
Mortgages ...	1,000,566	51·06	4·233
Railway Debentures ...	280,000	14·29	4·913
Dock Debentures ...	50,000	2·55	4·
Loans on Policies ...	69,449	3·54	5·
Cash at Bankers', House, &c....	29,661	1·51	
Total capital ...	1,959,906	100·	4·229

Metropolitan Life, 4th April 1857.

Investments	Amount	Per-cent of Total	Bearing Interest per-cent
	£		
£1,170 Annuities for Terms ...	3,175	·42	4·441
£15,000 New 3 per Cents ...	14,619	1·92	2·989
£40,000 Turkish Bonds... ..	41,685	5·46	3·725
£12,000 Exchequer Bills ...	12,019	1·58	3·686
	71,498	9·38	
Mortgages ...	352,764	46·24	4·537
City of London Bonds... ..	45,000	5·90	4·584
Railway and other Debentures	188,752	24·74	4·304
On Life Annuities ...	2,174	·28	4·278
Reversionary Interests ...	11,956	1·57	5·002
On Policies of Society ...	65,668	8·61	4·838
Cash, and House and Furniture	25,014	3·28	
Total capital ...	762,826	100·	4·303

An examination of these figures will show how extremely limited was the field covered. Although it is obvious that the opportunities for a wider range of investments were at that time insignificant when compared with the facilities of the present day, still it does appear to me extraordinary that the investment of these large sums should have been confined to so limited an area.

There can, I think, be no question but that the practice of the present day is to spread investments and to increase the field; however, speaking generally, there is still considerable room for extension. It is in most cases very difficult to form any accurate opinion from the published balance sheets as to how far progress has been made in this direction by the various offices, owing to the grouping of securities; still, a careful examination of the figures shows that, although some are still very backward in this respect, others have made considerable progress, and as far as one is able to judge, with satisfactory results. On broad grounds, however, I think we may assume that the principle of extension of investments has been financially successful, for it would appear to be mainly owing to this practice that the average rate of interest was kept at so satisfactory a figure during the years of falling rates. Further, the troubles arising from the recent heavy fall in prices of securities would have been much greater had the old practice continued, for then the investments would have been mainly confined to high class Home Securities in which the decline has been most marked.

The question is extremely difficult, and one on which it is impossible to obtain definite information, but from the investigations I have made, I believe that I am right in saying that those offices which have spread their investments the most, and at the same time have gone furthest a-field, have suffered the least from depreciation and have also been the ones to obtain the most satisfactory returns.

Whatever the principles decided upon by an office for the investment of its funds, it must be admitted that for best carrying them into effect it is necessary to have some one who has not only made a special study of the subject, but who, above all, has had practical experience. A special training is required for the investment work of an office just as much as for the actuarial or managerial work, and although in smaller offices it is customary to combine certain appointments, there can be no doubt but that the investment work not only deserves

but requires the undivided attention of an expert. The duty of such an expert would be to carry out the skilful investigations necessary and to express his opinions to the management, who in turn would advise the directors with whom the final decision rests.

It may be argued that the expense would be considerable, and whilst fully admitting the truth of this, I feel sure not only that such expense is necessary and ought to be incurred, but that the amount would be insignificant when compared with the resulting gain. I believe that the lack of expert knowledge in this branch of our work in the past was to a great extent the cause of the limited field covered by the investments of assurance companies.

It may also be necessary on occasions to obtain expert opinions in regard to investments from outside sources exactly as is now done in regard to real estate in the case of a possible mortgage or purchase. In order to make myself clearer I will deal with an example which recently came under my notice. One of the larger American Offices was approached in regard to the purchase of a block of bonds of a railway in the United States. After investigation the finance committee decided that the proposition seemed a sound one according to the then financial state of the railway, but they wished to assure themselves not only that the line itself was in good condition, but that the future of the undertaking was likely to be prosperous, thus tending to the increased security of the bonds in question. They therefore instructed certain experts to make an examination of the line and to report thereon. The bonds formed part of a new issue, and it was, of course, to the advantage of the railway to afford the necessary facilities. A complete examination of the undertaking was carried out, and since the experts in their report stated that the line was in excellent condition, and that in their opinion there was every prospect of increased prosperity in the future, the Company took a large block of the bonds. The sequel was rather curious, for some of the other companies who had previously refused to entertain the security also participated in the issue when they heard of the results of the investigation. In an investigation such as I have been recounting it must be recognized that it was only after having *prima facie* satisfied themselves that the security, as far as they were able to judge, was good, that the further investigation was undertaken with a view to confirming the original

opinion formed. The point I wish to emphasize is that sufficient material must be available in order that a fairly good opinion can be formed as to the security of any investment, and that one must not rely too much upon the opinion of an outside expert, but use such opinion for the purpose of confirmation only. For example, to take an extreme case, it would be inadmissible for an insurance company, on the opinion of experts alone, to buy the bonds of a company formed for the purpose of constructing a railway, even though such experts were appointed by themselves. It is absolutely essential that the undertaking be to a great extent a "proved proposition." There may, however, in the case of a new railway, be guarantees by existing railways or by a government, &c., which would entirely remove the security from the class of "unproved propositions." I merely cite this example in support of my argument, it having recently come under my notice, and it was probably one more particularly suited for American Insurance Companies who were on the spot, and therefore perhaps better able to make the selection of experts, but I trust that it will serve my purpose. In any case similar propositions, *mutatis mutandis*, constantly arise.

It is perfectly true that an investigation similar to the above would be a somewhat expensive process, and would only be possible provided a large block of bonds were under consideration (in the case I am referring to an investment of \$1,000,000 was made) and therefore quite unsuitable for any but an office with comparatively large funds. This, however, brings me to the consideration as to whether it may not be possible for arrangements to be entered into between several offices by which the cost of such an investigation could be borne in proportion to the relative amounts proposed to be taken. This suggestion has, I believe, been previously made in the discussions here, and has perhaps to some slight extent been taken advantage of, but I believe the principle is capable of very much wider application, and would undoubtedly be of very great mutual benefit. Not only would a wider field of investments be available, but by taking large blocks of securities as is suggested, such securities would be obtained on very favourable terms, more particularly so in those cases where the dealings were made direct without the usual intermediaries. The latter operation would, of course, require that the legal title should be duly investigated, which would necessitate the employment of expert

legal advice in the country of origin of the particular investment, but this would not, I submit, be any serious bar. What I may perhaps call foreign legal advice is surely very often required by those offices carrying on insurance business abroad, and I have never heard that there has been any very great difficulty in securing competent advisers, but on this point there will be many here who can speak from actual experience. A case which recently came under my notice, and one which showed the necessity for, and benefit of, obtaining legal advice in regard to foreign investments, may be of interest to members of the Institute, and I will therefore set it out here. The investments under consideration were bonds issued by certain Canadian towns, and in addition to the ordinary type which may perhaps be termed direct obligations, what were described as School bonds were also offered. These bonds yielded a considerably higher rate than the direct obligations and enquiry was made for the purpose of ascertaining whether, in substance, their security was inferior. The School bonds were of two kinds (1) Roman Catholic Schools, and (2) Protestant Schools. It is the practice in many Canadian towns to have these two classes of Schools, the expenses being met by the levy of a rate in the one case on the property of the Roman Catholics, and in the other on that of the Protestants. The enquiry elicited the opinion that so long as there were any Roman Catholics or Protestants possessing property in the towns a rate could be levied to the amount of the assessed value of the respective properties for the purpose of the Schools. The School authorities could call upon the Municipal Authorities to collect these rates, and if payment could not be obtained the Municipality would have to make good the deficit from the Municipal Fund. It will thus be seen that if either or both of the Roman Catholic or Protestant portion of the population were considerable, the security for the respective School bonds would approximate to that of the direct obligations. This enquiry was made in regard to the bonds of a town in Ontario and the opinion may be considered as only applicable to towns in that province. Somewhat similar laws, however, govern the issuance of such bonds in the other provinces.

A very burning question with us all at the present time is that of depreciation in the value of investments. It has been fully brought home to us by the terrible fall which has taken place in the prices of many classes of securities and has been still further emphasized by recent legislation.

I refer, of course, to the certificate which now has to be appended to the balance sheet by virtue of the Act of 1909. Although an examination of the trend of prices in recent years shows, generally speaking, a considerable depreciation in values, the fall is much more marked in some classes of securities than in others, whilst in a few groups an actual appreciation is shown. The conclusion one may safely draw from such an examination is that a Company by spreading its investments over a large area would have acquired some of each of these classes of securities and the rise in some would to some extent have minimized the fall in others. In considering the investments of an assurance company, however, it is not reasonable to confine an investigation to a short period of years. In order to form any reliable opinion it is necessary to examine the trend of prices for many years, and also to make some assumption as to the amounts invested year by year. As I have previously stated, an investigation such as this depends so largely upon the personal element that I do not feel it desirable to introduce examples into this paper; further, I am sure that it is only by personal investigation that one is in a position to form a reliable judgment. A survey of the price of securities over a long series of years will in any case enable one to see that the funds of an assurance company could have been invested year by year over a large area, and in such a manner that any trouble from temporary depreciation would have been avoided. In fact, such a scheme could be devised by which it could be shown that a considerable appreciation in gross would have resulted. It is, of course, to be assumed that any excess interest or profit from sales has been applied to writing down capital values. I know that I am laying myself open to the argument that I am wise after the event, but I feel sure that if any impartial person will make such an investigation as I have indicated, it will go a long way towards satisfying him that, by a careful selection of investments over as wide an area as possible, both in regard to the different classes of investment and to their geographical distribution, satisfactory results could have been attained. The degree of success must, of course, depend upon the skill exercised in the selection, and also upon buying and selling at the right time, and I propose therefore to make a few remarks in regard to each of these points.

The first question, of course, to be decided is the scheme of investment. The chief points to be considered are mainly in regard to the field to be covered and the relative amounts to be

[illegible]

invested. (a) in each geographical division, and (b) in the various classes of securities of such geographical divisions. These decisions are so much a question of individual opinion that I am afraid it would be quite out of place for me to put forward any scheme, but I set out opposite a skeleton schedule which may, perhaps, be found a convenient form in which to summarize results.

The schedule is, I think, self explanatory, except perhaps the columns headed "Amount Authorized" and "Amount Held." The first of these gives the amount that it is thought desirable should be invested in the particular class of securities of the country to which it refers, and the second shows the amount actually held. Thus to take a particular case the letter A represents the amount that it is proposed should be put into Home Railway Securities, and B the amount already invested therein. The geographical divisions do not, of course, admit of much variation, but the number of groups of classes of stocks can be considerably extended at the will of the investigator. It will also be found convenient in practice to introduce percentage columns, but for the sake of simplicity I have not inserted them here.

I have described this schedule as a Summary, but if we replace the monetary columns by one column for each class of security, it may be used as an index for a book in which the various securities are set out in detail.

The book itself may be made out somewhat in the manner set out below :

Description	Amount authorized	Amount held	Amount authorized	Amount held	
	£	£	£	£	
GREAT BRITAIN.					
Government					
Securities					
Consols ...					
" ...					
" ...					
" ...					
&c. ...					&c.
Railways—					
Lon. & North					
Western					
Debenture...					
Preference...					
" ...					
" ...					
" ...					
&c. ...					

The form also is self-explanatory, the monetary columns having the same meaning as in the summary. The additional columns are inserted for the purpose of making up the book from time to time. It would not be found convenient in practice, nor is it necessary, to alter the book each time as purchases are made, but at settled intervals, say once a year or once in six months, it could be made up by making use of two more columns. The only alterations that are really required are when any authority is extended or a new authority is inserted. The endeavour in practice should be to follow more or less closely in the proportion of the purchases the proportions of the amounts authorized, but it is impossible to keep very strictly to this rule over short intervals. The figures necessarily represent ideals, and though an endeavour should be made to attain them, the exigencies of business will not permit of exactitude.

It will be necessary to have an additional book, which may be called a stock book, in which the purchases from day to day are entered up under their respective headings, the amount of authority for each class of stock being also inserted in a conspicuous position. These books have been put forward as a suggested aid for enabling a more or less scientific distribution of one's assets to be effected, but they also serve a further purpose to which I now propose to refer.

It will often be found desirable, if not actually necessary, in order that the best results may be obtained, to give an immediate answer when an investment is offered. As an example I may cite the underwriting of a new issue, although this is only one of many instances. It is customary for the underwriting to be commenced say, at mid-day, and for it to be completed the same evening. It is therefore obvious that if a reply cannot be given at once a favourable opportunity for investment may often be thrown away. As I have before stated the final decision in regard to investments must necessarily be with the Directors of the Company, and it is very often impossible to get into touch with them at a moment's notice. Now to a great extent the scheme I have indicated obviates this necessity, as it will often be found that the required authority is available.

I now wish to deal with the question of the selection of investments; I do not propose, however, to go *seriatim* through the various classes, but merely to make a few general remarks,

although I should like first of all to deal with a particular class, viz. : that of Trustee Securities.

Trustee Investments may, I think, be defined as a class of securities hall-marked by a paternal Government for the investment of trust funds which come under the administration of trustees who may possibly be either dishonest, ignorant, lazy, or too busy to devote the time required for the proper selection of investments. It is needless to say that none of these descriptions is applicable to those having the administration of insurance funds, and since an artificial market is created for these securities and the yield is on this account very low, they are on the whole quite unsuitable for the investment of our funds. Though this is, I believe, generally admitted, it is often argued that a certain amount of them should be held for a purpose which is most commonly described either as "giving a tone to the Balance Sheet" or "Window Dressing." This may to some extent be desirable, but I think that too much weight is given to this consideration. A good maxim in regard to investments is to buy nothing, but on the other hand to refuse nothing, without investigation. Acting on this principle it will be found that some trustee securities are worth purchasing on their own merits, and I may instance 3 per-cent redeemable securities which stand at a considerable discount and are of relatively short terms. These stocks are unsuitable for trustees, the immediate income being too small, and on this account they may often be obtained on favourable terms.

I have previously pointed out that in any investigation it is desirable if not absolutely necessary, to have sufficient information to satisfy oneself, *prima facie*, that the investment is suitable ; additional, or what may be termed outside, information must be used with caution and chiefly as a means of confirmation. Past history is an extremely valuable source of information, but must not, I think, be too implicitly relied upon. Thus take, for example, the case of an industrial debenture where the undertaking may have shown good and continued progress. The security of the debentures may be considered to be excellent, and the market price consequently will be comparatively high. Trading companies, however, do not last for ever, and it is necessary to consider whether the undertaking is at the zenith of its success. If this is the case it is probable that the investment would not show a high enough rate to cover the risk of a decline in the company's prosperity. Incidentally, it may be

pointed out that it is seldom, if ever, that the specific security for an industrial debenture can be relied upon to repay the capital in full in the event of liquidation on a failure of the undertaking. The valuation made at the time of an issue gives the value of the property as a going concern, and although this value may be considerably in excess of the debt, yet the amount realized on a forced sale will probably fall considerably short of the amount due for capital and interest in arrear. Even if this contention be admitted, I do not think it necessarily follows that these securities are unsuitable, for provided the yield is sufficiently high to cover the risk, the necessary steps can be taken to meet the possible contingency of the loss of part of the capital.

Although it is true that every endeavour should be made to find the weak points of an investment, or class of investments, it cannot be too strongly emphasized that having found defects one should carefully consider whether they are really material. It will often be found that what at first sight appeared to be material will, after a careful examination of all the facts, be considered quite harmless. Further, even though a material weakness be found, the investment should not be rejected without considering whether the defect cannot in any way be remedied, or whether there is not some *quid pro quo*, e.g., a relatively high yield, which over a number of such investments would be more than sufficient compensation. I may perhaps make my meaning clearer by an example that recently came under my notice. The security consisted of a fifty year bond of a Canadian Railway, the line being about 50 miles in length. It is obvious that on their own merits these bonds would be quite unsuitable as an investment for an insurance company. The interest, however, but not the principal, was guaranteed by the Canadian Pacific Railway which controlled the small railway by ownership of the majority of its stock. It might at first sight be thought that the lack of guarantee for the capital would vitiate the investment, but a further investigation will, I think, show that this is not the case. In the first place it may be argued that since the Canadian Pacific Railway Company have thought it worth while to obtain control of the line, it will still be of value to them at the time of the maturity of the bonds, and further, since the term is a long one of fifty years, a small additional rate of interest will at the end of that time provide considerable margin. It is probable that these two considerations would be sufficient to

satisfy one as to the security of the capital, provided the investment yielded a sufficiently high rate, but there was a further point which made assurance doubly sure. A clause in the mortgage provided that a sinking fund of $\frac{1}{2}$ per-cent per annum for the first five years and 1 per-cent thereafter should be applied either in drawings if the bonds were over par, or by purchase if the bonds were at or below par. It was further provided that this sinking fund was to be a first charge on the net earnings of the Company, so that if the line only earned sufficient to pay the sinking fund the whole of the interest would have to be paid by the Canadian Pacific Railway Company.

I have referred to the case of a guarantee, and it is perhaps convenient here to point out that bonds guaranteed both as to principal and interest can nearly always be purchased to yield a higher rate than the direct obligations of the guarantor, though in many cases the security of the former may be even better. A typical instance is that of the bonds of a land mortgage company having a government guarantee.

When one is satisfied that an investment offers sufficient security, judging by past history and any other information available, it is of vital importance that consideration should be given to the future, the object being to buy those securities that give promise of remaining stationary or increasing in value, rather than those one thinks likely to fall in price. It is needless to say that one cannot always be right, but attention to this point is, I believe, of the utmost importance.

This brings me to the question, "When is the time to buy a particular stock or class of stocks?" The obvious reply is when they are cheap, and although this may sound a perfectly simple operation, anyone who has to do with investments will appreciate how extremely difficult it is. I do not profess to be able to give the solution of the problem, but perhaps a few remarks in regard to the question may be of interest.

A careful study of the range of prices from day to day, which can most conveniently be done by means of charts, will enable one to ascertain immediately there is a fall in the price of any stock or class of stocks. If there is a marked fall it is necessary to consider the cause, for it may be that there is a material reason, and one has to judge whether this fall is greater than is justified or not. It will usually be found that a sudden fall due to some disquieting circumstance is far greater than is warranted, and this being so such an occasion will very often

prove a favourable time for investment. Apart, however, from sudden falls, it sometimes happens that for no very obvious reason a class of stock goes out of fashion, and the prices decline ; this also, I think, may be regarded as an opportunity for investing.

In a time of financial panic, such as the American crisis of 1907, which may perhaps be described as a money panic, it is necessary for many people to obtain gold, and naturally the most convenient manner to do so is by means of a sale of securities. The result is a slump in prices which, though general, is most marked in the higher class marketable group. The security in most cases is not one iota less, but the prices fall enormously ; thus, in the crisis above referred to, many gilt-edged railway bonds were obtainable at from 10 to 20 points lower than they could have been previously purchased. Such a time as this may be described as the "golden opportunity" and every penny available should then be invested in the particular class of security affected : it would in fact to my mind be a grievous error to buy anything else for the time being.

One should endeavour to enter a new field of investment at the first suitable opportunity. As an example of this I may mention the case of some of the smaller Canadian towns which, owing to their position on the railway or some other similar cause, might reasonably be expected to grow in importance. Only three or four years ago many of the bonds of this class of town could be purchased to yield about $\frac{1}{2}$ per-cent more than they do at the present time.

It is generally admitted that when prices are high the desirable class of investments are mortgages and short term securities. The supply, however, on reasonable terms will, I am afraid, generally be found woefully short of the demand, and the deficiency may perhaps be made good by the purchase of such securities as are not readily affected by monetary conditions. Thus, amongst American Railway Bonds—I refer more particularly to those now yielding from $4\frac{1}{2}$ per-cent to 5 per-cent—satisfactory investments can often be found, and an examination of the past prices will show that the above mentioned condition is to some extent fulfilled.

I now propose to deal with the question, "When should an Insurance Company sell securities?" I am not, of course, referring to sales for the purpose of realizing the money necessary to meet ordinary engagements.

The matter is, I am afraid, a rather vexed one. It has

sometimes in the past been seriously argued that an insurance company should never sell. With this contention I cannot for one moment agree, although I am strongly of opinion that jobbing in and out of stocks is a business altogether apart from the investment of assurance funds, and one only suitable for trust companies and other kindred institutions.

There are, however, some occasions when I think one ought to sell and others when it is desirable to sell. As examples of the first class I may mention (*a*) the case where an investment has turned out badly, and where you not only feel certain of a further loss but have also come to the conclusion that the security as you then know it is not suitable for your purpose, (*b*) where in your opinion the price of a stock is so high that you are no longer justified in holding it. Thus if a stock continues to rise in price there must come a point where you would not be prepared to increase your holding although you might be prepared to keep what you already hold. If the price rises still higher it is obvious that it must reach a figure at which you are no longer justified in retaining the investment. As examples of the second class I may mention (*a*) the case of bonds nearing maturity which can be discounted at a low rate of interest; (*b*) it may sometimes occur that considerable purchases have been made in a particular class of security so that its proportion to your total investments is in excess of that which it is considered desirable to hold. This might very possibly occur in such a case as I have just instanced, the purchase of American Railway Bonds during the 1907 crisis. It may be argued that one ought never to have increased one's holding to so great an extent, but to this I would reply that to miss such an opportunity would be contrary to the true spirit of investment. Further, it is only by taking advantage of such obvious opportunities that one is able to avoid the bugbear of depreciation.

Throughout this paper I have emphasized the fact that it is necessary to consider part of the interest yielded by many investments as a risk premium, and it is therefore desirable to consider how this portion of the income should be dealt with in the accounts. If it were possible to spread investments sufficiently and in a suitable proportion, it would I conclude be unnecessary, apart from temporary fluctuations in the market value, to deal specially with this surplus. It would automatically be available to meet losses which in the supposed case would occur in the same proportion from year to year. It is, however,

obvious that in actual practice this could never be the case, and it is therefore necessary to adopt some method of dealing with such accumulations. The first consideration is the rate with which one is prepared to credit oneself. The balance could then be carried to a special reserve together with any profit from the sale of securities, and this fund could be drawn upon from time to time to meet any losses that might occur. Instead of carrying these sums to a reserve fund, it is, of course, possible to use the amount year by year in writing down securities, thus creating a hidden reserve. Whichever course is pursued it would be possible to use part of this accumulated surplus from time to time for increasing the rate of interest, that is provided the opinion I have previously expressed that relatively high yielding securities provide an additional rate over and above that required to provide for any additional risk is correct. Any sum so appropriated would, of course, have to be spread and not used for the interest of any particular year.

Practically no paper, I believe, has been read at the Institute on Investments in which some reference has not been made to the five canons for investment laid down by Mr. A. H. Bailey in his paper of 1862. I think I may say that no paper would be complete without making such reference, for these canons are as true to-day as ever they were. One is very loth to tamper with any laws which have stood the test of time, but it is, I believe, necessary in order to suit the requirements of growth and progress to make additions to existing legislation, however beneficent it may have proved in the past. Since, therefore, the object of my paper is directed to the extension of the field of investment for Assurance funds, which object I firmly believe to be for progress, perhaps it will not be thought out of place if I venture to make some slight additions to Mr. Bailey's canons, although I must admit to doing so with considerable hesitation. Although the canons are so familiar to all of us, it will, perhaps, be convenient to set out each canon, and to place underneath the remarks I wish to make.

1. That the first consideration should invariably be the security of the capital.

As I have before indicated I believe that attention in the past has been mainly directed to the application of this canon to each individual investment, and although this is essential I venture to think that such application is not complete. It is absolutely impossible to assure oneself of the safety of the capital of any

single investment, and it is only in regard to the safety of the capital as a whole that one has a reasonable chance of success.

2. That the highest practicable rate of interest be obtained, but that this principle should always be subordinate to the previous one, the security of the capital.

In order to make this canon in agreement with my suggestions it is necessary to write "investment rate" for "rate", the surplus, as I have stated, being applied as a risk premium towards securing the safety of the capital.

3. That a small proportion of the total funds (the amount varying according to the circumstances of each individual case), should be held in readily convertible securities for the payment of current claims, and for such loan transactions as may be considered desirable.

The application of this canon becomes automatic if the investments are spread over a large area, for it is obvious that there would always be a considerable number capable of favourable realization.

4. That the remaining and much larger proportion may safely be invested in securities that are not readily convertible; and that it is desirable, according to the second principle, that it should be so invested because such securities, being unsuited for private individuals and trustees, command a higher rate of interest in consequence.

5. That, as far as practicable, the capital should be employed to aid the life assurance business.

In addition to these five canons it may perhaps be desirable to add:

That in order to minimize the result of temporary fluctuations and to secure the safety of the capital in the best way it is desirable to spread these investments over as large an area as possible.

In conclusion, I would like to say that although it has fallen to my lot to write this paper, many of the ideas embodied therein are equally attributable to Mr. Joseph Burn, with whom it has been my good fortune to work for many years past.

ABSTRACT OF THE DISCUSSION.

Mr. E. W. TOWNLEY said that the author had given tables analysing the investments of four leading companies in 1857, the main feature disclosed being the undue proportion of mortgages.

This feature was by no means peculiar to the offices cited, and as recently as 1885 the total funds of all British offices combined were invested in the following proportions :

Mortgages	61·8 per-cent.
Stock Exchange securities	24·7 „
Reversions and properties	7·4 „

From that year onwards the percentage represented by mortgages had continually declined, while that shown by Stock Exchange securities had steadily increased, the figures for 1910 being as follows :

Mortgages	41·2 per-cent.
Stock Exchange securities	43·4 „
Reversions and properties	10·7 „

The percentage lost by mortgages had practically been gained by stocks—a result due, no doubt, to a large extent to the competition of trustees for mortgages—and he thought it must be admitted that the distribution as between the three main classes was more satisfactory than at any previous time.

Passing to the question of the distribution of the Stock Exchange securities themselves, judicious spreading was undoubtedly one of the best means of minimizing depreciation of the funds as a whole, but this principle should, he thought, be applied with special caution to life office finance, where one of the essentials of success was elasticity. The argument for an elaborate distribution scheme was much more cogent when applied to a trust company than to a life office. A trust company traded with a fixed capital ; it dealt largely in securities with a limited market ; and it was therefore bound to watch the distribution of its purchases closely, since it was difficult afterwards to redress the balance without realizing—usually at a loss. But the position of a life office was essentially different. A life office could afford to take the fullest advantage of every investment opportunity that offered, because it could rely on the normal growth of the funds to correct any temporary disproportion that might result.

For that reason there was room for considerable difference of opinion as to the advantage of drawing up an elaborate cut-and-dried scheme of distribution at all. Mr. May, however, had given a specimen book which went so far as to suggest what amount should be held of each class of stock in every country. Without raising the large question of how these limits were to be fixed, it seemed to him that with conditions constantly changing and fresh opportunities arising, particularly in the way of new issues, the value of such a scheme crystallized at a particular point of time was very doubtful. There was a further point—the bearing of the suggested scheme of limits on dealings with new issues. In order to minimize depreciation, it would seem just as important to buy on the most favourable terms as to spread the purchases over a wide area. That undoubtedly explained why new issues played so

important a part in the policy of progressive offices. Apart from the question of underwriting, a new issue usually afforded an opportunity of securing an investment at the bottom price.

On the whole, he could not help thinking that when once the broad lines of the investment policy had been settled, the better plan was to allow the actual distribution to shape itself from the opportunities that arise. Occasionally, when exceptionally heavy purchases have been made in one direction, such as the American market, over a long period, it might be necessary to consider whether the time had not come to invest in other directions, and for this purpose an up-to-date geographical analysis of existing investments would be useful.

The question of the treatment of securities in the books was of increasing importance, having regard both to the cosmopolitan policy of investment now being adopted and to the increasing percentage of funds invested in Stock Exchange securities. The author suggested that an investment rate should be arbitrarily fixed and the surplus over that rate carried to a special reserve or applied to writing down; and that if the investment rate were found, in fact, to err on the safe side, it would be possible, afterwards, to put back part of the surplus into interest, thus raising the yield. That seemed a somewhat artificial and roundabout way of treating the case. In the first place, how was the investment rate to be fixed? If they took a uniform rate for all investments it would produce anomalous results in some cases, as, for instance, where a security was intrinsically first-rate, but had a limited market and was under-valued accordingly. If, on the other hand, each case were to be treated separately on its merits, the position, though more logical, became extremely complicated. For these reasons, he ventured to suggest as preferable what had been described as the counsel of perfection—the simple plan of never writing up and always writing down. It avoided the arbitrary and artificial splitting of the interest. It also gave the nearest approach of any method yet devised to what might be termed a scientific reserve. It placed the office in a position of growing strength from year to year, because it gave effect to the maximum depreciation of each security over an increasing term of years. If he correctly understood Mr. May's plan of writing down by the risk premium, it seemed that if the high yielding stocks appreciated, it would give the awkward result of book values and market values moving in opposite directions. If, to take the reverse case, market prices fell heavily, the writing down by the risk premium would probably prove inadequate and it might be necessary, after all, to take the market value.

Mr. May had given an instructive case, showing the benefits of taking legal advice in regard to foreign investments. A somewhat similar instance came before him recently. The bonds of a gas and electric company, having a monopoly in the capital of one of the Western States, were on offer on very favourable terms. A point arose, however, as to the terms of the company's franchise. This was originally limited to a term of fifty years, and was subject

to revision or repeal by the terms of the constitution of the State which granted it. Legal opinion showed that neither point was of practical importance. The charter life of the company could be extended from time to time by a simple voluntary action on the part of the directors : while, as regarded the second point, it transpired that a similar reservation existed in the franchise of practically every gas and light company in the State. The risk of repeal was, therefore, purely academic, so far as the vested rights of existing companies were concerned, in view of the widespread disaster which any exercise of the right of repeal would create.

Turning to the general question of high-yielding investments, he thought it would be agreed that what a life office required was not so much a high yield as a permanent one. A railroad bond producing $4\frac{1}{2}$ per-cent for fifty years was obviously preferable to a first-rate industrial debenture yielding even 6 per-cent or more for ten years, because by the choice of a long-term security the office could render itself independent of the general course of interest for perhaps the next forty or fifty years. When that principle was applied it ruled out a large number of higher-yielding securities. The debentures of trading companies in particular were, he thought, unsuitable for life assurance funds, as they depended very largely for their security on good management. Bad management usually meant liquidation in the long run, and that in turn meant a forced sale, with the result, as the author stated, that the property would probably realize considerably less than the amount of the debt and arrears of interest. Mr. May went on to say that, even so, it did not follow that the securities of trading companies were unsuitable, provided the yield was sufficient to cover the risk. But that raised the question—What is the risk? If only the risk could be measured the premium could be fixed. The great difficulty was the difference of opinion existing among auditors themselves as to the correct method of drawing up the assets side of a trading company's balance sheet. Too frequently the assets side merely gave a record of how the capital had been spent, and was absolutely useless as an index to what the properties would fetch at the present time. In that connection he might cite the classical case of the Gigantic Wheel Company, at Earl's Court. The wheel, the principal asset of the company, figured in the balance sheet year by year at £29,300—the cost price—until the company went into liquidation, when it realized £300. That was, of course, an exaggerated instance. He mentioned it only because it showed the possibilities of an industrial balance sheet.

If an office at the present time wished to obtain a satisfactory yield on debentures of the industrial type, he thought it could not do better than consider the important class of public utility companies found in America. These companies supplied gas, electric light and power. They were not at the mercy of competition, as they almost always had a monopoly in the area they covered : they had large tangible assets, usually in important and growing centres : and the question of management did not affect them to anything like

the same extent as the ordinary trading company. They were rather in the position of our Parliamentary companies, but their bonds had the advantage of showing a very satisfactory yield of round about 5 per-cent, usually for a long term. Where that yield could be obtained on good security, he thought industrial debentures with their many problems must be ruled out on practical considerations.

Towards the end of the paper, the author gave an instructive list of instances in which it would be desirable or necessary to sell. An interesting question that arose in that connection was whether an office would be justified in stimulating the rise in its interest yield by selling the lower yielding stocks, even at present prices, in order to free a larger sum for investment on the more favourable terms now offering. That question was one of considerable practical importance to a number of offices which had until recently been hampered in their investment policy by restrictive deeds of settlement, and consequently held an abnormally heavy percentage of funds invested in British securities of the gilt-edged type. It seemed to him that sales of these low yielding stocks were perfectly legitimate, and ought in fact to be a feature of a progressive policy. It might be argued that if sales were effected now, the office would lose the benefit of any future appreciation in capital value. He thought that difficulty was not a serious one, and could readily be overcome by the investment of part of the proceeds of sale in the purchase of reversions to the same type of stock as had been sold. The present time offered a very good opportunity for such purchases. Ten years ago, buyers of reversions in the open market had to give prices which were practically 4 per-cent values based on the high prices then ruling for gilt-edged stocks. At the present time, market prices for reversions were represented, broadly speaking, not by the 4 per-cent, but by the $4\frac{1}{2}$ per-cent values, and with stocks standing at their present level, a purchaser had every reason to expect an appreciation in capital value of the fund itself by the time the reversion fell in. The point he wished to emphasize, however, was that by holding the reversions, the office stood to benefit to precisely the same extent, in the event of capital appreciation, as if it held the stocks themselves. In addition to this it would receive until the reversions fell in, not the $3\frac{1}{2}$ per-cent yielded by direct investment in the stocks, but $4\frac{1}{2}$ per-cent, the yield on the reversions.

As the paper dealt largely with the problem of the geographical distribution of risk, he wished to mention, in conclusion, another direction in which he believed reversions would have in the future an increasingly important bearing on a forward policy. The tendency at present was to invest almost exclusively abroad, and it was not difficult to imagine an office which had been vigorously investing on those lines reaching a point, in course of time, where it might be considered desirable to increase the holding of British securities. The difficulty which at once arose was that of investing here in sound securities and at the same time securing a satisfactory yield. Industrial debentures usually

afforded the yield without the security. On the other hand, the low return on gilt-edged stocks at once precluded direct investment in them. He thought the solution of the problem lay in the purchase of reversions to the average type of well invested fund.

Mr. A. T. WINTER said there was one sentence in the paper which to a large extent seemed to be a reversal of one of the chief tenets of the old investment creed, namely, "The investment of funds in relatively high yielding securities carefully selected not only gives a more satisfactory return than that shown by the lower yielding stocks, but, on the average, gives greater security for the capital." In their younger days actuaries were taught that if they wanted high yielding securities it was necessary to sacrifice at least some of the security. They were now told by the author that not only could a high rate of interest be obtained, but in connection with that high rate greater security would be obtained than in low yielding investments. It was easy to show that during the last fourteen years the course of prices had been decidedly favourable to what he might call second-grade investment securities, especially in comparison with the enormous depreciation in first-class securities. He thought perhaps the author's opinions had been unduly influenced by these facts. In such a matter observations should be taken over a longer term of years and due weight must be given to the earlier course of prices prior to the year 1897, when movements of prices were in quite the opposite direction from those which have occurred since. He thought, also, that before coming to any definite conclusion on the point, weight should be carefully given to certain special political and economic events which had affected the price of gilt-edged stocks in the last fourteen years, some of which events were not likely to recur. Another way of looking at the matter was to consider the position which existed, say, fourteen years ago, and to ask whether one could at that time reasonably have come to the conclusion which the author had arrived at as to the merits of high yielding investments? A long period of years had then been passed through in which gilt-edged securities had steadily appreciated in value. The main anxiety of investors at that time was the falling rate of interest. The position was, he thought, fairly well reflected in a paper read by Mr. J. Burn before the Institute in 1899, in the course of which he said—"There is, of course, the possibility of a rise in the rate of interest, and the consequent depreciation of marketable securities; but this is for the most part considered so improbable that it may practically be left out of consideration in any extended view of the future." It was well-known, of course, that that forecast was falsified almost immediately, owing to events which neither Mr. Burn nor anybody else could have foreseen. For instance, there was the War in South Africa; there was another great war in the East; and many other unlooked-for political and economical events occurred which, to a large extent, accounted for the depreciation in first-class securities. Past records of prices were an important factor, but unless due effect were given to special disturbing elements—political and economic—they might be misleading.

He thought a good deal of useful information could be obtained as to the course of prices in second-class investment securities by examining the history of the Financial Trust Companies. He had been looking at the prices of the deferred stocks of such companies for the period from 1890 to 1910. Such deferred stocks fairly well represented the course of prices of second-grade investment securities. He found that in 1890 the average price of the deferred stocks of ten of the largest financial Trust Investment Companies was 108 per-cent. That price steadily went down, till the average price in 1896 was 44, a decrease of 64 points. From that year it steadily rose year by year until the average in 1910 was 102. He thought the important fact to notice was that the low-water mark in the price of second-grade investment stocks, viz., that recorded in the year 1896, was coincident with the high-water mark in the price of the best class of investment securities, and that the decline of the price of the second-rate securities was exactly concurrent with the appreciation of the highest class of investments. He thought that these opposite movements had an important bearing on the future outlook. Gilt-edged securities were at a very low ebb at the present time, but everyone hoped, and he thought it was even probable, that in the course of time, while they would possibly not exactly revert to the position which existed in 1896, there would be some recovery in the prices of those securities. He thought that might reasonably be assumed, because it was known that a number of untoward circumstances had occurred in the past 14 years which partly accounted for the depreciation that had taken place. If that happened, judging from the history of the past, it was not unreasonable to anticipate some decline in the price of high yielding securities. Bearing these facts in mind it was perhaps not a judicious policy to invest to a very large extent in second-grade securities at the present time. Although in regard to the merits of high interest securities he thought the author had perhaps given too much weight to the progress of values in the last 14 years, he was by no means saying that there was not a great deal to be learned from the history of that period. He believed one of the important lessons to be learned was the necessity for further geographical distribution of their investments. Many opportunities had existed for doing so owing to the enormous number of issues in recent years in the Colonies and abroad, and the companies were, he believed, largely extending the area of their investments by taking advantage of these issues.

Not only should they distribute their investments abroad more generally, but he thought they should also carefully watch to see that they were not over-loaded in any particular country. It was well-known that all rapidly-developing countries were liable to great reverses at times, and consequently very strict watch should be kept on the amounts insurance companies had at stake in them. It was also necessary for the companies to keep in view the amount they held in any particular class of undertakings. In fact, the principle of limitation of risks, which had been regarded as of vital importance on the liability side of the accounts, was usually being extended now

to the question of investments. In that respect British companies were very much better off than foreign and colonial companies. It would be seen from the investments of those latter offices that they were almost entirely in the particular country in which they were operating. That feature was very marked in the insurance companies of the United States, whose enormous funds were almost entirely invested in the States, except to the extent of the deposits they had to keep abroad. Not only were the funds invested in the United States, but a great proportion of them were invested in one particular class of security, viz., American Railway bonds. Taking three of the principal companies, he noticed that about 50 per-cent of their total assets were in American Railway bonds. Actuaries well knew the violent movements that took place at times in those bonds, even of the best class. The author had referred to the American crisis of 1907, when he believed an average depreciation of something like 14 points took place in the course of two years. With the geographical distribution which British insurance companies were now adopting, he hoped it would be impossible for their funds to suffer such enormous variations in such a short time. In concluding he desired to thank the author for his very interesting paper. In some respects his opinions might not be entirely orthodox, but he did not know that that was a demerit in the introduction of a discussion on investments. Even if Mr. May did not convince all the members that his opinions were correct, he had led them to reconsider their position, and to enquire whether their old views were still tenable.

Mr. F. SCHOOLING thought that the author might fairly be congratulated on having placed before the Institute a most useful and practical paper on a difficult, but at the same time most important subject. The few remarks he desired to make were of a general character. In the first place, he wished to say that he thought Mr. Townley had misunderstood Mr. May's suggestion as to dealing with interest over a certain rate. He did not think the author meant to say for one moment that the writing-down of securities, when necessary, should not be insisted upon in addition to dealing with interest over a certain rate. Mr. Townley devoted a large amount of time to that particular point, but he really did not think that the criticism arose upon the paper. Events of recent years had taught actuaries that safety and immunity from loss were not to be obtained by simply investing in so-called gilt-edged securities, but rather that a wide field of investment should be adopted by distributing the funds in a large number of varying interests all the world over. That knowledge had, he thought, come not only to insurance companies, but to the investing public in general, and it was a factor, perhaps a large factor, in producing the present price of Consols. People generally recognized that there was now a world-wide field of investment open to them, and he did not think they would be cajoled into putting their money into Consols by any scheme for the purpose of raising the price unless such a scheme also had the effect of raising the interest yield; and it was extremely difficult to see how

such a plan could be formulated without loss to the country. He thought the author showed clearly in his paper that it was of the first importance that actuaries who had to advise their companies as to investments should be experts of great ability, who had the power of clearly setting out all essential facts in connection with the particular investments under consideration. The proper investigation and analysis of securities offered was undoubtedly one of the most important matters that had to be considered in the management of an assurance company. He regarded such papers as the author's as of great value, and he was sure Mr. May would agree with him in the opinion that practical experience alone could give the necessary qualification for the investment expert.

Mr. O. T. FALK said that he had been struck with the remark made in the paper, that hitherto all actuaries had attempted to deal with the investments of life offices practically without any special training for the work. That seemed to him to be the outstanding weakness of the present management of insurance companies. How many of those present, from top to bottom, had anything more than a book knowledge of the investments with which they were dealing? How many were in any real contact with the market? How many had any first-hand experience or knowledge of the countries and the concerns in which the money of their companies was being invested? The criticism might not be a fair one if it could be equally well applied to all other investing bodies, but it could not. It was not, for example, made with equal force with regard to those who were responsible for the investment of Trust Companies' funds. He knew there was one reply which in many cases was a very satisfactory one, but it did not hold for all companies, namely, that investments were primarily the concern of boards of directors. If that was to be given as a final answer, he thought that probably the Institute would not be discussing the subject that evening. As a matter of fact, the modern tendency was, he believed, to consider that officers of insurance companies should be to a large extent responsible for their investments, and he imagined that that was a step in the right direction.

That led him to the second point in the paper which seemed of outstanding importance, namely, the reference to the possibility at some future date of the formation of a General Trust Company by a combination of insurance companies. That held out, he thought, some hope at last for the life offices of assuming that position of power and importance in the financial market and the City generally which they had not held in the past. It was no exaggeration to say that to a large extent assurance companies, with their millions of funds, have been the laughing-stock of the City. In some cases the offices had men of power on their boards, whose opinion was respected, and who were enabled through the control of large sums of money to have some weight in the City, but for the most part, as was said very commonly by big bankers and other people in the City, the power of assurance companies was not used to the best advantage. As one who felt that every actuary was under some disadvantage in

that respect, he thought a most important point had been made in the paper that it was very necessary in the future that those who were to be responsible in some degree for the investment of life office-funds should have better opportunities than they had had in the past for learning the most important side of their business.

Mr. GEOFFREY MARKS said he did not know that he would have intruded in the discussion had it not been for what Mr. Falk had just said. He felt with Mr. Falk that the system of finance which had been applied to life assurance offices had hitherto been somewhat machine-made, and for that reason he would like to utter a word of warning in regard to the author's paper, because it seemed to him the system there advocated was in some respects an extension of the principle he had just described. It was within the recollection of most of those present that, even within their own time, many of those industries which were considered most stable—the securities of which were considered most desirable for the investment of life assurance and other funds—had been partly superseded. For instance, gas had been succeeded by electric light; telegraphs had given way largely to telephones; telephones, cables and telegraphs would possibly be superseded by wireless telegraphy; railways had been to some extent damaged by tramways; and both tramways and railways would no doubt be further damaged by motor traffic. All those instances led him to utter the warning which he had in mind, that if assurance companies applied to their finance certain rules and regulations which tended to stereotype pre-conceived ideas, they were going a long way towards limiting their own opportunities and destroying the usefulness and the importance of the position which they ought to occupy in the world of finance. There was no doubt that a certain amount of training was desirable and necessary for anybody who hoped to make any sort of mark in the financial world. In spite of Mr. Falk's remarks, in no other profession than that of the actuary was any attempt made to give systematic education in finance, and in making that statement he did not forget the Chartered Accountants and the Bankers.

He desired to refer to the scheme which Mr. Falk had mentioned. He had discussed it with the President and had referred to it on many previous occasions, both in public and private. The scheme, which was one that was very near to his heart, consisted in the formation of some sort of Financial League amongst the life offices. He believed the time was coming when better financial education, not only amongst the responsible actuaries and managers of companies, but also what was much more important in that connection, amongst their boards, would have a tendency to show how vitally important it was, if assurance companies were to maintain their place in the world of finance, that there should be some sort of practical union among them. It must be evident to all that the enormous waste of power which took place from day to day, almost from moment to moment, in their financial dealings with the issuing houses was such as to render insurance companies the laughing stock of the City—he thought Mr. Falk's expression was hardly too strong

to use. All those practical financiers, many of them men of eminence, to whom he had suggested the possibility that the scheme he had mentioned would come into existence, had welcomed the idea as something perfectly new, and as one which might have the most important influence on the finance, not only of this country, but even of the world, since probably no other body could bring to bear on financial questions the weight of such enormous funds as the insurance companies possessed, if they would only act in unison. He hoped the mention of the scheme at the Institute might be the means of bringing it in a more practical form before the public generally, and those interested in insurance work particularly.

Mr. J. R. HART, in concluding the discussion, said that the author had raised the important question of the geographical distribution of investments. This had really been forced on the companies by several well-known causes, namely, the depreciation in home investments; the improvement in foreign and colonial securities; and the increase of the companies' funds. It was not necessary for him to say much about depreciation, because that point had already been referred to in the course of the debate, but nobody, he supposed, now referred to Consols as the premier security, and life offices only held about 1 per-cent of their total funds in such securities. He thought, however, the depreciation in trustee securities hardly justified the remarks made by the author when he said that "Trustee investments may, I think, be defined as a class of securities hall-marked by a paternal Government for the investment of trust funds which come under the administration of trustees who may possibly be either dishonest, ignorant, lazy, or too busy to devote the time required for the proper selection of investments." Except within recent years it must be admitted that those powers had been most valuable, and when companies had been buying reversions they had attached considerable importance to the consideration whether the powers were restricted in the way referred to.

Coming to the question of the improvement in foreign and colonial investments, it was necessary to consider the remarks of the author as to the investments of the companies in 1857. Canada, South America, China and Japan, in which most assurance companies now held securities, were, he supposed, absolutely out of the field in 1857, and it seemed to him that the investments of the companies at that time were really admirably chosen. The mortgages produced $4\frac{1}{2}$ per-cent, and the other securities were investments which afterwards appreciated. He was not quite clear in reading the paper whether Mr. May recommended investments carrying a higher rate of interest, which were really more risky, or investments that carried a higher rate of interest due to special selection and facilities of investment. If he meant the former, he thought the discussion had shown that most of those present would not agree with him, but if he meant the latter, it was clear, he considered, that life offices did possess facilities and powers of getting a higher rate than the general public. In that connection he was

interested in reading a paper by the Public Trustee, to see how life offices compared with that official in the matter under consideration. The Public Trustee laid great stress on the fact that he could get the opinion of a number of brokers. He thought most of the members who had to do with brokers knew that their opinion from an investment point of view was not worth very much; they very seldom had the time or the organization to analyse an investment in the way insurance companies required. Brokers could give an opinion as to the public demand or supply, and in that respect no doubt their opinion was very useful, if insurance companies chose to avail themselves of it. But, as compared with the Public Trustee, life offices could supply the funds at any time, and could allow their investments to remain for any period. Moreover, they had boards of directors; and although he could conceive cases where a director's opinion might have more weight than it really should have, still some value accrued to the offices when they placed the evidence they had carefully collected before a dozen business men for their decision.

With regard to Mr. May's "expert", the extension of investments had led to the necessity for studying a great deal of literature on the subject, but it seemed to him that the expert who remained at home and studied such literature was not the expert insurance companies required. The expert must be one who knew the real conditions abroad, and unless he knew those conditions he was not very likely to be successful. With regard to the question whether the expert should be an actuary, although an actuary was not more competent to forecast than anybody else, still in many respects he was best fitted to analyse investments. He noticed that in a paper read in 1896 an actuary was described as "A prophet or seer who looks back to see how money values have been rising and falling, and whose observations serve him as a guide, more or less reliable, in his prophecies for the future." He did not think anyone would give that definition now, but that did not alter the argument that the actuary's care and power of analysis must be of value. For that very reason the actuary was probably the worst speculator. He desired, in conclusion, to make one remark about mortgages. The author had referred to the supply of mortgages being woefully small, and he personally thought there were obvious reasons for that state of affairs. The results of the past ten years had led offices to rule out of their range of investments a number of securities that some of them took years ago, such as flats, public houses, theatres, and large basement houses, and other securities which might have been taken formerly with guarantees. The author evidently contemplated lending on mortgage abroad, because he included such a column in his schedule, but that seemed to be a direction in which more than ever the expert must be one who knew the exact conditions abroad. It was of no use if he endeavoured to obtain his knowledge simply by reading books. Such a paper as Mr. May's was very welcome to the Institute, because it enabled them to put on record in the *Journal* the circumstances under consideration at the time with regard to investments.

and it was extremely interesting to be able to look back and see how they changed.

The PRESIDENT said that at the close of what he was sure all the members would regard as a very interesting debate, he rose to propose a hearty vote of thanks to the author of the paper, and he desired to add his own personal congratulation to Mr. May for having employed his time and leisure to such excellent advantage on behalf of the Institute. The paper dealt with a subject which lay at the very foundation of the prosperity of life assurance, and it was timely and suitable that the Institute should have a discussion on the subject, some considerable period having elapsed since the matter was last debated within their walls. He was aware that sometimes the cloak of authority was denied to actuaries as experts in finance, and there might be something to be said for that point of view if they were dealing with the broad realm of finance in its entirety. But when it was remembered that actuaries chiefly claimed to be regarded as financial experts in regard to life assurance companies, then he thought the difference would be immediately appreciated, and the strictures which were passed upon actuaries for posing as financial experts would be seen to be baseless. It was necessary to remember that the finance of a life assurance company was something apart from the general scheme of finance. It usually required special training, technical knowledge, and, it might be said, a different standpoint from that of finance in general. From the time when the founders of the Institute put their heads together to devise the future course of the profession, it had been regarded as right and proper that finance should take a prominent part in the Institute's proceedings. Hence those papers which had been referred to by the author, and others by Mr. Jellicoe and what might be called the "Old Masters" of the Institute, would be found to refer to the question of finance as one which very properly belonged to actuarial territory. The paper and discussion had dealt very largely with the course of future prices of securities and he desired to advise students to consider the matter in a broad and general spirit. It was quite clear that the course of prices must depend upon a large number of factors; upon the preservation of the peace of the world; upon the development of trade; upon the cultivation of the earth's surface; upon the increasing output of gold; and upon the maintenance of the Bank Act of 1844. If consideration were given for a moment to the effect which would be brought about upon the whole course of investment prices by a repeal of legislation governing the free receipt of gold at a fixed price in accordance with the Bank Act, he thought it would immediately be seen that the course of future prices was but the last link of a long chain of cause and effect. He, therefore, thought that in discussing the subject the members should have regard to broader considerations than had been brought before them.

In dealing with the general question of the suitability of particular forms of securities for life office investments, it was very difficult to find any underlying principle which made it possible to discuss the matter without being unduly diffuse. He remembered the

chairman of one of the great life offices at the present time telling him that the investments of his company were made to be held and never to be sold. That policy, as a matter of fact, had answered exceedingly well; but if actuaries could differentiate between those financial advisers who selected investments with a view to their being permanently held, and those others who proceeded more upon the lines of the trust companies, taking second-class securities in small bundles and having an eye to the course of prices and the profits which might from time to time be realized, he thought two different schools of ideas, based on distinct underlying principles, could be set up in regard to finance. If any of those present were disposed to accept the principle of only advocating investments which they were prepared to hold permanently, a great many investments which might, by the glitter of their high rate, appeal to an investor would have to be set on one side. Whether that, in the next twenty years, would prove to be more advantageous to individual companies than the free acceptance of second-class investments having a high yield nobody could tell. The course of the last fourteen years had completely changed actuaries' views with regard to the investment of the funds of life offices, and it was not inconsistency on their part, but rather a wise adaptability, which made it necessary and desirable that they should review their position in the light of recent experience. If actuaries carefully and intelligently studied the course of finance, a great many points would occur to them from time to time with regard to the legitimate effect of the continually changing circumstances. Personally, he believed that credit was now much better understood and played a much more important part in life assurance finance. In the old days it was held that a large part of the securities of a company should be kept in a liquid state. Now it was quite well understood that that was not so necessary; that so long as credit was possessed by a company it was not essential that a large proportion of the funds should be invested in gilt-edged or liquid securities. Moreover, actuaries had come to learn that what used to be regarded as the principal class of liquid securities, such as Consols and Government securities, could no longer claim to possess that quality.

The varying points of view of finance could be illustrated by the example of the independent companies transacting marine insurance. If the past reports of marine insurance companies were inspected, it would be found that interest was not counted upon as being a very important contributor to their profits, but the fact that the investments should be of a very high class and liquid character was looked upon as all important. Surely that consideration ceased to be a fair ruling principle when a marine insurance company associated itself with one of the large composite companies which possessed a very large fund of credit, and so made it quite unnecessary that the funds of the marine department should, to the same extent, be invested in liquid securities. He did not profess to lay down any rule for their guidance, to express any original views, or to offer any universal panacea for past troubles, in making

those general remarks, but he thought it was necessary to keep a very open mind in regard to the future course of finance in their country. They must bear in mind how difficult, and indeed impossible, it was to plot out the course of events, particularly when they turned upon so many circumstances which themselves were in a very fluid and unfixed condition.

The resolution of thanks was then put and carried with acclamation.

Mr. G. E. MAY, in reply, thanked the members for the very kind way in which they had received his paper, and particularly those who had so ably discussed it. When he wrote the essay it was his endeavour to base it on actual experience : in other words, what he wanted to write was a business paper : and it was, therefore, a source of great satisfaction to him to receive the commendation of the President. Mr. Schooling had also spoken very kindly of his efforts, and as he had worked under Mr. Schooling for many years he desired to take this opportunity of publicly thanking him for the encouragement and assistance which had always been so kindly given. He thought, perhaps, his paper had been a little misunderstood in some respects. Thus, Mr. Marks, basing his remarks possibly on the schedules submitted, referred to the method suggested as machine-made. These had, however, been introduced into the paper merely for the purpose of demonstrating the distribution of the assets at various times and not with the object of laying down any very strict law, to be adhered to in all circumstances, in regard to the manner in which the funds should be invested.

Mr. Townley referred to the question of the percentage of the funds held in mortgages, which he showed was gradually falling : probably that was to a great extent owing to the issue of mortgage debentures, whereby securities previously included under mortgages had been transferred to Stock Exchange securities. Mr. Townley had remarked that it would be difficult to put a different investment rate of interest against each security. He did not suggest that that should be done. If a fair rate of interest for all investments of this class was assumed and the requisite amount carried to the interest account of the company, then whether the security yielded 5s., 10s., or 15s. more did not matter, because such surplus interest would be accumulated for the purpose of meeting any losses that might occur.

In regard to investing in the higher-yielding securities he did not suggest that those securities should be risky securities in the sense apparently applied to that expression by Mr. Hart, that is, securities which he would prefer to describe as highly speculative. There could be no question but that in every investment there was a certain element of risk and his contention was that in the case of the higher-yielding securities, carefully selected, there was a margin of interest which, on the average, was far more than sufficient to provide for any loss that might happen. They had seen in recent years that losses of great magnitude had occurred in the low-yielding gilt-edged securities, and the misfortune had been that there was no margin of interest to make good the loss. He was not suggesting for one minute that

a company should put all its funds in such higher-yielding securities, but only that a certain proportion should be so invested, and, in making that proposition, he anticipated he would meet with a certain amount of opposition.

As the President had said, it was extremely difficult to forecast the future. The investor must be wrong in some cases, but the great desideratum in investing money was to accommodate oneself to changing conditions. It was, however, necessary, in accommodating oneself to changing conditions to act quickly, although with due consideration. To act slowly was generally to lose the opportunity. For instance, if a fall of price occurred, owing to some catastrophe, such fall was nearly always greater than was justified, and if the investor waited for a week or fortnight to think about it he would nearly always find he had missed the market. In investment work it was particularly necessary to have the courage of one's convictions and stand by them. To act up to this standard was not always so easy as it appeared, and in time of panic the general atmosphere of complete distrust was most difficult to resist. For instance, during the American panic of 1907, the man whose duty it was to advise on big investments, when he saw prices continually falling, had some difficulty in persuading himself (and others) that a great recovery was bound to come.

Mr. Marks had referred to the formation of an Insurance Trust Company, but lest there should be any misconception of the remarks made in the paper he wished to observe that he had no intention of suggesting so extensive a scheme as that indicated. He was quite in agreement with Mr. Hart in saying that it was absolutely essential that the investment expert should have practical experience.

MR. LIDSTONE'S METHOD OF APPROXIMATING TO THE VALUES OF
JOINT-LIFE AND LAST SURVIVOR ANNUITIES (*J.L.A.*, xlvii,
pp. 1-64.)

I.

*Note by MR. D. C. FRASER, with reference to the case of three
lives.*

THE object of the present note is to examine a feature referred to by Mr. Lidstone in paragraph 23 of his paper, where he points out that in certain cases better results are obtained by using two terms of his formula than by the employment of three terms.

1. It will be convenient in the first place to investigate the relation between Σa^2 , Σa^3 , and Σa^4 , when $\Sigma a = 0$ and Σa^2 has a known value, for the special case when three quantities only are included in the summations.

Put

$$a^2 + b^2 + c^2 = 6:2$$

Since $a + b + c = 0$ it follows that if a be the greatest of the three quantities, a , b , c , irrespective of sign, they must be of the form a , $-\epsilon a$, $-(1-\epsilon)a$, where ϵ ranges between the values 0 and 1.

Therefore $2a^2(1-\epsilon+\epsilon^2) = 6:2$

and
$$a^2 = \frac{3:2}{1-\epsilon+\epsilon^2}$$

The expression $1-\epsilon+\epsilon^2$ can be written in the alternative form $1-\epsilon(1-\epsilon)$, and $(\epsilon-\frac{1}{2})^2 + \frac{3}{4}$. The value of ϵ being limited to the range 0 to 1, an inspection of the first form shows that the maximum value of $1-\epsilon+\epsilon^2$ is 1; while an inspection of the second form shows that its minimum value is $\frac{3}{4}$. Hence the minimum value of a^2 is $3:2$, and its maximum value $4:2$. Thus a may have positive values ranging from $\sqrt{3:2}$ to $2:2$, or negative values ranging from $-\sqrt{3:2}$ to $-2:2$; and we can write $a = \pm \sqrt{3+k}:2$, where k ranges from 0 to 1.

The value of the product abc can be found as follows:

$$\begin{aligned} 2bc &= (b+c)^2 - (b^2+c^2) \\ &= a^2 - (6:2 - a^2) = 2a^2 - 6:2 = 2k:2 \end{aligned}$$

Therefore $abc = ak:2 = \pm k \sqrt{3+k}:2$.

It is easy to prove that $\Sigma a^3 = 3abc$, and $\Sigma a^4 = \frac{1}{2}(\Sigma a^2)^2$, and hence

$$\Sigma a^3 = \pm 3k \sqrt{3+k}:2$$

$$\Sigma a^4 = 18:4$$

2. It follows at once that when three lives are involved the general interpolation formula up to fourth powers can be written in the form

$$A + B6:2 \pm C3k \sqrt{3+k}:2 + D36:4$$

Since k can take any value from 0 to 1, this formula represents a set of curves all having the same ordinate at the origin. That ordinate is a maximum ordinate of all the curves if B is $-ve$, and a minimum ordinate if B is $+ve$. The curves lie side by side without intersecting or meeting (except at the origin), and they are bounded by the two limiting curves

$$A + B6:2 + C6:3 + D36:4$$

$$A + B6:2 - C6:3 + D36:4$$

3. For given central ages, x , the values of the coefficients A, B, C, D , can be determined if we know four values of the function not all belonging to the same curve.

Taking central ages 50, and the following values of last survivor annuities,

$$\text{Ages } 50, 50, 50 : a = 20.103$$

$$38, 56, 56 : \beta = 21.280$$

$$44, 44, 62 : \gamma = 21.144$$

$$30, 60, 60 : \delta = 22.874$$

the equations to find the coefficients are

$$a = A$$

$$\beta = A + 216B - 1296C + (216)^2D$$

$$\gamma = A + 216B + 1296C + (216)^2D$$

$$\delta = A + 600B - 6000C + (600)^2D$$

Here a, β, γ , lie on the curve obtained by giving $\pm k\sqrt{3} + k$ the value 2, and δ lies on the curve obtained by giving to the same expression the value -2 .

Solving the equations,

$$A = 20.103$$

$$10^2B = -.571$$

$$10^3C = -.0525$$

$$10^4D = -.0268$$

4. The formula employed by Mr. Lidstone (par. 19) can be written

$$a + \frac{\beta - a}{216} \Sigma a^2 + \frac{\gamma - \beta}{2592} (\Sigma a^3 + 6 \Sigma a^2)$$

and it gives accurate values of ordinates on the two limiting curves when $\Sigma a^2 = 0$ or 216, that is when $z = 0$ or 6 and when $k = 1$.

From our equations for the coefficients we see at once that

$$\frac{\beta - a}{216} = B - 6C + 216D$$

$$\frac{\gamma - \beta}{2592} = C$$

Put $\frac{\beta - a}{216} = B'$. Then the complete formula to fourth powers can be written

$$A + B'6z^2 + C(36z^2 \pm 3k\sqrt{3+k} \cdot z^3) + D36z^2(z^2 - 36)$$

the first three terms of which constitute Mr. Lidstone's formula.

Remembering that $a = \pm \sqrt{3+k} \cdot z$ the formula last obtained may be expressed as

$$A + B'6z^2 + C(12 + ak) \cdot 3z^2 + D36z^2(z^2 - 36)$$

where it should be noted that a, k may take any value from $2z$ to $-2z$.

5. The error involved in using two terms of Mr. Lidstone's formula is

$$C(12 + ak)3z^2 + D36z^2(z^2 - 36) = P.$$

The error involved in using three terms is

$$D36z^2(z^2 - 36) = Q.$$

We have to examine the conditions under which P is numerically $< Q$.

6. If P is numerically $< Q$ the two terms which occur in P must be of opposite sign, and $C(12 + ak)3z^2$ must be between 0 and $-D72z^2(z^2 - 36)$.

Therefore $\frac{C}{D}$ must lie between 0 and $\frac{24(36 - z^2)}{12 + ak}$.

7. The expression $12 + ak$ is always $+ve$, because ak cannot have a larger $-ve$ value than $-2z$; and the expression $36 - z^2$ is always $+ve$ if $z < 6$; therefore, if C and D have opposite signs, the conditions under which $P < Q$ are not satisfied; and it follows then that in all cases where the sum of the squares does not exceed 216 ($= 6^2$) the 3-term approximation will be better than that obtained by using two terms.

8. If C and D have the same sign it may happen that the conditions under which $P < Q$ can be satisfied. It will be sufficient to examine three cases.

I. Take $ak = -2z$; so that a, b, c , are of the form $-2z, z, z$.

Then $\frac{C}{D}$ must lie between 0 and $\frac{24(36 - z^2)}{12 - 2z}$

or between 0 and $12(6 + z)$.

In the numerical example we have taken, C and D have the same sign, and $\frac{C}{D} = 20$ approximately. It follows that

the 2-term approximation is always better than the 3-term approximation for last survivor annuities on three lives when the ages are of the form $50 - 2z, 50 + z, 50 + z$.

II. Take $ak = 2z$; so that a, b, c , are of the form $2z, -z, -z$.

Then $\frac{C}{D}$ must lie between 0 and $\frac{24(36 - z^2)}{12 + 2z}$

or between 0 and $12(6 - z)$.

Putting $\frac{C}{D} = 20$, we find that $12z < 52$ or $z < 4\frac{1}{3}$; and therefore $6z^2 < 113$. Thus, in our example, if the ages have the form $50 + 2z$, $50 - z$, $50 - z$, the 2-term approximation gives the better results so long as $\Sigma a^2 < 113$.

- III. Take $ak = 0$: so that a , b , c , are of the form $\sqrt{3}z$, 0 , $-\sqrt{3}z$. Then $\frac{C}{D}$ must lie between 0 and $2(36 - z^2)$. Putting $\frac{C}{D} = 20$, it follows that z^2 must be < 26 , and $6z^2 < 156$. Thus, if the ages have the form $50 + a$, 50 , $50 - a$, the 2-term approximation will be the better so long as $\Sigma a^2 < 156$.

9. These results are capable of a simple geometrical interpretation. In the cases we have examined the first two terms of Mr. Lidstone's formula represent a parabola which in the first part of its range lies below the set of curves represented by the formula when completed to 4th powers. It cuts the lowest curve of the set when $\Sigma a^2 = 113$: the mean curve when $\Sigma a^2 = 156$: and the uppermost curve when $\Sigma a^2 = 216$.

Up to the points of section, the ordinates of the parabola are less than the corresponding ordinates of the respective curves. The correction $C(36z^2 + 3akz^2)$ is essentially negative up to the points of section, and alters the value obtained from the parabola in the wrong direction.

For the uppermost curve the correction $C(36z^2 + 3akz^2)$ becomes $C(36z^2 - 6z^3)$ which is $-ve$ when $z < 6$, and $+ve$ when $z > 6$. The parabola cuts this curve when $z = 6$, and its ordinates are too small up to that point and too great afterwards. In each case the correction increases the error, so that, for the uppermost curve, the 2-term approximation is always the better.

II.

Remarks by MR. LIDSTONE in regard to the discussion on his Paper and to MR. FRASER'S Note.

(The paragraphs are numbered in continuation of the original paper.)

45. IN paragraph 23 of the paper it was remarked—in relation to last survivor annuities on three lives—that “a curious feature which the writer cannot satisfactorily explain is that in the majority of cases . . . the first (2-term) approximation is actually “closer than the second (3-term).” After the paper was prepared for press, but before it was read, it became evident to me that the

apparent anomaly depended on the relative sign and magnitude of the 3rd and 4th terms in the expansion, though I was not able, at the time, fully to work out the matter. Having since had the advantage of reading Mr. Fraser's very interesting memorandum—with which I fully concur—and of considering the matter further, I have arrived at a complete explanation in a form somewhat different from Mr. Fraser's, and the results are here given. The investigation will probably be easier to follow if we first consider the analogous case of an ordinary interpolation of a function of a single variable, in which case a numerical example can easily be given.

46. Consider then a function u_x of the 3rd degree, in which $\Delta^3 u_x$ is constant. We have, writing u for u_0 , and Δ'' for $\Delta'' u_0$

$$u_x = u + x\Delta + \frac{x(x-1)}{2} \Delta^2 + \frac{x(x-1)(x-2)}{6} \Delta^3$$

which is an *exact* formula in the case supposed. If we take a 2-term approximation, $u + x\Delta$, the error, which we may call ϵ_2 , will be

$$\epsilon_2 = \frac{x(x-1)}{2} \Delta^2 + \frac{x(x-1)(x-2)}{6} \Delta^3 = \frac{x(x-1)(x-2)}{6} \Delta^3 \left(\frac{3}{x-2} \cdot \frac{\Delta^2}{\Delta^3} + 1 \right)$$

If we take a 3-term approximation, the error ϵ_3 will be

$$\epsilon_3 = \frac{x(x-1)(x-2)}{6} \Delta^3 (1)$$

47. Now obviously if $\frac{3}{x-2} \cdot \frac{\Delta^2}{\Delta^3}$ is positive, ϵ_2 will be numerically $> \epsilon_3$ (the sign, which is immaterial, being that of $x(x-1)(x-2)\Delta^3$ in each case). But if $\frac{3}{x-2} \cdot \frac{\Delta^2}{\Delta^3}$ is negative and numerically < 2 , then ϵ_2 will be numerically $< \epsilon_3$ (the sign being that of $x(x-1)(x-2)\Delta^3$, while $\frac{3}{x-2} \cdot \frac{\Delta^2}{\Delta^3}$ is between 0 and -1 , and the contrary sign when $\frac{3}{x-2} \cdot \frac{\Delta^2}{\Delta^3}$ is between -1 and -2).

48. Take as an example $u_x = 100 + 11x + x^3$. Here $\frac{\Delta^2}{\Delta^3} = 1$, and

x	u_x		
0	100		(if x is $< \frac{1}{2}$), $\frac{3}{x-2} \cdot \frac{\Delta^2}{\Delta^3}$ is negative and numerically
		12	< 2 , so that ϵ_2 is numerically $< \epsilon_3$, <i>i.e.</i> , the
1	112	6	2-term approximation is better than the 3-term
		18	one. Try $x = \frac{1}{4}$, giving $102\frac{49}{64}$ for the exact
2	130	12	value, $u_{\frac{1}{4}}$.
		30	
3	160	18	
		48	
4	208		

$$\text{2-term approx.} = 100 + \frac{1}{4} \cdot 12 = 103 \quad \epsilon_2 = \frac{15}{64} < \epsilon_3$$

$$\text{3-term approx.} = 100 + \frac{1}{4} \cdot 12 - \frac{3}{32} \cdot 6 = 102 \frac{28}{64} \quad \epsilon_3 = \frac{21}{64} > \epsilon_2$$

49. Passing to the case of a three-variable function and reverting to par. 18 of the paper, it will be found that $\Sigma a^4 = 18t^4$, and writing down the values of α , β and γ , to four terms, we shall have

$$\alpha = A$$

$$\beta = A + 6t^2B - 6t^3C + 18t^4D$$

$$\gamma = A + 6t^2B + 6t^3C + 18t^4D$$

or writing $6t^2B = b$, $6t^3C = c$, $18t^4D = d$

$$\alpha = A$$

$$\beta = A + b - c + d \quad \beta - \alpha = b - c + d$$

$$\gamma = A + b + c + d \quad \gamma - \beta = 2c$$

50. Thus the general expression for the value of the function taken to four terms instead of three will be

$$\begin{aligned} & A + \frac{\Sigma a^2}{6t^2}b + \frac{\Sigma a^3}{6t^3}c + \frac{\Sigma a^4}{18t^4}d \\ &= A + \frac{\Sigma a^2}{6t^2}(b - c + d) + \left(\frac{\Sigma a^2}{6t^2} + \frac{\Sigma a^3}{6t^3}\right)c - \left(\frac{\Sigma a^2}{6t^2} - \frac{\Sigma a^4}{18t^4}\right)d \end{aligned}$$

or remembering that in the case of three variables $\Sigma a^4 = \frac{1}{2}(\Sigma a^2)^2$

$$= A + \frac{\Sigma a^2}{6t^2}(b - c + d) + \left(\frac{\Sigma a^2}{6t^2} + \frac{\Sigma a^3}{6t^3}\right)c - \left(\frac{\Sigma a^2}{6t^2} - \frac{(\Sigma a^2)^2}{36t^4}\right)d$$

Now the "1st coefficient" $= \frac{\Sigma a^2}{6t^2} = K_1$, say,

and the "2nd coefficient" $= \left(\frac{\Sigma a^2}{12t^2} + \frac{\Sigma a^3}{12t^3}\right) = K_2$, say,

so that the last expression may be written

$$A + K_1(b - c + d) + 2K_2c - (K_1 - K_1^2)d$$

51. The 2-term approximation, $A + K_1(\beta - \alpha) = A + K_1(b - c + d)$ and the error, which we may call ϵ_2 will be

$$\epsilon_2 = -2K_2c + (K_1 - K_1^2)d = (K_1 - K_1^2)d \times \left(1 - \frac{2K_2}{K_1 - K_1^2} \cdot \frac{c}{d}\right)$$

52. The 3-term approximation,

$$\Lambda + K_1(\beta - z) + K_2(\gamma - \beta) = \Lambda + K_1(b - c + d) + K_2 \cdot 2c$$

and the error is

$$\epsilon_3 = (K_1 - K_1^2)d$$

Hence $\epsilon_2 = \left(1 - \frac{2K_2}{K_1 - K_1^2} \cdot \frac{c}{d}\right)\epsilon_3$, and ϵ_2 will be numerically less than ϵ_3 if the factor is numerically < 1 , i.e.,

if $\frac{2K_2}{K_1 - K_1^2} \cdot \frac{c}{d}$ lies between 0 and +2

if $\frac{K_2}{K_1 - K_1^2}$ or $\frac{K_2}{K_1(1 - K_1)}$ lies between 0 and $\frac{d}{c}$

53. This criterion is of course identical with Mr. Fraser's, though in a very different form, which appears to be more convenient where a table of coefficients is available. The criterion may be tested by applying it to the case considered by Mr. Fraser, viz., the examples given in the paper, p. 16, for last survivor annuities on 3-lives of average age 50. The following table gives the required values of $\frac{K_2}{K_1 - K_1^2} = \frac{K_2}{K_1(1 - K_1)}$:

<i>m</i>	<i>a</i>	K_1	$1 - K_1$	$K_1(1 - K_1)$	K_2	$K_2 \div K_1(1 - K_1)$
0	5	·077	·923	·071	·049	·69+
0	10	·309	·691	·214	·240	1·12+
0	15	·694	·306	·212	·636	3·00
0	20	1·235	·235	·290	1·303	4·49
5	5	·077	·923	·071	·028	·39+
5	10	·231	·769	·178	·116	·65+
5	15	·540	·460	·248	·377	1·52+
5	20	1·003	·003	·003	·877	292·00
10	10	·309	·691	·214	·069	·32+
10	15	·540	·460	·248	·163	·66+
10	20	·926	·074	·069	·463	6·71
15	15	·694	·306	·212	·058	·27+
15	20	1·003	·003	·003	·126	42·00
20	20	1·235	·235	·290	·068	·23+

54. Referring to Mr. Fraser's memorandum, par. 3, it will be seen that our *c* is his C multiplied by 216×6 : and our *d* is his D multiplied by 216×216 ; hence our $\frac{d}{c}$ will be

$$\frac{36D}{C} = \text{about } \frac{36}{20} = 1·8.$$

The cases in which $K_2 \div (K_1 - K_1^2)$ fall between 0 and 1.8 are marked with a dagger † in the foregoing Table, and it will be seen that they correspond precisely with the * which mark the cases where the 2-term approximation is the better in the examples on p. 16 of the paper.

55. When the disparities of age fall within the extreme range $3t (= 18)$ years on which the Tables are based, $(K_1 - K_1^2)$ and K_2 , and therefore $K_2 \div (K_1 - K_1^2)$, will always be positive; thus $K_2 \div (K_1 - K_1^2)$ cannot fall between 0 and $\frac{d}{c}$ unless $\frac{d}{c}$ is positive, *i.e.*, unless c and d have the same sign. Therefore, within that range of ages, the 2-term result cannot be better than the 3-term result if $\frac{d}{c}$ is negative, *i.e.*, if c and d have different signs, a result which is found independently by Mr. Fraser. As an illustration we may take the 3-joint-life values for average age 50 (paper, p. 14). Here $\frac{d}{c} = -.44$, and all the 2-term results are worse than the 3-term ones.

56. It is to be observed that there is always a choice of *two* 2-term approximations, one based on α and β (as in the examples given in the paper), and the other based on α and γ .

In this latter case we have $\gamma - \alpha = b + c + d$ and $\beta - \gamma = -2c$, and proceeding as before we shall obtain the following results:

$$\text{Complete value} = A + K_1(b + c + d) - \left(\frac{\sum a^2}{6t^2} - \frac{\sum a^3}{6t^3} \right) c - (K_1 - K_1^2)d$$

$$\text{or writing } 2K'_2 \text{ for } \frac{\sum a^2}{6t^2} - \frac{\sum a^3}{6t^3}, \text{ i.e., } K'_2 = \left(\frac{\sum a^2}{12t^2} - \frac{\sum a^3}{12t^3} \right)$$

$$\text{Complete value} = A + K_1(b + c + d) - 2K'_2c - (K_1 - K_1^2)d$$

$$\text{2-term approximation} = A + K_1(b + c + d)$$

$$\text{3-term approximation} = A + K_1(b + c + d) - 2K'_2c$$

whence we have

$$\epsilon'_2 = 2K'_2c + (K_1 - K_1^2)d = (K_1 - K_1^2)d \times \left(1 + \frac{2K'_2}{K_1(1 - K_1)} \frac{c}{d} \right)$$

$$\epsilon'_3 = (K_1 - K_1^2)d$$

So that ϵ'_2 is numerically $< \epsilon'_3$ if $\frac{K'_2}{K_1(1 - K_1)}$ lies between 0 and $\frac{d}{c}$,

i.e., if $\frac{-K'_2}{K_1(1 - K_1)}$ lies between 0 and $-\frac{d}{c}$. Now it may be shown

that $-K'_2$ for disparities of age m and n is equal to the tabular value of K_2 for disparities $n - m$ and n . Hence the values of the criterion may be written down from those already calculated thus:

m	n	$n-m$	n	Criterion	m	n	$n-m$	n	Criterion
0	5	5	5	-·39+	5	20	15	20	+ 42·00
0	10	10	10	-·32+	10	10	0	10	- 1·12
0	15	15	15	-·27+	10	15	5	15	- 1·52
0	20	20	20	-·23+	10	20	10	20	- 6·71
5	5	0	5	-·69	15	15	0	15	- 3·00
5	10	5	10	-·65	15	20	5	20	+ 292·00
5	15	10	15	-·66	20	20	0	20	+ 4·49

57. In the case of three joint-life annuities (*vide* pp. 14-15 of the paper) the 2-term result there given is always worse than the 3-term result; but the above investigation suggests that this may be because, in the absence of the criterion since determined, the less advantageous 2-term result was chosen. It therefore becomes of interest to test the other set of 2-term results, obtained by an interpolation between α and γ instead of between α and β . Mr. A. E. King has accordingly prepared the following Table for average age 50, where the 2-term results are found from α and γ .

EXAMPLES.

Three Joint Lives. $x:x+m:x+n$.

$O^{M50} 2\frac{1}{2}$ per-cent.

Youngest Age	MEAN AGE 50. $a_{50:50:50}=9\cdot045$.						
	m	n	True Value	New 2-term Approximation	Error	Second Approximation	Final Error
48 ¹ _{61·2}	0	5	8·959	*8·957	-·002	8·966	·007
46 ¹ _{61·2}	0	10	8·692	*8·690	-·002	8·711	·019
45 ¹ _{61·2}	0	15	8·244	*8·248	-·004	8·265	·021
43 ¹ _{61·2}	0	20	7·637	*7·627	-·010	7·606	-·031
46 ¹ _{61·2}	5	5	8·966	8·957	-·009	8·972	·006
45 ¹ _{61·2}	5	10	8·801	8·780	-·021	8·816	·015
43 ¹ _{61·2}	5	15	8·454	8·425	-·029	8·475	·021
41 ¹ _{61·2}	5	20	7·929	7·894	-·035	7·932	·003
43 ¹ _{61·2}	10	10	8·755	8·690	-·065	8·765	·010
41 ¹ _{61·2}	10	15	8·527	8·425	-·102	8·543	·016
40 ¹ _{61·2}	10	20	8·120	7·982	-·138	8·126	·006
40 ¹ _{61·2}	15	15	8·435	8·248	-·187	8·445	·010
38 ¹ _{61·2}	15	20	8·165	7·894	-·271	8·166	·001
36 ¹ _{61·2}	20	20	8·044	7·627	-·417	8·032	-·012

58. It will be seen that the new 2-term results are in some cases (marked *) better than the 3-term results. These cases correspond completely to those marked † in the criterion table in par. 56, namely, the cases when the criterion falls between 0 and -·44, which is the value of $\frac{d}{c}$. The complete success of the

criterion in all the curves tested shews in these examples—and it may well prove to be generally the case—that the addition of the fourth term must represent the value of the function very closely ; Mr. A. E. King deals with this point in his remarks, p. 182.

59. A similar criterion could be obtained for 4-variable functions, but it would not take the same compact form since there is not, in the case of four variables, any means of expressing Σa^4 in terms of Σa^2 . It is possible, however, that further investigation might shew that in this case, as in the case of three variables, one or other of the alternative 2-term approximations may in many cases be superior to the 3-term approximations.

60. I am obliged to Mr. Newling for calling attention to the more elegant method of obtaining the values of the symmetrical functions of the deviations a, b, c . If terms of the higher orders were required—which, however, is not the case in practice—it would be advisable to use that interesting method, but I doubt whether the average actuarial student would find that method simpler than that given in par. 43 of the paper for the terms which are actually required in practice.

61. Further consideration of the very interesting remarks made by Mr. Todhunter in the discussion has convinced me that the ingenious plan suggested by him supplies the extension of my method which I had sought in vain at the time of preparing the paper, and further, that it is probably equally applicable to 3-life and 4-life cases, where it is more urgently required. In the pressure of other engagements, I have been unable to investigate the matter in any detail, but Mr. A. E. King has done so (as regards 2-life cases) in consultation with me, and the results are given in the following note, in which he deals also with some other points of interest which arose when the paper was being prepared.

III.

Remarks by MR. A. E. KING.

A FEW notes on various points which mainly arose while Mr. Lidstone's paper was being prepared may be of general interest, and they may also help anyone who is investigating the subject further, and save the labour of travelling over ground which has already been found unproductive. Constant reference is made to the original paper, and the paragraphs and pages mentioned refer to that paper.

Some of the most interesting sections of the paper centre round the choice of the selected values which form the basis of the interpolation. It will be seen as regards two lives, upon referring

to §13, that the value of n has been taken as $\frac{3}{2}$, *i.e.*, the standard disparities taken were 20 and 30 years, because that value of n was found, on the whole, to give the best results. This choice of a convenient value for n must be a matter of trial, and the advantage of choosing the value $n = \frac{3}{2}$ will be seen by adopting the same extreme range, *viz.*, 30 years, but putting $n = 2$ (instead of $\frac{3}{2}$) so that the annuities are taken for ages $x : x$, $x \pm 7\frac{1}{2}$, $x \pm 15$, *i.e.*, the standard deviations are 15 and 30 years. Calculating on this basis the approximate two joint-life annuity-values for average age 50, and comparing them with the examples in the paper, page 9, we shall find that the final errors for disparities of 0—15 years inclusive, are less, as may be expected, with a maximum error of .004 as against .011; but afterwards, for disparities 16—30 years inclusive, the errors are greater with a maximum error of $-.025$ as against $-.013$. Thus the small errors which are already unimportant are reduced, but the large errors are increased, so that on the whole the change is disadvantageous.

Mr. Todhunter, in the discussion, suggested that the range in the case of two lives could be further extended from 30 to 50 years, and from 50 years upwards, until the whole of the possible combinations of ages would be covered. The following figures shew the success of trials on these lines:

For Case I.— $a = a_{x-15 : x+15}$

For Case II.— $a = a_{x-15 : x+15}$

$$\beta = a_{x-20 : x+20}$$

$$\beta = a_{x-21 : x+21}$$

$$\gamma = a_{x-25 : x+25}$$

$$\gamma = a_{x-24 : x+24}$$

Joint Lives. Mean Ages 50.

0^M 2 $\frac{1}{2}$ per-cent.

Actual Ages	True Value	Case I Final Approx.	Error	Case II Final Approx.	Error
31 : 66	7.964	7.972	.008	7.973	.009
33 : 67	7.652	7.664	.012	7.665	.013
32 : 68	7.339	7.349	.010	7.353	.014
31 : 69	7.026	7.033	.007	7.036	.010
30 : 70	6.714	6.714	.000	6.719	.005
29 : 71	6.404	6.398	-.006	6.404	.000
28 : 72	6.097	6.086	-.011	6.093	-.004
27 : 73	5.795	5.781	-.014	5.790	-.005
26 : 74	5.497	5.486	-.011	5.497	.000
25 : 75	5.205	5.205	.000	5.218	.013
			$\Sigma = \pm .079$		
				$\Sigma = \pm .073$	

The errors over the same range of 30—50 years have also been found in the case of last survivor annuities to be even less than those shewn above for joint life-annuities—the total errors in the 10 approximate values amounted to $\pm \cdot 063$ as compared with $\pm \cdot 079$ in Case I above.

Passing to the case of three lives, if the results be obtained from values symmetrically situated on each side of the mean age, *i.e.*, with deviations of the form $(0, 0, 0)$, $(-t, 0, +t)$, $(-nt, 0, +nt)$, the formula at the foot of page 12 is altered to the form given at the foot of page 4 for two lives, and the values of annuities (given as examples on page 14), with the values of m and n $(0, 5)$, $(0, 10)$, $(0, 15)$, $(0, 20)$, will be respectively identical with those giving the values of m and n $(5, 5)$, $(10, 10)$, $(15, 15)$, $(20, 20)$, because the constants, C , E , &c., which are multiplied by Σa^2 , Σa^3 , &c., respectively, cannot be obtained from the standard values chosen. These standard values would therefore be impracticable for interpolation purposes unless only two terms of the formula were taken.

Considering further this question of choice of values, it will be noticed that the results are even better when interpolation is made between values in the same line of interpolation, because such an interpolation is really in respect of one variable only, namely (actual deviation): (standard deviation), *e.g.*, when deviations are of the form $(-2k, +k, +k)$, it would be better to interpolate between values with deviations of $(0, 0, 0)$, $(-2t, +t, +t)$, $(-2nt, +nt, +nt)$, and similarly, when deviations are in the form $(-k, -k, +2k)$, it would be better to interpolate between values with deviations of the form $(0, 0, 0)$, $(-t, -t, +2t)$, $(-nt, -nt, +2nt)$. The values of a , β , and γ , can then correspond to ranges of 0 , $3t$, and $3nt$, where a convenient value of n is taken, just as in the case of two lives. This, however, only applies to cases when two of the ages are equal. A third class (furthest removed as regards similarity of expression) is that in which the deviations are of the form $(-k, 0, +k)$: here we can interpolate between values with deviations of $(0, 0, 0)$, $(-t, 0, +t)$, $(-nt, 0, +nt)$, and again we have a single variable only: the ranges can again be extended, and the coefficients given in Table "A" on page 54, for two lives, could be employed, provided the same standard ranges are adopted.

Although we have noted three important classes, namely, functions with deviations of the form $(-2k, +k, +k)$, $(-k, -k, +2k)$, $(-k, 0, +k)$, the great majority of combinations of ages in practice do not take either of the forms mentioned, and it would be impracticable to have a fresh formula for each intermediate form of expression, which would involve not only different coefficients but greatly extended calculations of annuity-values. It may be stated, however, that with a fixed range, and taking only three terms of the general formula, slightly better results would be obtained if we divided all annuities into three cases according as they approximate to either of the three forms mentioned above. For example, taking a fixed range of eighteen

years, we could classify all three life annuities on ages $x, x+m, x+n$, according as (1) $(n-m)$ is $> m$, (2) $(n-m)$ is $< m$ (for the present purpose the case $(n-m) = m$ can be assumed to fall into either (1) or (2)), and we could use for class (1) the constants B and C obtained from annuity-values with deviations $(0, 0, 0), (-9, 0, +9), (-6, -6, +12)$, and for class (2) from annuity-values with deviations $(0, 0, 0), (-9, 0, +9), (-12, +6, +6)$. We should have as a result values which are on the whole slightly more accurate than those given in the paper, but the very slight increase in accuracy would not compensate for the considerable extra labour involved in preparing the Tables.

The remarks in §18 as to the selection of the deviations $-12, +6, +6$, and $-6, -6, +12$, raise a point of interest as to the effect which would be produced if other standard deviations were chosen. As mentioned in Mr. Lidstone's paper, the above deviations produce the maximum values of Σa^2 and Σa^3 for a range of 18 years. The following Table shows the effect of adopting what may be termed intermediate standard values for β and γ , and in this example the disparities taken were $-10, +2, +8$ for β and $-8, -2, +10$ for γ , giving the same extreme range, 18 years.

Three Joint Lives. $x, x+m, x+n$.

$0^{M(5)} 2\frac{1}{2}$ per-cent.

	Mean Ages 50. $a_{50:50, 50} = 9.045$						
Youngest Age	m	n	True Value	2 term Approximation	Error	3-term Approximation	Final Error
48 $\frac{1}{2}$	0	5	8.959	8.967	.008	8.963	.004
46 $\frac{1}{2}$	0	10	8.692	8.733	.041	8.705	.013
45	0	15	8.244	8.343	.099	8.248	.004
43 $\frac{1}{2}$	0	20	7.637	7.797	.160	7.573	-.064
46 $\frac{1}{2}$	5	5	8.966	8.967	.001	8.971	.005
45	5	10	8.801	8.812	.011	8.812	.011
43 $\frac{1}{2}$	5	15	8.454	8.500	.046	8.465	.011
41 $\frac{1}{2}$	5	20	7.929	8.031	.102	7.908	-.021
43 $\frac{1}{2}$	10	10	8.755	8.733	-.022	8.761	.006
41 $\frac{1}{2}$	10	15	8.527	8.700	-.027	8.535	.008
40	10	20	8.120	8.109	-.011	8.109	-.011
40	15	15	8.435	8.343	-.092	8.438	.003
38 $\frac{1}{2}$	15	20	8.165	8.031	-.134	8.154	-.011
36 $\frac{1}{2}$	20	20	8.044	7.797	-.247	8.021	-.023

It will be noted that the weakness of these approximations is shown in the cases where Σa^2 and Σa^3 are considerably greater than the sum of the corresponding powers of the standard deviations and this, no doubt, was the reason which prompted Mr. Lidstone to adopt in his paper the forms of β and γ which gave the maximum values to Σa^2 and Σa^3 .

Throughout Mr. Lidstone's paper the first two or three terms of

the general formula were employed in illustrating the possibilities of the new method of approximation, but it is interesting to note the exact effect of introducing the fourth term, namely, $D\Sigma a^4$.

As stated in p. 4, § 9, we somewhat sacrifice the simplicity of calculation, and very considerably increase the labour of calculating the Tables if we go beyond the third term. An additional Table of coefficients would not, however, entail much labour, and, moreover, in the case of 3-life functions we have the useful relation that $\Sigma a^4 = \frac{1}{2} (\Sigma a^2)^2$, which gives a simple method of calculating the third coefficient.

It is shown in Mr. Lidstone's further memorandum, §50, that the 4-term approximation takes the form

$$A + \frac{\Sigma a^2}{6f^2} (b - c + d) + \left(\frac{\Sigma a^2}{6f^2} + \frac{\Sigma a^3}{6f^2} \right) c - \left(\frac{\Sigma a^2}{6f^2} - \frac{\Sigma a^4}{18f^4} \right) d,$$

and it will be found that, taking in a further standard annuity-value λ with deviations $-t$, 0, $+t$, we have

$$A = a$$

$$b = \beta - a + \frac{\gamma - \beta}{2} - \left\{ \frac{3}{2} \left(\frac{\beta + \gamma}{2} + 2a - 3\lambda \right) \right\};$$

$$c = \frac{\gamma - \beta}{2}$$

$$d = \left\{ \frac{3}{2} \left(\frac{\beta + \gamma}{2} + 2a - 3\lambda \right) \right\}$$

The correction to the 3-term formula

$$a + \frac{\Sigma a^2}{6f^2} (\beta - a) + \left(\frac{\Sigma a^2}{12f^2} + \frac{\Sigma a^3}{12f^3} \right) (\gamma - \beta)$$

is therefore

$$- \left(\frac{\Sigma a^2}{6f^2} - \frac{(\Sigma a^2)^2}{36f^4} \right) d$$

which reduces to (1st coefficient)(1 - 1st coefficient) d

where

$$d = \frac{3}{2} \left(\frac{\beta + \gamma}{2} + 2a - 3\lambda \right)$$

Taking the case of the last survivor annuity examples, p. 16, for mean ages 50, the sum of the final errors for the 14 annuity-values is reduced by 88 per-cent by using this further correction, *i.e.*, from $\pm .364$ to $\pm .045$, and the maximum error is reduced from .044 to .007.

All that is necessary to do, if it be desired to benefit by this

further correction, is to form a table of (1st coefficient) $(1 - 1st \text{ coefficient})$ from Table D, pp. 36-8, and to calculate the value of d , i.e., $\frac{3}{2} \left(\frac{\beta + \gamma}{2} + 2a - 3\lambda \right)$ after λ has been calculated for each year of life. The calculation of λ , of course, greatly increases the total labour, especially in the case of last survivor annuities.

It is not, of course, essential that λ should be based on the deviations $-t, 0, +t$ (where $t=6$), for any similar symmetrical* values with deviations of the form $-m, 0, +m$ are equally convenient, and in fact very accurate results have also been obtained when λ has been taken with deviations $-\frac{t}{2}, 0, +\frac{t}{2}$, in one case, and $-\frac{3t}{2}, 0, +\frac{3t}{2}$, in another case, the sum of the final errors in these two cases being $\pm .057$ and $\pm .059$ respectively.

Passing to the "Further Development", it may be noted that, for the O^M Table, remarkably good results can be obtained by using only the first uniform seniority correction, even over a range of 50 years. For example, if but one correction were used, the largest error in the final column of the Table at the head of p. 29 of Mr. Lidstone's paper would only be .015.

Should it be convenient to assume that Gompertz's Law holds, or to use some other approximation familiar to one in practical work, a similar further development could be employed in order to correct the errors resulting from the assumptions made. Probably one correction would suffice, and all that it would be necessary to tabulate would be the standard errors for a disparity of, say, 20 or 30 years.

* Values which are not symmetrical are not so convenient.

IV.

Note by MR. W. PALIN ELDERTON on his suggested approximation to the values of Last Survivor Annuities.

THE method of approximation is as follows :

$$a_{\overline{bed} \dots (n)} = a_{\overline{fff} \dots (n)}$$

where

$$\frac{1}{\mu_{t+a}} = \frac{1}{n} \left\{ \frac{1}{\mu_{b-a}} + \frac{1}{\mu_{c+a}} + \frac{1}{\mu_{d+a}} \dots n \text{ terms} \right\}$$

and a in the last equation is roughly the value of the annuity required.* If it cannot be estimated closely at a first trial the formulæ can be applied a second time.

The method requires the tabulation of last survivor or joint-life annuities for equal ages, but this would not entail much labour and.

* Cp. The method of *J.I.A.*, xlv, p. 293, *et seq.*

as has been shown,* the joint-life annuities for equal ages are useful even when Makeham's law does not hold.

EXAMPLES.

Two lives—

$$\begin{aligned}
 & a_{37:63} = {}^{(O)M}2\frac{1}{2}\frac{5}{5} \\
 \text{reciprocal of } & \mu_{37+20} = 43.78 \\
 & \therefore \mu_{63+20} = 5.41 \\
 & 49.19 \div 2 \\
 & 24.60 = \text{reciprocal of } \mu_{45+20} \\
 \therefore & a_{37:63} = a_{45:45} = 20.559
 \end{aligned}$$

The method does not pretend to extreme accuracy and the nearest integral age has been used throughout for the substituted life. The true value is 20.422.

Three lives—

$$\begin{aligned}
 (1) \quad & a_{25:35:45} = {}^{(O)M(5)}2\frac{1}{2}\frac{2}{5} \\
 \text{reciprocal of } & \mu_{25+25} = 66.05 \\
 & \therefore \mu_{35+25} = 34.97 \\
 & \therefore \mu_{45+25} = 16.23 \\
 & 117.25 \div 3 \\
 & 39.08 = \text{reciprocal of } \mu_{33+25} \\
 \therefore & a_{25:35:45} = a_{33:33:35} = 26.243.
 \end{aligned}$$

The true value is 26.284.

$$\begin{aligned}
 (2) \quad & a_{40:55:55} = {}^{(O)M(5)}2\frac{1}{2} \\
 \text{reciprocal of } & \mu_{40+20} = 34.97 \\
 & \therefore \mu_{55+20} = 10.73 \\
 & \therefore \mu_{55+20} = 10.73 \\
 & 56.43 \div 3 \\
 & 18.81 = \text{reciprocal of } \mu_{48+20} \\
 \therefore & a_{40:55:55} = a_{48:48:48} = 20.925.
 \end{aligned}$$

The true value is 20.948.

* *Loc. cit.*

Four lives—

$$\begin{array}{rcl}
 a_{\overline{33:45:49:53}} & O^{M(5)} 2\frac{1}{2} \% & \\
 \text{reciprocal of } \mu_{33+23} & = & 45.99 \\
 \text{.. .. } \mu_{45+23} & = & 19.06 \\
 \text{.. .. } \mu_{49+23} & = & 13.78 \\
 \text{.. .. } \mu_{53+23} & = & 9.86 \\
 & & \hline
 & & 88.69 \div 4 \\
 & & \hline
 & & 22.17 = \text{reciprocal of } \mu_{43+23}.
 \end{array}$$

$$\therefore \overline{a_{\overline{33:45:49:53}}} = a_{\overline{43:43:43:43}} = 23.756.$$

The true value is 23.970.

These examples have been taken at random, but I do not think three-life annuities would always show such close results.

It is now necessary to consider how one might approximate to the last survivor annuity when the lives are not subject to the same mortality (*e.g.*, one male and one female), and it seems that the method of substituting a life or lives by using the annuity-values is sufficient for all, or nearly all, practical purposes.

Example : $a_{\overline{(m,50):(f,40)}}$ O^{am} and O^{af} $2\frac{1}{2}$ per-cent

$$\begin{array}{rcl}
 a_{[f,40]} & = & 19.622 = a_{[m,38]} \\
 \therefore a_{\overline{(m,50):(f,40)}} & = & a_{[m,50]:[m,38]} \\
 \text{reciprocal of } \mu_{[50]+21}^* & = & 15.74 \\
 \text{.. .. } \mu_{[38]+21} & = & 38.28 \\
 & & \hline
 & & 54.02 \div 2 \\
 & & \hline
 & & 27.01 = \text{reciprocal of } \mu_{[43]-2} \\
 \therefore a_{\overline{(m,50):(f,40)}} & = & a_{\overline{(m,50):(m,38)}} \\
 & = & a_{\overline{(m,43):(m,43)}} \\
 & = & 21.568
 \end{array}$$

The true value is 21.731.

The method when there are more than two lives is similar.

It will, I think, be found that the underlying principle of the method of approximation indicated in this note is the assumption of a constant value of μ_x , and by taking the values of the force of mortality a years hence we are averaging the values of μ .

* Tables of μ_x for the O^{am} Table are only given in "Principles and Methods"; if they are not at hand, q_x may be used.

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., *Barrister-at-Law.*

Assurance
Companies Act
1909. Subsidiary
company.

THE case of *In re Lancashire Plate Glass, Fire and Burglary Insurance Company*, which came before the Courts recently, is of some interest to all assurance companies as throwing light on the question as to what constitutes a subsidiary company within the meaning of the provisions of section 16 of the Assurance Companies Act, 1909, relating to the winding-up of subsidiary companies.

The company concerned was formed in 1903 for the purpose of carrying on most branches of the business of insurance, except life assurance. In 1909 a provisional agreement was entered into for the sale of the company's undertaking to the National Provincial Insurance Corporation, Limited, but this agreement was not carried out owing to the opposition of certain shareholders in the Lancashire company. An arrangement was then made under which, in 1909 and 1910, the National company purchased the bulk of the Lancashire company's shares from individual shareholders. They also guaranteed the policies of the Lancashire company, and entered into a treaty of mutual re-insurance in respect of the two companies' policies.

In July 1911, a petition to wind-up the National company was presented, and on 9 August, a compulsory order was made thereon. On 18 September 1911, the National company presented a petition for the winding-up of the Lancashire company on the ground that the latter was a subsidiary company to the former within the meaning of section 16 of the Assurance Companies Act, 1909, and that it was just and equitable that the two companies should be wound-up together.

The case came before Swinfen Eady, J., in November last, and he then held that on the facts stated the Lancashire company was not a subsidiary company to the National company. In delivering judgment to this effect, he said: "In my opinion the petitioners have not established any case within section 16 of the Assurance Companies Act, 1909. I cannot find that there was any transfer of assurance business. A treaty of mutual reinsurance is not a transfer of assurance business. A guarantee of policies is not a transfer of assurance business. Again, the purchaser of shares in an assurance company does not acquire a transfer of any part of its assurance

“ business, even though he obtains control of the company by
 “ acquiring the bulk of the shares and appointing the directors.
 “ Section 16 is therefore inapplicable, and the Lancashire
 “ company cannot be wound-up as a subsidiary company.”

Policy subject
 to forfeiture on
 assured going
 abroad.
 Application for
 return of
 premiums.

The recent case of *Sparenborg v. The Edinburgh Life Assurance Company* [1912] 1 K.B. 195, was concerned with the effect of a condition, endorsed upon certain policies of life assurance, relating to the assured going abroad. The policies were issued in June, 1894, and one of the conditions named certain limits and circumstances within and in which the assured could go abroad without special permission. The condition went on to state that “ if the assured shall go beyond
 “ those limits . . . without obtaining license from the company,
 “ the assurance shall be void, and the premiums paid shall be
 “ forfeited.” A further condition provided that “ If during the
 “ first five years from the date of the commencement of the
 “ policy no extra premium shall have been incurred . . . the
 “ assured shall be at liberty, after the expiry of the said period,
 “ to travel or reside in any part of the world, or to engage in any
 “ occupation, without obtaining license from the company.”

In 1897 the plaintiff went on a voyage to India and back, stopping eight days at Calcutta. This was outside the limits allowed by the policy, but he did not obtain a license for the purpose, either not knowing or overlooking the fact that such a license was required. The fact of his having made this voyage was unknown to the assurance company. The plaintiff continued to pay the premiums down to the end of 1910, and early in 1911 he proposed to sell the policies, when the intending purchaser raised the question as to their being world-wide and free from restrictions. This called the plaintiff's attention to the conditions of the policies, and he then drew the attention of the assurance company to his voyage to India, and requested them to endorse the policies as being world-wide and free from restrictions.

Upon receiving this application, the assurance company, in letters dated 14 and 21 February 1911, pointed out that, strictly speaking, the policies were void by reason of the breach of the condition, but that if the extra premium that would have been charged for the voyage to India, had they known of it, together with interest, amounting in all to £42, were now paid, they would

waive the forfeiture and make the policies world-wide. To this the plaintiff's solicitors replied that since the policies became void in 1897, the premiums received by the company since that date should be refunded, with interest. The assurance company then offered, in the circumstances, to forego the payment of the extra premium for the license, and to make the policies world-wide. The plaintiff, however, persisted in his claim for the return of the premiums, and on 13 March, 1911, commenced an action to recover £5,664, the amount of the premiums paid since 1897, as money had and received by the defendants to the use of the plaintiff, upon the ground that the policies became void in February, 1897, or became voidable at the option of the defendants, and the latter, by their letters of 14 and 21 February, 1911, declared the policies void.

The case came before Bray, J., on 7 November last, and he decided in favour of the assurance company. In the course of his judgment, he said : " In this case a number of points have
" been raised, but it is only necessary to decide one, namely,
" the point which is set up in answer to the claim for the recovery
" back of the premiums paid, and which arises upon Condition 6,
" endorsed upon the policies. That condition says that if the
" assured shall travel beyond certain limits without obtaining the
" license of the defendants, the assurance shall be void, ' and the
" ' premiums paid shall be forfeited.' It is contended on behalf
" of the defendants that those words cover all the premiums
" paid since the breach by the assured of the condition. . . .

" The question turns upon the meaning of ' the premiums
" ' paid.' It must have been in the contemplation of the parties
" that the question of the avoidance of the insurance might
" probably not arise until a considerable time after the assured
" had gone abroad outside the authorized limits, inasmuch as the
" insurers would probably know nothing of it unless the assured
" told them, and in the interval the premiums would have been
" paid. The words in their natural meaning would include the
" premiums so paid. It is said that they refer to the premiums
" paid before the act which caused the forfeiture, namely, the
" assured going abroad. I do not think that I have any right to
" insert those words. The words ' the premiums paid ' mean all
" the premiums paid, and I have no right to limit the meaning
" by holding that they only refer to the premiums paid before
" the act causing the forfeiture, more especially as in the body
" of the policy there is a similar clause in which the words ' all

“ ‘moneys paid on account of this insurance’ must mean all the
 “ premiums paid. I, therefore, reluctantly come to the con-
 “ clusion that no action can be brought to recover back the
 “ premiums as money paid under a mistake of fact or without
 “ consideration, even upon the assumption, as to which I express
 “ no opinion, that the policy became, *ipso facto*, void as soon as the
 “ assured went abroad outside the specified limits without the
 “ license of the defendants.”

The learned judge also refused to make the declaration, asked for by the plaintiff as an alternative, that the policies were valid and effectual and world-wide, but the defendants agreed that there should be inserted in the judgment a statement that, upon the plaintiff paying the premiums due in June, 1911, they would endorse the policies “admitted world-wide and effective.”

Two or more
 policies on same
 life. Assumed
 knowledge of
 company.

The case of *Marcovitch v. The Liverpool Victoria Friendly Society*, 1912, xxviii. T.L.R. 188, was concerned with the validity of a policy of life assurance in the following circumstances: The policy contained a clause stating that “one policy only is allowed to be in force on the life of the
 “ person assured and named therein, unless special permission
 “ be obtained from the committee of management for any
 “ additional policy created, and should any such additional
 “ policy be obtained without the knowledge and consent of the
 “ committee (which consent shall be evidenced by an endorse-
 “ ment on the policy signed by the secretary of the society)
 “ such policy other than the first shall, if discovered during the
 “ life of the assured, be rejected, or if discovered after death, be
 “ null and void, and the sum or sums assured forfeited to the
 “ society.” The policy in question had not been endorsed as required by this clause, although at least three other policies had previously been effected on the life. The society disputed the policy on this ground, but the Court of Appeal, reversing the decision of the Divisional Court, gave judgment in favour of the plaintiff.

Lord Alverstone, C.J., in delivering judgment, said: “It
 “ was contended that unless there was an endorsement on the
 “ policy, signed by the secretary, giving the society’s consent,
 “ the existence of any other policy would render this policy void.
 “ He could not take that view. There might well be a direct
 “ consent, as indeed perhaps there was in this case, without any

“such endorsement being made. There the facts were that three times during a year or so before the present policy was issued. the assured had insured her own life, or insurances had been effected on it by other persons. The three policies were nominated away to different people, and they were all treated by the society as valid, and the amounts due on them had been paid. Under these circumstances, in his opinion, the burden of proof was on the defendants to show that they had not consented to more than one policy being in force.

“It was suggested that the defendants did not keep any record of the policies which they issued, except in their collecting books. It was difficult to believe this, and he did not think it was open to an insurance company to say that they did not know the names of the persons whose lives they had insured. He came to the conclusion that the appeal must be allowed.”

Absence of insurable interest. Can the premiums be recovered on the ground of failure of consideration? The case of *Evanson v. Crooks and Others*, 1912. xxviii. T.L.R. 123. was an action brought against the trustees of the Liverpool Victoria Legal Friendly Society for the return of premiums, on the ground that the policy in respect of which they were paid was void by reason of the absence of insurable interest. The only point of interest in the case is to be found in one of the pleas put forward on behalf of the plaintiff, under which, after failure to prove fraud, it was sought to recover the premiums on the ground that, as the policy was invalid, there had been no consideration for their payment.

In giving judgment for the defendants, Hamilton, J., said : “The remaining point was whether, in view of the fact that the plaintiff got nothing for his premiums, he could recover as on a total failure of consideration. It had been contended on his behalf that, although an action would not lie for their recovery on the ground of payment under a mistake of law, or as money alleged to have been paid under a gaming contract, the same money might be recovered as on a total failure of consideration. It was obvious that that would give the go-by to the defence of wagering and gaming. The claim sought to be set up in this action was one of money had and received to the plaintiff’s use—that is, that *ex æquo et bono* he ought to have the money back. He did not think these different ways of stating this claim were mutually independent of one another. When an Act of Parliament had said that a transaction was void,

“the defendant was not under an obligation *ex æquo et bono* to repay moneys received by him under it. Therefore, he must give judgment for the defendants, with costs if the defendants asked for them.”

Power to invest in securities of British colonies. Does this extend to colonial provincial securities?

The case of *In re Sir S. M. Maryon-Wilson's Estate*, [1911] 2 Ch. 58, dealing with the question as to whether power under a will to invest in the stocks or securities of any British colony or dependency included power to invest in the stocks of any of the provinces of the Dominion of Canada, was referred to in these Notes when it came before the Chancery Division of the High Court (*J.I.A.* xlv. 585). Mr. Justice Eve then decided that such securities were not included in the powers of investment contained in the will, and the Court of Appeal has now confirmed that decision ([1912] 1 Ch. 55). It is not necessary to give the judgment of the latter Court, as it agreed closely with that of the Court below. It will suffice to quote a few words from the judgment of Cozens-Hardy, M.R., dealing with the general principle applicable to such a case. He said: “A clause of this nature, enlarging the powers of investment beyond what the general law sanctions, ought, I think, to be construed strictly. It is for those who seek to include a particular investment to prove beyond all reasonable doubt that the words of the clause cover it. In the present case I am not satisfied that the provinces of the Dominion of Canada are either colonies or dependencies.”

Remission of income tax in respect of double endowment assurance premiums.

It is, doubtless, within the knowledge of many readers of these Notes that the Inland Revenue authorities have recently objected to the whole of the premium, in the case of double endowment assurances, being taken into account as a life assurance premium for the purpose of obtaining the remission of income tax given by section 54 of the Income Tax Act, 1853. The contention of the Crown has been that only that portion of the premium representing the temporary assurance on the life is available for the purposes of remission, and that, therefore, before any allowance could be made, the premium must be divided into its component parts. The assurance companies have steadfastly resisted this demand on the part of the Inland Revenue authorities, and, at the instance of the

Prudential Assurance Company, the matter was brought before the Court in respect of one of their policies of the description in question.

The action was brought in the form of a special case stated for the opinion of the Court, by way of appeal from a decision in favour of the Crown given by the Commissioners at Derby, and was heard by Mr. Justice Hamilton on 23 February last, under the name of *Gould v. Curtis* (Surveyor of Taxes). A report of the case will be found in *The Times* newspaper, of 24 February, but as it has not yet been reported in any of the recognized Law Reports, it is not proposed to deal with the matter at any length on this occasion. It will be sufficient to state that Mr. Justice Hamilton decided in favour of the appellant, holding that the whole of the premium came within the exemption of section 54. A fuller account of the case will be given when it appears in the Law Reports.

It is, perhaps, hardly necessary to point out that the real importance of the case lies in the fact that the arguments used by the Crown in favour of dividing the premium into its constituent parts would apply equally to the ordinary endowment assurance, and, that had they been successful in their contention, there is little doubt but that this latter form of assurance would have been treated in the same manner as the double endowment in the near future.

Moneylenders
Act, 1900. Position
of transferee
without notice
where a charge
is void under
that Act. Money-
lenders Act, 1911.

Attention was called recently in these Notes to the position of a *bonâ fide* transferee for value without notice, when the mortgage of which he is the transferee is declared void under the Moneylenders Act, 1900. In the case of *In re Robinson, Clarkson v. Robinson* ([1910] 2 Ch. 571, [1911] 27 T.L.R. 182, *J.I.A.*, xlv, 241) it was, in effect, held that the transferee in such circumstances stood in no better position than his transferor, and that the fact that he was an innocent purchaser for value without notice did not make his security valid. The difficulty revealed by this decision has now, however, been remedied by the Moneylenders Act, 1911, which, by section 1, provides as follows :

“ 1 (1) Notwithstanding anything in section 2 of the “ Moneylenders Act, 1900 :

“ (a) Any agreement with, or security taken by, a money-
“ lender shall be, and shall be deemed always to have

- “ been valid in favour of any *bonâ fide* assignee or
 “ holder for value without notice of any defect due
 “ to the operation of that section, and of any person
 “ deriving title under him ; and
 “ (b) Any payment or transfer of money or property made
 “ *bonâ fide* by any person, whether acting in a
 “ fiduciary capacity or otherwise, on the faith of the
 “ validity of any such agreement or security, without
 “ notice of any such defect, shall, in favour of that
 “ person, be, and be deemed always to have been as
 “ valid as it would have been if the agreement or
 “ security had been valid ;
 “ but in either such case the money-lender shall be liable to
 “ indemnify the borrower or any other person who is prejudiced
 “ by virtue of this enactment, and nothing in this enactment
 “ shall render valid an agreement or security in favour of an
 “ assignee or holder for value who is himself a money-lender.
 “ (2) A person shall not be deemed to have had notice of a
 “ defect in an agreement or security by reason only that a
 “ search in the register established under the Moneylenders
 “ Act, 1900, would have disclosed the defect or shown that
 “ the agreement or security was effected with a money-lender ;
 “ and for the purposes of this Act and the Moneylenders Act,
 “ 1900, the provisions of section 3 of the Conveyancing Act,
 “ 1882, shall apply and be deemed always to have applied as
 “ if the expression ‘ purchaser ’ included a person making any
 “ such payment or transfer as aforesaid.
 “ (3) Nothing in this section shall render valid for any
 “ purpose any agreement, security, or other transaction which
 “ would, apart from section 2 of the Moneylenders Act,
 “ 1900, have been void or unenforceable, nor any agreement or
 “ security which has, before the commencement of this Act,
 “ been declared void by a Court of competent jurisdiction.”

*On Aids to Calculation, with special reference to some recently
 published Tables. By H. C. PLUMMER, M.A., Assistant at
 the University Observatory, Oxford.*

THE labour of numerical calculations, irksome yet not without its
 fascination for a certain order of mind, falls chiefly on two classes
 of men—the statistician, including in particular the actuary, and the

astronomer. The former is concerned more generally with the application of the simpler rules of arithmetic, addition, multiplication, and the like; while the latter must needs use the numerical values, as a rule in tabular form, of various mathematical functions, and chiefly the trigonometrical functions. But apart from this—and the difference after all is not very great, because the true meaning of a table in no way affects its technical application—it might be expected that the practice of both classes would conform very closely. Whether it does so or not we have little means of knowing because in this country at any rate there has been little interchange between the two professions. It is quite conceivable that a greater divergence of method exists here than, for example, in the Scandinavian countries where the more lucrative careers offered by insurance practice have attracted several men of distinct mathematical promise and even of distinguished achievement, to the severe loss of pure science. Be that as it may, the masses of data go on accumulating, the need for more refined treatment increases, and the burden of what remains to be done grows heavier with time. Hence there has been in recent years a great stimulus to the production of aids to calculation of all kinds, both mechanical and tabular, and in response there have appeared a number of works of very varied merit and utility, but generally possessing some feature of interest for the professional computer.

The difference between the point of view of a statistician and an astronomer is suggested by two tables of logarithms and anti-logarithms, the one to five places by E. Erskine Scott (a new and much cheaper edition of the older work) and the other to four places by Major-General T. C. Hannyngton, both published by Charles and Edwin Layton. The arrangement of both tables is essentially the same. Both give the argument to the full five and four figures respectively, and thus there is no occasion for interpolation. Perhaps it is unfair to consider these works from the standpoint of an astronomer, for whom they do not profess to be intended. And yet his view, such as it is, may be of interest. Now he, probably without the slightest hesitation, would prefer to use tables in which the last figure of the argument was suppressed and to interpolate by the differences. He would also sacrifice the anti-logarithms entirely without regret. It is true that he would thus have a little extra trouble in taking out each logarithm. But on the other hand his tables would be reduced in bulk to one-twentieth, and in this fact he would find ample compensation. What this means is most strikingly illustrated in the case of 4-figure logarithms. Instead of seeking the required entry on one of 36 pages, he will have all that he wants on two pages at a single opening. He will have no need to turn a page, and it is in this very fact that the main advantage of 4-figure work lies. Similarly the 5-figure tables can be reduced without real loss from 360 to 18 pages. It may be answered that the interpolation, besides being troublesome, introduces an uncertainty in the last figure. This is true, but it must be remembered that an uncertainty is equally introduced by the simplest operation,

such as the addition of two logarithms, and the plain fact of the matter is that if any importance attaches to the precise figure in the last place too rigid economy is being practised, and the calculation ought to be made with tables to an extra place.

It should be added that the logarithms and the anti-logarithms in Erskine Scott's table are distinguished by being printed on white and green paper respectively. In the 4-figure table no such difference is made. If it would not shock the book producer's sense of propriety another device might be suggested which would serve in a very simple way to keep the two halves of a book distinct. If the width of the page in the second half were reduced by a quarter of an inch, the computer would open the book mechanically in the section which he requires.

These books have been discussed at some length chiefly for the reason that they have appeared most recently. But the opportunity may be taken to mention a number of works which have come under our notice during the last few years. Calculating machines like the Brunsviga have probably superseded to a large extent the multiplication tables formerly in use. But the latter will probably always possess one great advantage among others. They are silent. The tables of Crelle are too familiar to call for any description. They remain as invaluable as ever. But it may be mentioned that two new features have been introduced in the latest edition (1908). The factors ending in 0 are no longer omitted in their proper sequence, and the table is thus made complete from 1×1 to 999×999 . Also when the last two figures of the product (in the right-hand margin) exceed 49, the preceding part of the product in the several columns is followed by an asterisk, thus making it unnecessary to look at the margin in order to see whether the final figure should be raised when the last two places of the product are not actually required.

Until recently we were content with tables giving the products of numbers with 3×3 places. Now we have also the tables of Dr. Peters (Berlin: Reimer, 1908) which give the products of numbers with 4×2 places. Their arrangement and general appearance are identical with those of Crelle, with this difference. The heading of the different columns no longer indicates the first place (*i.e.*, the hundreds figure) of the factor of which the last two places appear in the column on the left, but it represents the first place (*i.e.*, the thousands figure) of the other factor which is plainly marked at the top. The products in one horizontal line thus have the last three places common, and these three figures are placed in a column on the extreme right in the way to which we are accustomed by Crelle.

In spite of their great value and general merits the tables of Crelle and Peters have one drawback. By the nature of the case they are undeniably bulky. Hence there is room for the more compact Calculating Tables of Zimmermann (Asher & Co., 1904), which give the products of factors of three figures by factors of two, of which the first fifty appear on the left-hand page, and the second fifty on the right-hand page. The products for the successive factor extending to 999 are placed in consecutive vertical columns. These

tables are convenient when products of 3×2 figures are required. The bulky form of Crelle's table is the only excuse for the appearance of Kühtmann's *Rechentafeln* (Dresden: Kühtmann, 1911), which profess to serve the same purpose. In these a separate small table in two parts is given for each of the factors to 999. In the upper part is given a double entry table of products less their two end figures, the hundreds of the second factor appearing in a vertical column on the left and the tens in a horizontal line at the top. The last two figures for all the products in any vertical column are thus the same, the last in fact being 0. In the lower part of the table are given the products for the units figure (placed in a vertical column on the left) increased by the number needed to complete the products in the same vertical column of the upper part of the table. Thus any required product is to be found by adding two numbers which appear in the same column of the upper and the lower table. It will scarcely ever happen that a product can be taken direct from the tables without an addition, and it is obvious that these tables are practically useless. It may be said generally that extended multiplication tables are of little value unless the required result can be taken from them either directly or at any rate with no more trouble than interpolation at sight. Otherwise it becomes far simpler to resort either to logarithms or to the calculating machine.

It has been said that the accuracy of astronomical observations corresponds to the use of 7-figure logarithms. If this be taken to mean that the astronomer always uses logarithms of this order it is a misconception. Though astronomy may be on the whole the most accurate of the sciences, an astronomer with judgment will avail himself of any tables which will give the desired result with the greatest economy of labour. On occasion he need not disdain even 3-figure tables. Preliminary orbits based on the earliest observations of a newly-discovered planet or comet are commonly calculated with 6-figure tables, generally Bremiker's, and other problems are treated by what experience has shown to be the appropriate means. But it is quite true that until recent years the highest demands of the practical astronomer were adequately met by 7-figure tables. Thanks to the advancing precision of his methods, however, the need for convenient 8-figure logarithms has lately made itself felt, and the want has now been supplied by the new German tables of Bauschinger and Peters (Leipzig: Engelmann, 1910-11), prepared with the help of subsidies from the Academies of Berlin and Vienna. It is the first volume which will be of value to the statistician, the second containing the logarithms of the trigonometrical functions for every centesimal second of arc. In the first are given the logarithms of every number from 100,000 to 200,000 and from 20,000 to 100,000, arranged on a plan very similar to that of Schrön's 7-figure tables. Proportional parts are supplied in the margin for every difference throughout the work, with the exception of a few pages following the change to 20,000, where the differences change too rapidly to make this possible. Even here, however, proportional parts are tabulated at the least for every

alternate difference. The excellence of the work in every practical detail seems to leave nothing to be desired.

A word on the preparation of these tables may be of interest. As a foundation the original 14-figure logarithms of Briggs were adopted. A preliminary examination showed that they contained singularly few errors, and these were corrected without difficulty. They were then reduced to 12 figures, the last figure of the original being in any case untrustworthy. The next step was to interpolate to the required intervals between the intervals of the original, and finally, after this had been done, the tables were contracted to eight places. The point of interest lies in the manner in which the interpolation was performed. It was carried out (in duplicate) by a specially constructed calculating machine which was capable of taking into account second differences and printing the results. The action of the machine throughout was irreproachable, and the authors pay just acknowledgment to the skill of the maker, Herr Hamann of Friedenau, Berlin. When we consider the costly and unsuccessful efforts of Babbage to solve this problem we may well be impressed by the advances which have been made in the art of mechanical calculation. It is not simply that the problem which baffled the pioneer has now been solved, but that the production of a novel and complex machine of this kind to order has actually become an ordinary commercial transaction.

Though the recently published tables of Professor Andoyer are not likely to be of any use to the statistician, they are not altogether without interest for any one who is concerned in methods of calculation in their more general aspect. These tables contain the logarithms of the trigonometrical functions for every ten seconds of arc to fourteen places. This labour has not been independently repeated since the original tables of Briggs were published to which Bauschinger and Peters had recourse as stated above. The repetition was therefore desirable, but one would have supposed that the task was one of the most formidable kind. In fact no volunteer has undertaken it in a period of nearly three centuries. Yet Professor Andoyer, in the midst of pressing academic duties, succeeded in carrying out the whole of the calculations in the space of about twenty months, single-handed, and without any mechanical aid whatever. The author modestly says that a natural taste for work of this kind and the devotion of a few hours daily to the work enabled him to accomplish his task in the time stated. But we cannot withhold a tribute to a piece of computation which constitutes a veritable *tour de force*.

REVIEW.

Insurance Law. By E. J. MACGILLIVRAY. (Sweet and Maxwell, Ltd., London.)

THE life assurance world has long felt the need of a satisfactory text book on the law relating to life assurance. It is true that there

are in existence comparatively recent editions of the well-known works on the subject by Bunyon and Porter. Both of these books, however, notwithstanding their many good points, leave much to be desired, and the treatise by Crawley, which in many respects was superior to either of those named, is now thirty years old, and, to a great extent, out of date. In these circumstances, high hopes were entertained when it became known that a comprehensive treatise on the law of insurance, including life insurance, was being written by Mr. E. J. MacGillivray, than whom no one is better qualified to deal with what the writers named, and others, have found to be a difficult and complicated subject.

The book has now appeared, and we must confess to a slight feeling of disappointment with regard to it. The feeling is accentuated by the evidence, of which the book is full, that the author has both the ability and the material to produce the ideal treatise on the law of insurance for which we have all looked so long. It is always more pleasant to praise than to blame, and since it is necessary to indicate the respects in which Mr. MacGillivray's treatise falls short of expectation, it will be better and more convenient to deal with those matters now, leaving its many excellencies to be noticed later on.

The primary ground of complaint which we have against the book, and one which directly or indirectly involves most of the others, is its unwieldy size, which is due mainly to two causes, namely, the scope of the work, and the method of treatment. As the title indicates, it professes to deal with the law relating to all forms of insurance other than marine, and this fact leads at once to the criticism that the scope is too comprehensive, and that the distinction between life assurance and other forms of insurance is so deep and fundamental that any attempt to deal with them in the same treatise almost inevitably yields unsatisfactory results. Some justification for the combination may, perhaps, be sought in the fact that a somewhat similar mixture, though on a smaller scale, is contained in the Assurance Companies Act, 1909. This fact is, however, rather an aggravation than a mitigation of the offence, if such it be, for while some features of that Act have been praised, and others blamed, expert opinion has, we believe, been almost unanimous in holding that life assurance should, from its nature, have been made the subject of an Act by itself. Moreover, the book fails to cover, at any rate in the way in which it deals with life and fire insurance, some of the subjects contained in its title, for although it groups together in its title life, fire, accident, guarantee, burglary, third party risks, and employers' liability insurance, the only special classes of insurance that are dealt with on an adequate scale are those of fire and life, each of the other forms, apart from general principles, being relegated to a few pages. We venture to think that a much better method of dealing with the subjects embraced by this book would have been to have separated the life assurance portion from the rest and made it a distinct treatise. Then, if it were desired to write a complete treatise on the law of insurance, such a treatise might

consist of four volumes, each of moderate size, namely, on life assurance, fire insurance, (including the other forms of insurance dealt with in this book except employers' liability,) marine insurance, and employers' liability (including in this last volume both the law relating to the general subject and the special insurance law relating to such risks).

Another point calling for notice is the method of treatment adopted by the author. There are three main classes of books on law, all of which have their uses and may be said to be complementary to one another. These are, collections of Statutes, reports of cases, and text books. Each of these is, in certain circumstances, and for certain purposes, useful, and indeed, indispensable, but when an attempt is made to combine in one volume all three of these classes, the result may easily be to present the unsatisfactory features which usually characterize a compromise, and this book has not altogether escaped that fate.

The author has introduced copious extracts from Acts of Parliament and Rules of Court, all more or less connected with the subject of his work, but many of them less rather than more. For example, nearly nine pages are taken up by giving the whole of the text of the Married Women's Property Acts of 1882, 1893, 1907 and 1908, although as regards a large part of those Acts, there seems to be no more justification for introducing them into a book on Insurance Law than there would be for introducing a large portion of the Statutes relating to criminal law, on the ground that signatures to insurance documents are sometimes forged and insurance officials sometimes embezzle the funds of their companies. As regards the Rules of Court introduced, these can only be for the use of the practising lawyer, and he may be presumed to have his Annual Practice always at hand. With regard to the reports of cases, which form so striking a feature of the book, these will, no doubt, be useful; but it is open to question whether it was worth while to take up so much space with them, since for the lay reader a short statement of the effect of the case would, perhaps, be sufficient, and those using the book for professional purposes will still have to refer to the full reports of the cases.

After what has been said, the question will, perhaps, be asked: Is the book good, bad, or indifferent? The answer to this question is that what has been said relates to form, rather than to substance, and that the book itself is not only a good one on the law of insurance, but that it is so good as compared with existing treatises on the subject as to stand almost alone.

The author commences with a discussion of the law relating to insurance companies, extending to upwards of 100 pages, and this may be said to constitute in itself a fairly complete treatise on the subject, with special reference to the Assurance Companies Act, 1909, and with excursions into the region of the law of friendly societies and industrial assurance companies. On this last mentioned matter, it may be pointed out that certain of the author's statements as to the law relating to industrial assurance companies

and collecting societies apply only to the latter, and not to both, as suggested in the book. An example of this is to be seen at the top of page 23. The subject of friendly societies is, necessarily, dealt with very briefly, and it is open to question whether it was worth while to introduce the subject into a book such as this.

The statement on page 27 that "an endowment policy assuring a sum payable in the event of the assured attaining a specified age is a policy of assurance upon human life whether or not any sum is payable in the event of earlier death", made on the strength of the decision in the case of *Prudential Assurance Company v. Commissioners of Inland Revenue*, needs some little qualification, and should certainly be limited by adding the words "within the meaning of the Stamp Act, 1891." As a matter of fact, the only justification for the words quoted is to be found in certain dicta of the learned judge who decided the case, which were not necessary for the decision actually given. This case must now be considered in connection with that of *Curtis v. Gould*, referred to in another part of this number of the *Journal*.

The author then proceeds to deal with the subject of insurable interest generally, and devotes a section to insurable interest in lives which appears to be a complete and satisfactory statement of the law on the subject. The third chapter, dealing with the formation of contract, is a very useful one, particularly those portions relating to the authority of agents, proposal and acceptance, and execution and delivery of policy.

Under the general head of "Duration of Risk", the author in Chapter IV deals with the question as to when the risk attaches, what constitutes the payment of the premium, and difficulties connected with renewal premiums and days of grace. This is followed by a chapter on "Voidable Policies", in which, in addition to a general discussion of the subject, adequate treatment is given to such matters as Fraud and Misrepresentation, Non-disclosure and Warranties.

The chapter of most interest to life assurance officials is Chapter VI which is entitled "Life Assurance, Claims and Title to Policy." This chapter may be said to give, in some 250 pages, the best account of most parts of the law of life assurance that is at present available. In it the author deals with every aspect of the law that is likely to trouble a life assurance company in connection with the payment of its claims or surrenders, and does so in a full and lucid manner. It is not possible, in the space available, to give any adequate idea of the contents of this chapter, but one or two points may be noticed. Under the head of "Claimant's Title", among other things, the right of the assurance company to retain the documents of title is discussed, and it is pointed out that, in accordance with the decision *In re Palmer*, the company cannot so retain the documents. By some oversight, apparently, this case is omitted from the Table of Cases at the commencement.

Under the head of "Voidable Assignments", the author deals both with void and voidable assignments, and refers to the danger arising under the provisions of the Moneylenders Act, 1900, as illustrated

by the case of *In re Robinson*. He points out, however, in the preface, that this difficulty was in the course of removal by legislation, and this has now been accomplished by the Moneylenders Act, 1911, to which reference is made in another part of this number of the *Journal*. The author deals very fully with the subject of Mortgages in connection with Life Policies, and cites the case of the *Life Interest and Reversionary Securities Corporation v. The Hand-in-Hand Insurance Society* in its bearing on the question of power of sale being properly exercisable. This case must now, however, be considered in connection with Section 5 of the Conveyancing Act, 1911, which has been passed to deal with the difficulty disclosed by that case.

Another very useful section is that relating to Married Women and Settlement Policies, in which the law relating to this somewhat difficult subject is fully set out in some 35 pages. We notice that the author takes the view which, we believe, is very generally held, that the case of *Robb v. Watson* was wrongly decided. He states, however, on page 559, on the authority of *Cleaver v. Mutual Reserve Fund Life Association*, that where the objects of the trust fail, there is a resulting trust in favour of the assured and his representatives, "even although not named in the policy." This may be so, but the case quoted can hardly be cited as an authority for the proposition, since, as will be seen from the report of the case which follows almost immediately after the statement, in that particular case the policy was expressed to be in favour of the wife, if living at the death of the assured, and otherwise to his legal representatives.

The remaining chapters of the book deal with Fire Insurance Claims, Claims for Premiums, Stamp Duties, and Clauses and Conditions, the last mentioned being an especially valuable chapter, dealing as it does with the legal effect of the conditions contained in all forms of policies, and giving particulars of the clauses which have been judicially construed. It is difficult to speak too highly of the value of this chapter to insurance officials. The book concludes with an Appendix containing the text of the Assurance Companies Act, 1909, and the Statutory Rules relating thereto.

Whatever objections may be taken to the form which the book has assumed, there can be no two opinions as to its value, alike to the practising lawyer and to the insurance official, and it is no exaggeration to say that any information relating to the law of life assurance which is likely to be required will be found here. Perhaps, however, the means of finding it might be improved, for in using the index, three particular subjects which were sought for there, namely, "Custody of Documents", "Public Policy", and "Underwriters", were not found under those headings, although, of course, they are dealt with in the book and are to be found under other titles in the index.

A. R. B.

CORRESPONDENCE.

FRIENDLY SOCIETY RESERVES AND MARGINS
OF SURPLUS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—It has already become evident and the point will be particularly appreciated by those who have had the advantage of hearing the last of Mr. A. W. Watson's Lectures on Friendly Societies that the National Health Insurance Scheme will give rise to a new class of problem as to the relative reserves for sickness benefits on different bases, and as to the surplus margins which will be available.

It may therefore be of interest to point out that two existing and very general investigations, ostensibly applying only to insurance benefits, may be applied with little or no modification to the case of sickness benefits, or even to a combination of sickness, insurance and other benefits.

1. In appendix B to a paper "On the Distribution of the Divisible Surplus of a Life Assurance Company" (*J.I.A.*, xxxii, p. 105) I gave a general expression for the present value of the future profit or loss from mortality and interest.

2. In a paper on "Changes in Pure Premium Policy-Values consequent upon variations in the rate of interest or the rate of mortality, or upon the introduction of the rate of discontinuance" (*J.I.A.*, xxxix, p. 209), I discussed a series of problems which are sufficiently indicated by the title of the paper, and arrived at some very general results.

Both these investigations extended to the case where the sum assured varied from year to year, and both provided for p and q being entirely independent, these quantities being defined as follows:

p_n = probability that if the status be in existence at the beginning of the n th year, it will survive to the end of the year.

q_n = probability that if the status be in existence at the beginning of the n th year, the sum assured, S_n , will become payable at the end of the year.

Thus $(q_n S_n)$ is the expected amount of claim during the year, and it will be found that it always enters into the investigations as one quantity, which might have been represented by a single symbol, say β_n , defined as the expected claim at the end of the n th year for each status in force at the beginning of the year. If, therefore, β represents the expected cost of sickness, or the expected cost of sickness plus death benefit plus any other benefit, the whole of the above general investigations will still apply, practically without alteration, except that for "mortality profit" we must substitute "combined profit from mortality, sickness and other benefits."

It is possible that this extension of results, which were already very general, may be of some use in future investigations.

I am, Sir,

Your obedient servant,

G. J. LIDSTONE.

Mansion House Street, E.C.

February, 1912.

INTERNATIONAL CONGRESS OF ACTUARIES, 1912.

THE Seventh International Congress of Actuaries will be held in Amsterdam from the 2nd to the 7th September next. We give below the official programme of the subjects to be dealt with.

SUBJECTS FOR DISCUSSION.

1. Reassurance in Life Assurance.*

- (a) Methods of Reassurance pursued in different countries ; General principles underlying them ; Reassurance Contracts : Customs governing Reassurance.
- (b) The theoretical and practical grounds for determining the maximum sum to be retained by a Company.
- (c) The experience of Companies of the value of Reassurance arrangements (such as the financial results, the results in respect of mortality, surrender, &c.) with a comparison between these different results, and a comparison with the results of direct business.

2. The organization in Public Administration of old-age pensions.
It would be important to institute a comparison of the methods pursued by the several Public Administrations in providing for the Costs of these pensions.

3. The incontestability of Life Assurance Policies.

Would it be feasible for a Life Assurance Company to comprise in its conditions of Assurance the absolute guarantee of the payment of the sum assured without concerning itself to ascertain where, when, in what manner and under what circumstances death has occurred, so that the restriction of the Company's liability as is now the practice where suicide takes place, may be expunged ?

Would it be practicable to exact an extra premium to cover incontestability, and, if so, on what technical basis could this extra premium be properly assessed ?

4. The course, since 1800, of the mortality of assured persons, distinguishing, if practicable, the sexes, professions, types of assurance, &c.

* We understand that since the programme was issued this subject has been extended to include other branches of Assurance.—ED.

At the fourth International Congress at New York the prolongation of the average length of life among the population of different countries was discussed.

Would it not be the more appropriate plan for Life Assurance Companies to investigate this prolongation among assured lives by means of tables of experienced mortality?

It is clear that it would be desirable to classify the assurances into certain principal categories, such as whole life assurances, endowment assurances, annuities, &c.

5. The question of the loading of premiums. The calculation of Office premiums.

SUBJECTS FOR PAPERS WITHOUT DISCUSSION.

1. Tables of mortality for the assurance of infantile lives.

The tables of experience in use for Endowments (immediate or deferred annuities, deferred capital payments, &c.) rarely commence below the ages of 10 or 20.

Actuaries are generally compelled, in consequence of this limitation, to employ for such assurances tables of mortality of the entire population, which indicate too heavy a mortality, with the result that the pure premium is not sufficiently high.

The formation of tables of mortality which are free from this defect is evidently important.

2. The influence of the climate of tropical regions upon the rate of mortality.

3. The course of legislation, of the teaching of assurance and of the development of the contract of assurance since the Congress at Vienna.

4. The importance, calculation and application of independent probabilities and their relations to other statistical measures.

If the exits of the members of a community A_x of persons of the age x be attributable to several causes (death, continued ill-health, marriage, &c.) direct observation shows the probabilities q_x , i_x , h_x , so that the number A_{x+1} of the persons existing in that community at the beginning of the ensuing year of age will be represented by the product $A_x \cdot F_x$, in which $F_x = 1 - q_x - i_x - h_x$. The theory of independent probabilities (Karup) substitutes for the total $1 - q_x - i_x - h_x$ the product $(1 - q'_x)(1 - i'_x)(1 - h'_x)$ in the development of the formulæ for determining the ideal probabilities q'_x , i'_x , h'_x , which are then employed in the same manner as is done with the factors of variations which are independent of one another.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the principal provisions of the Law of Bankruptcy in England, with references to some decisions of interest to Life Insurance Companies. By N. J. CARTER, F.I.A., Assistant Actuary of the Eagle Insurance Company.

[Read before the Institute, 26 February 1912.]

IT is with considerable diffidence that I submit this paper to the Institute seeing that there are many members who are far better qualified to deal with the subject.

However, it was suggested that a paper of this character would be acceptable, and I have endeavoured to make it as complete as possible without transgressing reasonable limits of space.

In the first place I have given a general account of the law of bankruptcy, following, in the main, the general arrangement of the subject adopted by Ringwood in "The Principles of Bankruptcy", and I hope sufficiently full to make the paper useful to Students for Part IV of the examination syllabus. This has inevitably made it somewhat long and formal.

In dealing with those parts of the subject that are of more interest to Life Assurance Companies, and especially where the legal position is uncertain, *e.g.*, "after acquired property", I have referred to various decisions bearing on the points and called attention to those matters upon which it is difficult to form a definite opinion, though, no doubt, considerable light will be thrown on these questions in the course of the discussion.

Various matters of practical interest have already been dealt with by Mr. Barrand in volumes xxxiii, xli and xliv of the *Journal*: Dr. Sprague dealt with the questions of what constitutes notice of bankruptcy in volume xxxiii, and Mr. Lidstone's paper in volume xxxv goes fully into the position of a purchaser of an interest under a voluntary settlement. In order to make the paper complete, I have, to some extent, gone over the same ground again.

In the body of the paper, Sections, where mentioned, refer to the Act of 1883, unless otherwise stated.

The law of bankruptcy was created by statute in the reign of Henry VIII. Bankruptcy was formerly considered a crime, and in the early Acts the debtor was called "the offender." The law was frequently altered by statute, and gradually bankruptcy came to be regarded as a remedy for traders in embarrassed circumstances; but, prior to the Act of 1861, only traders were subject to bankruptcy law. The law is now contained in the Bankruptcy Act of 1883, amended principally by the Preferential Payments in Bankruptcy Act, 1888, and the Bankruptcy Act, 1890, and the Bankruptcy Rules, 1886 and 1890, made under the powers of the Act of 1883. As a general rule all debtors are now liable to be made bankrupt.

PERSONS WHO CAN BE MADE BANKRUPT.

A creditor cannot present a bankruptcy petition against a debtor, unless the debtor is domiciled in England, or, within a year before the date of presentation of the petition, has ordinarily resided or had a dwelling-house or place of business in England. A foreigner is included under this section; but

Foreigner. a foreigner resident abroad cannot be made bankrupt in England, though he has a place of business and has contracted debts and acquired assets in England, unless he commits an act of bankruptcy here; and he cannot be served with a bankruptcy notice abroad. An infant, as a rule, cannot

Infant. be made bankrupt, as by the Infants' Relief Act, 1874, he is not liable on contracts entered into by him in the course of trade or otherwise, except for necessaries. The effect of fraudulent representation that he is of full age would create an equitable liability which, though not a proper petitioning creditor's debt, might be proved for if the infant become bankrupt when of full age. [*Ex p. Jones, In re J.* 18 Ch. D. 109.]

A married woman cannot be made bankrupt except where

- Married woman. (1) She trades as a *feme sole* by the custom of London,
 (2) Her husband is civilly dead,
 (3) She is judicially separated from her husband,
 (4) She carries on a trade separately from her husband, in which case by Sec. 1 (5) of the Married Women's Property Act, 1882, she is, in respect of her separate property, subject to the bankruptcy laws in the same way as if she were a *feme sole*.

Lunatic. A lunatic may be made bankrupt in respect of an act of bankruptcy committed whilst sane; but, in respect of acts committed after inquisition, it seems that bankruptcy proceedings cannot be taken without leave of the Court of Lunacy.

Company. Corporations are not liable to bankruptcy law, nor any partnership or association registered under the Companies Act, 1862, nor limited partnerships formed under the Limited Partnership Act, 1907. These may be wound up under the Companies Acts.

Other partnerships are liable.

A surety may be made bankrupt even though the petitioning creditor holds security from the principal debtor. [*Re Hodges, ex. p. Matthews* 3 Man. 329.] When a debtor is made bankrupt, he gives all his property to his creditors to be divided amongst them according to the rules of bankruptcy; and, if his behaviour has been free from serious blame, he obtains a discharge from past liabilities with certain exceptions mentioned later.

Until his discharge, however, a bankrupt is disqualified for sitting or voting in Houses of Parliament, acting as a Justice of the Peace, or holding Municipal and various other offices. Such disqualification ceases on annulment of bankruptcy or discharge with a certificate from the Court that the bankruptcy was caused by misfortune without any misconduct.

By the Act of 1890, Sec. 9, no disqualification is to exceed a period of 5 years from the date of discharge.

A bankruptcy petition must be presented either by the debtor, or by some creditor or creditors whose debt or debts amount to £50, in which case it must be grounded on an act of bankruptcy.

ACTS OF BANKRUPTCY (Sec. 4).

A debtor commits an act of bankruptcy

- (a) If in England, or elsewhere, he makes a conveyance
Assignment for benefit of creditors. or assignment of his property to a trustee or trustees for the benefit of his creditors generally.

There must be an actual conveyance or assignment, and not merely a declaration of trust of the whole, or substantially the whole, of the debtor's property; but, where the property consists partly of leaseholds, it will be sufficient if debtor covenants to deal with them as the trustee may direct. If the leaseholds are assigned, the trustee is liable for rent until adjudication, if proceedings are commenced against him before adjudication even though the assignment would be void as against a trustee in bankruptcy.

The assignment must be for the benefit of all creditors and not of a class, *e.g.*, trade creditors. It is unnecessary to show an intent to defraud and delay creditors because it is an endeavour to take the property out of the operation of the bankruptcy laws.

The words "or elsewhere" apply to the case of a person subject to English law making abroad such an assignment which is to operate according to English law; but a person domiciled abroad, and not subject to English law, cannot be made bankrupt here by reason of a disposition of property executed in and to operate according to the law of a foreign country. Except where executed by a foreign debtor abroad, or by a limited company, such an assignment must be registered under the Deeds of Arrangement Act, 1887; but want of registration, though making the deed void for the purpose for which it was executed, will not prevent its being utilized as an act of bankruptcy. [*In re Hollinshead ex p. Heapy*, 1889, 37 W.R. 415.]

- (b) If in England, or elsewhere, he makes a fraudulent
Fraudulent conveyances. conveyance, gift, delivery, or transfer of his property, or of any part thereof.

Under this heading are included all dispositions of property fraudulent by common law or by the statute 13 Eliz. c. 5. Such dispositions are utterly void except in cases where made upon good consideration to persons without knowledge of grantor's fraudulent intent.

An assignment of the whole of the debtor's property to one or more creditors to the exclusion of others for past debts is fraudulent because it prevents others from issuing execution; but a

debtor may mortgage all his property by way of security for even a small present advance, also partly for past debts and partly for present or future advances, where the lender had reasonable ground for believing that the loan would enable the debtor to carry on his business.

A *bona fide* sale of the whole of a trader's property is not of itself an act of bankruptcy, the effect being merely to change the nature of the property and an innocent purchaser will be protected even where sale is intended to enable debtor to defeat or delay his creditors and therefore an act of bankruptcy.

Assignment of
after acquired
chattels.

If a bill of sale purports to assign, in the body of the instrument, after-acquired chattels by way of security, it will be void altogether as not being in accordance with the form prescribed by the Bills of Sale Act, 1882. Bills of Sale are referred to later in this paper.

Fraudulent
preference.

- (c) If in England, or elsewhere, he makes any conveyance or transfer of his property, or any part thereof, or creates any charges thereon, which would under this or any other Act be void as a fraudulent preference if he were adjudged bankrupt.

A fraudulent preference is defined by Sec. 48. Shortly expressed, anything done or suffered by a person unable to pay his debts as they become due in favour of any creditor, or person in trust for any creditor, with a view of giving him a preference over the other creditors, shall, in the event of his being adjudged bankrupt on a bankruptcy petition presented within three months, be deemed fraudulent and void as against the trustee in bankruptcy. This does not affect rights of anybody making title in good faith and for valuable consideration through or under a creditor of the bankrupt.

Onus of proof lies on the trustee who may be defeated by creditor proving that transaction was the result of *boni fide* pressure.

But there will be no fraudulent preference where a debtor makes a payment, not with the dominant view of preferring a creditor, but

- (1) Believing on reasonable grounds that he is under legal and not merely moral obligation to pay,
- (2) To save himself from exposure or criminal prosecution,
- (3) To revive undisputed debt so that it may not be barred by statute,
- (4) To protect a surety from liability,

- (5) To make reparation for past wrong, *e.g.*, breach of trust.

It should be noted that a transaction which could not be set aside under Sec. 48 may still be deemed an undue preference, and this might affect the bankrupt's discharge under Sec. 8 of 1890 Act. For instance, it is an undue preference if on eve of bankruptcy an insolvent debtor pays a creditor in full even though he would probably get a similar preference in the bankruptcy. [*In re Bryant* 1 Q.B. 429.]

- (d) If with intent to defeat or delay his creditors, he does any of the following things, namely, departs out of England, or being out of England, remains out of England; or departs from his dwelling-house, or otherwise absents himself; or begins to keep house.

Evasion
of creditors.

As to these acts of bankruptcy an intent to defeat or delay creditors must be shown, and this is often a matter of inference.

- (e) If execution against him has been levied by seizure of his goods under process in an action in any Court, or in any civil proceeding in the High Court, and the goods have been either sold or held by the sheriff for 21 days.

Execution
orders.

This sub-section is modified by Sec. 1 of 1890 Act, which enacts that where an interpleader summons has been taken out with regard to the goods seized, the time elapsing between the date at which such summons is taken out and the date at which the sheriff is ordered to withdraw, or any interpleader issue ordered thereon is finally disposed of, shall not be taken into account in calculating the 21 days.

By Sec. 45 a creditor is not allowed to retain the benefit of an execution, except both seizure and sale take place before date of receiving order and before notice of presentation of bankruptcy petition or of the commission of any available act of bankruptcy.

By Sec. 11 of 1890 Act the rights of an execution creditor may in all cases be defeated if a receiving order is made against the debtor before sale of goods or satisfaction of the execution, and where the judgment is for a sum greater than £20, within 14 days of such sale or satisfaction for which period the sheriff must hold the proceeds.

Persons who purchase in good faith under a sale by the sheriff get a good title against the trustee in bankruptcy. See Sec. 46 (3).

Petition
by debtor. (f) If he files in the Court a declaration of his inability to pay his debts, or presents a bankruptcy petition against himself.

Judgment
debt. (g) If a creditor has obtained a final judgment against him for any amount, and, execution thereon not having been stayed, has served on him in England, or, by leave of the Court, elsewhere, a bankruptcy notice under this Act, requiring him to pay the judgment debt in accordance with the terms of the judgment, or to secure or compound for it to the satisfaction of the creditor or the Court, and he does not, within 7 days of the notice, in case the service is effected in England, and in case the service is effected elsewhere, then within the time limited in that behalf by the order giving leave to effect the service, either comply with the requirements of the notice, or satisfy the Court that he has a counterclaim, set-off, or cross demand which equals or exceeds the amount of the judgment debt, and which he could not set up in the action in which the judgment was obtained.

Any creditor may present a petition grounded on this act of bankruptcy within, of course, 3 months from date of expiration of the 7 days; that is to say, not necessarily the creditor who obtained final judgment.

Suspension
of payment of
debts. (h) If the debtor gives notice to any of his creditors that he has suspended, or that he is about to suspend, payment of his debts.

The notice need not be in writing, but it must be given formally and deliberately, and a mere casual conversation will not suffice.

A notice given "without prejudice" is admissible as proof of an act of bankruptcy.

The test as to whether a notice is an act of bankruptcy seems to be, what effect would the notice produce on the mind of the creditor receiving it as to the intention of the debtor with regard to his creditors? It would not be sufficient that, under the circumstances, suspension is very probable.

A further act of bankruptcy is set out in Sec. 103 (5).

Bankruptcy in
lieu of committal
order. "Where, under Sec. 5 of the Debtors' Act 1869, "application is made by a judgment creditor to a "Court having bankruptcy jurisdiction, for the

“committal of a judgment debtor, the Court may, if it thinks
 “fit, decline to commit, and in lieu thereof, with the consent of
 “the judgment creditor, and on payment by him of the pre-
 “scribed fee, make a receiving order against the debtor. In
 “such case the judgment debtor shall be deemed to have
 “committed an act of bankruptcy at the time the order is made.
 “And by Sec. 20 of 1890 Act the bankruptcy under this section
 “will be deemed to commence at time of the order or at the time
 “of the first of any previous acts of bankruptcy proved to have
 “been committed by him within 3 months before the order.”

CONDITIONS ON WHICH CREDITORS MAY PETITION.

- (1) The petitioning creditor's debt, or if two or more join in the petition, the aggregate amount of debts, must amount to £50.
- (2) The debt is a liquidated sum, payable either at once or at some certain future time.
- (3) The act of bankruptcy on which petition is grounded has occurred within 3 months before presentation of petition.
- (4) Debtor is domiciled in England, or within a year before presentation has ordinarily resided, or had dwelling-house or place of business in England.

**Secured
creditor.**

If the petitioning creditor is a secured creditor, he must either state he is willing to give up his security for the benefit of the creditors in the event of the debtor being adjudged bankrupt, or give an estimate of the value of his security. In the latter case he would be admitted as a petitioning creditor for the balance of his debt as if he were an unsecured creditor.

If he makes an estimate which is a real one, but on adjudication he does not prove, the trustee cannot claim to redeem at the estimated value. If he does prove, he will be bound by the estimate, at all events in the absence of a mistake.

If he states in petition he is willing to give up his security, he cannot prove in the event of adjudication without giving it up.

A surety cannot petition against a co-surety for an amount claimed as contribution until he has himself paid more than his share, provided that the co-surety has not been released by the principal creditor. [Wolmershausen *re* Gullich, 1893, 2 Ch. 514.]

A creditor who has notice of an act of bankruptcy committed

by his debtor is justified in refusing to accept payment of his debt and in proceeding to petition.

A creditor's petition must be verified by affidavit of the creditor, or some person on his behalf having knowledge of the facts, and served in the prescribed manner.

The Court requires proof of the debt, of service of the petition, and of the act of bankruptcy or of one act if more than one, and if satisfied will make a receiving order.

If the proofs are unsatisfactory, or debtor is able to satisfy the Court that he is able to pay his debts, or if for other sufficient cause no order should be made, the Court may dismiss the petition.

Examples of sufficient cause.

Dismissal of
Petition. Debtor's sole asset (with which he had proposed to pay a composition to his creditors) would be destroyed by his bankruptcy. Debtor is an undischarged bankrupt and no chance of anything being available for creditors in a second bankruptcy. Debtor had offered before petition, a composition to his creditors and petitioning had tried, as a condition of assent, to extort special terms for himself.

It is not sufficient cause that there will be probably nothing to distribute.

In the case of a debtor's petition it is sufficient if it allege that debtor is unable to pay his debts.

No petition may be withdrawn except by leave of the Court.

RECEIVING ORDER.

Effect of
receiving order.
(Sec. 9.) On the making of a receiving order an official receiver shall be thereby constituted receiver of the debtor's property and thereafter, except as directed by the Act, no creditor to whom the debtor is indebted in respect of any debt provable in bankruptcy shall have any remedy against the property or person of the debtor in respect of the debt or shall commence any legal proceedings unless with the leave of the Court and on such terms as the Court may impose.

This section does not affect the power of any secured creditor to realize or otherwise deal with his security.

A receiving order does not divest a debtor of his property, but merely protects it. At any time after presentation of a petition and before a receiving order is made, the Court may appoint an Official Receiver to be interim receiver of the whole

or part of the debtor's property if necessary for the protection of the estate.

Notice of every receiving order stating name, address and description of the debtor, date of the order, the Court by which it is made, and the date of the petition is to be gazetted and advertised in a local paper.

**Annulment of
receiving order.**
(Sec. 14.)

A receiving order may be annulled if it appear that majority of creditors in number and value reside in Scotland or Ireland, and that from situation of debtor's property, or other reason, his estate ought to be distributed according to the laws of those countries. A receiving order will also be annulled where the Court sanctions a composition or scheme under Sec. 3 of 1890 Act; or if it ought not to have been made; or if debtor or his friends pay the creditors in full.

**Duties
of debtor.**
(Sec. 16.)

On the making of a receiving order, the debtor must submit to the Official Receiver a statement of his affairs, verified by affidavit, and showing particulars of his assets, debts and liabilities, names, addresses and occupations of his creditors, securities held by them respectively, dates when securities were given, and such further information as may be prescribed or as Official Receiver may require.

If the order is made on debtor's petition this statement must be submitted within 3 days of the date of the order, if made on a creditor's petition within 7 days; but the Court may for special reasons extend the time. If the debtor, without reasonable excuse, fails to submit a proper statement, the Court may, on the application of Official Receiver or any creditor, adjudge him bankrupt.

**Public
examination.**

Whether proceedings result in a composition, or in bankruptcy, debtor must submit himself for public examination. The Court may dispense with the public examination in cases of illness or lunacy; and, for the purpose of approving a composition by joint debtors, with the examination of one of such joint debtors in case of his illness or absence abroad.

The examination is held as soon as convenient after expiration of time for the submission of the statement of affairs, it may be adjourned from time to time, and when the Court considers that the debtor's affairs have been sufficiently investigated, it declares the examination concluded, but not until after the day appointed for the first meeting of creditors.

The Official Receiver must take part in the examination ; he may, if authorized by the Board of Trade, employ a solicitor with or without counsel ; any creditor who has tendered a proof, or his representative authorized in writing, may question the debtor, and the debtor, who is sworn, must answer all such questions as the Court may put or allow.

Meeting of creditors. The first meeting of creditors is to be summoned for a day not later than 14 days after the date of the receiving order, unless the Court for any special reason appoint a later day, and the debtor must attend unless prevented by sufficient cause.

The Official Receiver, who, or whose nominee, is chairman at the first meeting, summons it by giving not less than 7 days' notice in the *Gazette* and a local paper. He is also to send as soon as practicable to each creditor mentioned in the statement of affairs a summary of the statement including the causes of debtor's failure and any observations thereon he may think fit to make, also notice of time and place of the first meeting.

COMPOSITION OR SCHEME OF ARRANGEMENT.

If a debtor, after the receiving order, desires to offer a composition or scheme of arrangement he must follow the procedure laid down in Sec. 3. B.A. 1890.

The Official Receiver must hold a meeting of creditors before public examination is concluded, having beforehand supplied each creditor with a copy of debtor's proposal with a report thereon. At meeting any creditor who has proved may vote in person or by letter, and if majority in number and three-fourths in value of such creditors approve the proposal, it will be binding on all the creditors if and when approved by the Court.

Before approving, the Court will consider the Official Receiver's report, any objections made by or on behalf of any creditor, and, generally, the interests of the creditors and the conduct of the debtor.

The Court will refuse to approve in any case where, if debtor were adjudged bankrupt, the Court would be required to refuse discharge.

The Court will not refuse approval for slight acts of misconduct on the part of debtor if 7s. 6d. in the £ is secured. [*In re E. A. B.* 1902, 1 K.B. 457.]

An approved scheme is binding on all creditors so far as relates to any debts due from debtor and provable in bankruptcy ;

but it does not release debtor from liability under a judgment against him for seduction, or as a co-respondent in a matrimonial case, except as expressly ordered by the Court.

If default is made in payments under the scheme, or the Court is satisfied that for any sufficient reason the scheme cannot proceed without injustice or undue delay to creditors or debtor, or that approval was obtained by fraud (*e.g.*, assets grossly exaggerated), the Court may on application by the Official Receiver, or the trustee, or any creditor, annul the scheme and adjudge the debtor bankrupt, but without prejudice to anything done under the scheme. The annulment of a scheme discharges a surety for it.

No scheme will be approved which does not provide for priority in payment of all debts directed to be so paid in the distribution of a bankrupt's property.

The acceptance of a scheme by the creditors does not release any person who would not be released by the discharge of the debtor if he were adjudged bankrupt, *i.e.* any person who was a partner or co-trustee with the debtor, or who was jointly bound or had made any joint contract with him, or any person who was a surety, or in the nature of a surety for him.

A composition or scheme is not binding on any creditor so far as regards a liability from which the debtor, if adjudged bankrupt, would not be discharged by an order of discharge, unless such creditor consents to the composition or scheme. [*Ex p. Rogers re Rogers* 1884, 3 Q.B.D. 438.]

Where a secured creditor values his security and proves for the balance and the trustee does not redeem and the bankruptcy is annulled on the approval of a composition, the debtor can redeem the security upon payment of the assessed value and premiums (if any) with interest from the date of proof [*In re Pearce* 1909, 2 Ch. 492.]

If the creditors, after the adjudication entertain a proposal for composition or for a scheme of arrangement, and the Court approves, it may annul the bankruptcy and vest the property in the bankrupt or such other person as Court may appoint.

ADJUDICATION.

If the creditors at their first meeting or any adjournment thereof resolve that the debtor be adjudged bankrupt, or if they pass no resolution, or if they do not meet, or if

a composition or scheme is not accepted or approved within 14 days of the conclusion of the debtor's examination or such further time as the Court may allow, the Court shall adjudge the debtor bankrupt and thereupon his property becomes divisible

among his creditors and vests in a trustee who is
Trustee. appointed by the creditors. The person so appointed shall give security to the Board of Trade who, if satisfied with the security and the fitness of the person appointed, shall certify the appointment.

By Sec. 4 of 1890 Act a person is not deemed to be fit to act as trustee if he has previously been removed from the office of trustee in bankruptcy for misconduct or neglect of duty.

In default of appointment of a trustee by the creditors the Board of Trade may do so. But the creditors (or committee of inspection if authorised) may at any subsequent time appoint a person to be trustee in the place of the person appointed by the Board of Trade.

The creditors may at the first or a subsequent meeting appoint a committee of inspection of not more
Committee of inspection. than 5 or less than 3 persons, to superintend the administration of the bankrupt's property by the trustee. No creditor shall be qualified to act until his proof has been admitted.

Notice of an order adjudging a debtor bankrupt,
Notice. stating his name, address and description, and date of adjudication and the Court by which adjudication is made must be gazetted and advertised in a local paper.

Though this paper only deals with the law of bankruptcy in England, it may be useful to call attention to the differences between English, Scottish, and Irish law, as regards what constitutes notice of bankruptcy.

In England an assignee in bankruptcy must give notice like any other assignee; but if the office is aware of the bankruptcy, it is doubtful whether payment can be safely made to the assured, although the office has had no formal notice from the assignee in bankruptcy. The Canadian Court decided (*Crawford v. Canada Life Assurance Co.*) that the office must pay the surrender-value of a policy to the trustee in bankruptcy although it had been previously paid to the assured. Here the trustee had written to the office notifying them of the bankruptcy and claiming a certain policy on the bankrupt's life, but not the one subsequently surrendered.

In Scotland, the confirmation of the trustee in a sequestration operates as an intimated assignation of all debts due to the bankrupt, but there is a clause in the Scottish Bankruptcy Act which protects a debtor who makes a *bonâ fide* payment in ignorance of sequestration. The effect of that provision seems to be that a Company paying a claim would be protected, though not in granting a loan or paying a surrender-value.

As far as granting a loan is concerned the case of *Scottish Union and National v. Fairley* confirms this. Lord Low said *inter alia* "Insurance Company cannot say that they were not aware Fairley was bankrupt and that they advanced the money in good faith . . . sequestration is a public matter, ignorance of which cannot be pleaded."

In Ireland, notice of bankruptcy in the *Gazette* is notice to the office and a second payment to the trustee may be demanded even though payment was made to the assured so soon after the notice in the *Gazette* that it was impossible for the office to see the notice before making the payment.

Before making any payment under a policy to a person domiciled in Scotland or Ireland a search for bankruptcy should be made.

The trustee in bankruptcy is a statutory assignee and not an encumbrancer for value and he takes the estate subject to all equities. He cannot by giving notice get precedence of an equitable mortgagee who has omitted to do so. [*In re Wallis ex p. Jenks* 1902, 1 K.B. 719.]

Title of trustee. In order to establish his title, the trustee must produce the adjudication order and the Board of Trade's certificate of appointment of trustee.

In small bankruptcies dealt with under the procedure known as Summary Administration, the Official Receiver acts as trustee, and he should produce adjudication order, order for Summary Administration, and write a letter stating that no one other than himself has been appointed as the creditors can, by special resolution, appoint a trustee.

Office copies of these documents should be retained.

Relation back of trustee's title. Sec. 43. Whether the bankruptcy of a debtor takes place on debtor's petition or on that of creditor or creditors, the title of the trustee in bankruptcy relates back to the date of the first act of bankruptcy proved to have been committed by the bankrupt within 3 months prior to the presentation of the petition and where a receiving order is made

under Sec. 103 the trustee's title relates back to the date of the order unless the debtor is proved to have committed any previous act of bankruptcy, in which case the title relates back to the first act of bankruptcy committed within 3 months prior to order.

Exceptions.

- (1) Where a creditor has completed an execution or attachment before the receiving order, and before notice of the presentation of a bankruptcy petition, or of the commission of any available act of bankruptcy by the debtor, the trustee's title will not overrule that of the creditor; provided that, in a case where the execution is against the goods of a debtor in respect of a judgment for a sum exceeding £20, the sheriff receives no notice of a petition on which the debtor is adjudged bankrupt within the 14 days during which he is obliged to retain in his hands the balance of the proceeds of sale or money paid to avoid sale, after deducting costs of execution.
- (2) Subject to the provisions of the Act as to execution or attachments and the avoidance of certain settlements and preferences, the following transactions are protected though they take place after an available act of bankruptcy, provided that they take place before the date of the receiving order, and that the person dealing with the debtor had, at the time of doing so, no notice of any available act of bankruptcy committed by the debtor.
 - (a) Any payment by the bankrupt to any of his creditors.
 - (b) Any payment or delivery to the bankrupt.
 - (c) Any conveyance or assignment by the bankrupt for valuable consideration.
 - (d) Any contract, dealing, or transaction by or with the bankrupt for valuable consideration.

Protected
transactions.
Sec. 49.

The onus of proving the want of notice lies on the person seeking the protection of the section. The transaction must be completed before the date of the receiving order; a payment made afterwards in fulfilment of a contract entered into before the date of the order would not be protected.

A secured creditor who has notice of an available act of bankruptcy committed by the debtor, cannot safely hand back the security to him against payment. In an action for redemption

the Court will see that the security is kept available for a possible trustee till 3 months from the act of bankruptcy have elapsed. [*Ponsford v. Union of London and Smith's Bank* (1906) 2 Ch. 444.]

Payment made to trustee under a deed of assignment for the benefit of creditors within 3 months of date of deed is not protected against the trustee in bankruptcy. [*Davis v. Petrie*, 1905, 2 K.B. 528].

A case coming under Sec. 47 which deals with the avoidance of voluntary and other settlements, is not protected by Sec. 49. *In re Reis ex p. Clough* (1904) 1 K.B. 451 not reversed on this point by Court of Appeal (1904) 2 K.B. 769.

PROPERTY NOT DIVISIBLE AMONGST THE CREDITORS (SEC. 44).

- (1) The tools (if any) of the debtor's trade and the necessary wearing apparel and bedding of himself, wife and children, to a value, inclusive of tools, apparel and bedding, not exceeding £20.
- (2) Property held by the bankrupt on trust for any other person. This includes
 - (a) Express trusts—where the trust is the origin of the legal ownership of the bankrupt.
 - (b) Trusts *virtute officii*, e.g., executors and administrators.
 - (c) Trusts created by the bankrupt, who, retaining the legal ownership, has divested himself of the whole or part of the beneficial interest.
 - (d) When a man holds property as a mere agent or factor, he will be considered to hold it as a trustee and should he become bankrupt, his principal can claim it, so long as it is distinguishable from the mass of the bankrupt's property. But it must be capable of being traced and must not have been left with the bankrupt under such circumstances as would render him the reputed owner.

In equity an order given by A to B upon C having funds of A to pay B out of such funds is a binding equitable assignment of so much of the funds, and it is unnecessary for B to give notice to C in order to perfect his assignment, although this should be done in order that another assignee may not get

priority by notice, and in order to prevent the operation of the reputed ownership clause in the event of A becoming bankrupt, and the amount due to him by C being a debt due to him in the course of his trade or business.

But a promise to pay when the debtor receives a debt due to him from a third person is not an equitable assignment, for it does not charge the debt in the hands of such third person.

PROPERTY DIVISIBLE AMONGST CREDITORS (SEC. 44).

- (1) All property belonging to or vested in the bankrupt at the commencement of bankruptcy and which may be acquired by or devolve on him before his discharge.

“Property” as defined by the Act, includes money, goods, things in action, land and every description of property, whether real or personal, and whether situate in England or elsewhere: also obligations, easements, and every description of estate, interest and profit, present or future, vested or contingent, arising out of or incident to property as above defined.

Property belonging to a bankrupt at the commencement of bankruptcy vests absolutely in the trustee, and the bankrupt cannot deal with it; but, until the trustee intervenes, all transactions entered into by a bankrupt after his bankruptcy with any person dealing with him *bonâ fide*, and for value, with respect to his after-acquired property, whether with or without knowledge of the bankruptcy, are valid against the trustee. This doctrine was laid down in the case of *Cohen v. Mitchell* 25 Q.B.D. 262-7; but according to Kay, L.J., in *Re New Land Development Association v. Gray* (1892) 2 Ch. 138, it is meant for the protection of persons dealing with bankrupt and his personal estate in the ordinary course of business where bankrupt is carrying on business without intervention by the trustee. Also the case of *In re Rogers ex p. Woodthorpe*, 8 Mor 236, seems further to limit the application of *Cohen v. Mitchell*.

If the property consists of a chose in action which has been assigned for value to a person who has not given notice of the assignment, the trustee, on intervening, may, by giving notice, acquire priority over such assignee [*Mercer v. Tans Colina*, 1900, 1 Q.B. 130].

In *New Land Development Association v. Gray* (mentioned above) it was held that the doctrine does not apply to land, and a bankrupt, even before the intervention of the trustee, cannot convey to a *bonâ fide* purchaser for value after-acquired real estate, whether legal or equitable. [*O. R. v. Cooke*, 1906, 2 Ch. 661.]

But if a person advances to an undischarged bankrupt, without notice of bankruptcy, money to enable the bankrupt to purchase freehold or leasehold property, and there is a contemporaneous agreement that the lender shall have a charge on the property for the purchase of which his money has been used, the trustee takes the property subject to the charge [*Bird v. Philpott*, 1900, 1 Ch. 822]. If the lender had notice at the time of the loan he has no equity against the trustee, nor can he or the bankrupt claim to be indemnified against liabilities or losses incurred. [*Re Clark ex p. Kearley*, 6 Mor. 42.]

The doctrine of *Cohen v. Mitchell* does apply to leaseholds [*Re Clayton and Barclay's contract*, 1895, 2 Ch. 212], and the rule holds even though a *bonâ fide* assignee may not actually have received property assigned to him by a bankrupt before the trustee's intervention and the fact that he knew the trustee was unaware of its existence does not necessarily show *mala fides*. [*Hunt v. Fripp*, 1898, 1 Ch. 675.]

Property acquired by a bankrupt after his discharge may in certain circumstances belong to his creditors, *e.g.*, if an order is made granting discharge conditional on payment of a composition and property devolves on the bankrupt more than sufficient to pay the composition, the surplus belongs to the creditors in so far as may be required to pay them in full with interests and costs. [*Re Hawkins ex p.* O. R. 1892, 1 Q.B. 890].

In re Bennett, 1907, 1 K.B. 149, it was held that the doctrine of *Cohen v. Mitchell* only applies where transaction was for value, and that a person who receives after-acquired property from a bankrupt or his representative otherwise than for value cannot retain it against the trustee in bankruptcy. In this case an administrator, who had distributed amongst the next of kin, of which he was one, the monies under a policy effected by an undischarged bankrupt, had to pay over to the Official Receiver his own share and those he held for 2 infants, but was protected in respect of shares he had paid away in good faith. The other next of kin had, however, to refund his share.

Policy issued
on life of
undischarged
bankrupt.

It has been suggested that, relying on the principle laid down in *Cohen v. Mitchell*, an office can deal with the assured in respect of a policy issued on the life of an undischarged bankrupt and ignore the trustee in bankruptcy until he intervenes. In this connection it may be interesting to give particulars of a case submitted to Counsel in 1894.

In 1879 "A" effected a policy with a Company, and in 1881 mortgaged it to "B." In 1886 the Company advanced money to "A," "B" joining in the deed and postponing his mortgage. In 1893 "A" applied for a further loan on his policy, and in the course of investigations as to title in consequence of "B's" bankruptcy, it transpired that "A" had been adjudged bankrupt in 1871, that the trustee had been in receipt of part of "A's" pension from the War Office, out of which he had already paid the creditors 16s. in the £, and that "A" had not received his discharge. The trustee had not intervened with respect to the policy and was no doubt ignorant of its existence. Counsel was asked (*inter alia*) "whether the Company, knowing that 'A' is an undischarged bankrupt, can safely make the further loan [*Cohen v. Mitchell*]."

Counsel's opinion—

"I cannot advise that the Company will be safe in making the proposed loan. The case of *Cohen v. Mitchell*, though no doubt the wording of the judgment is very wide, related to a case where a bankrupt carried on business after his bankruptcy, practically with the sanction of the trustee, and the protected transactions related to goods acquired in his business. In such cases it has long been held that a bankrupt can deal as absolute owner until the trustee intervenes. The present case seems to me to be more like that of *New Land Development Association v. Gray*, 1892, 2 Ch. 138, and in my opinion it would be unsafe for the Company to advance money to 'A' on a policy as this is in no sense a dealing in the ordinary course of business, and complications might ensue should the trustee intervene on 'A's' death and require payment of the amount of the policy.

"The principle on which such cases as *Cohen v. Mitchell* rest is that the bankrupt is the agent of the trustee to carry on his business and to do whatever is usual or necessary in that position."

I should mention that "A" claimed to have maintained the policy out of the portion of his pension not appropriated towards

discharge of his liabilities, but the Company was advised that "the residue of his pension is for his personal maintenance, and "the creditors can, we think, claim any investment of accumulation."

In Halsbury's "Laws of England" it is stated that the limitation of the doctrine of "*Cohen v. Mitchell*" to the case of persons dealing with the bankrupt in the course of business by Cave, J., in *Re Rogers ex p. Woodthorpe* 1891, 8 Mor. 236. is inconsistent with the decision in *Re Bennett ex p. O.R.*, 1907, 1 K.B. 149. This supports the suggestion referred to above.

"*In re Hall ex p. O. R.*, 1907, 1 K.B. 875."

Policy effected on
debtor's life
prior to
bankruptcy.

Here the mortgagees, after notice of an available act of bankruptcy, advanced a further sum to the mortgagor, by paying the bulk of his creditors a composition on their debts, but they were not entitled in the mortgagor's bankruptcy to treat such sum as added to their security.

In a somewhat similar case "*In re Tyler ex. p. O. R.*, 1907, 1 K.B. 865", where at bankrupt's request and with the trustee's knowledge the wife paid premiums on policies and interest on overdraft from bankers to whom policies had been assigned, it was held by the Court of Appeal that, on death of bankrupt, the trustee must, out of balance of policy monies received by him from the bankers, recoup the widow for the payments she had made in respect of premiums and interest. It should be noted that in the case of *In re Hall* mentioned above, the lower Courts followed the principle laid down in *In re Tyler*, "that trustee in bankruptcy finding that he has in "his hands money which in "equity belongs to someone else ought to set an example to the "world by paying it to the person really entitled to it", but the decisions were reversed by the Court of Appeal.

These two cases were considered in *Tapster v. Ward*, 1909, 101 L.T. 25. Here bankrupt, who had paid one premium on a policy before his bankruptcy but did not disclose policy to trustee, continued to pay the premiums after his discharge. The Court decided that the policy monies belonged to creditors and laid down the principle that a debtor who makes payments in respect of property vested in trustee in bankruptcy, and belonging to creditors, cannot claim proceeds.

It frequently happens that a bankrupt has a policy on his life of so short a duration as to bear no surrender-value, and the trustee hands it back to him, but without assigning it. If the

bankrupt continues to pay the premiums and afterwards wishes to deal with it, but has not received his discharge, the question of title arises. In cases where it is impossible to get an assignment from the trustee it has been suggested that the policy might be treated as after-acquired property since its value may be said to result from premiums paid since bankruptcy. But this cannot be considered satisfactory, especially in view of *Tapster v. Ward*, for though in that case the policy had been concealed from the trustee, and it was not a question of dealing with a current policy, but paying a claim, yet Eve, J., in delivering judgment, appeared to set aside the question of the concealment, and decided on the assumption that the assured had acted in good faith.

There are certain words of Lord Watson in delivering judgment in the case of *Whyte v. Northern Heritable Securities Investment Company*, 1891, 18 R. (H.L.) 37, which give some support to the proposition that the assured has a title to the policy by abandonment. He said *inter alia* "The acts of the trustee and creditors " in relation to it may be such as to indicate that the bankrupt " is, according to their desire, to be deemed to be in future the " master or owner of the property, and that they have abandoned " or rejected it " ; but as Mr. Barrand points out in his paper (*J.L.A.*, xli, 171), it is stated on page 124 of Pollock and Wright's *Possession in the Common Law*, that " It is even doubted whether " it is possible for a possessor to divest himself of his possession " of a thing by wilful abandonment."

An undischarged bankrupt is entitled to retain money earned by his own labour or skill in order to support himself and family, but if he earn more than sufficient, the trustee will be entitled to claim surplus for benefit of creditors.

And where bankrupt is in receipt of a salary or income, half-pay pension, &c., the Court may, on application of the trustee, direct that whole or part be paid to the trustee to be applied as Court may direct [Sec. 53 (2)].

Where bankrupt is a beneficed clergyman the trustee may apply for a sequestration of the profits of the benefice. The bishop of the diocese has right to appoint to bankrupt, out of the profits, such sum as a curate would have received so long as the bankrupt performs his duties, and the profits are further liable for any salary due to any licensed curate for 4 months preceding date of receiving order not exceeding £50.

Bankrupt
clergymen.

The principle that the trustee cannot sue for personal earnings is confined to the proceeds of the bankrupt's personal and daily labour. It does not apply to the earnings of a bankrupt in a business, even though he would have to use his personal skill in the business [*Re Rogers ex p. Collins*, 1894, 1 Q.B. 425].

Where a bankrupt, though he had no clerk or offices, habitually earned commission by introducing purchasers of houses to vendors, such commission was not personal earnings within the exception, and so passed to the trustee. [*Mercer v. Fane Colina*, 4 Man 363.]

It would seem, therefore, that the trustee in bankruptcy could claim commission earned by an insurance agent after bankruptcy and before discharge. But he would be apparently entitled to such commission until the trustee intervenes. Renewal commission would seem to be in a different position from commission on new business introduced after the bankruptcy.

Personal earnings may lose that character where they are dealt with by the bankrupt as property [*In re Rogers ex p. Collins*, 1894, 1 Q.B. 425], and in the same case where "A" mortgaged the profits of a dental-surgeon business, and then became bankrupt, it was held that trustee and not mortgagee was entitled to the after-acquired profits.

I have recently heard of a case where the respective rights of the various parties concerned are very difficult to adjust. "A" gave his bankers a lien on his commission account to secure an advance, and on his subsequent bankruptcy it transpired that he was at the time of borrowing an undischarged bankrupt. The office concerned, acting on their Solicitor's advice, are, for the present, retaining the commission.

I believe that some offices avoid such difficulties by making all commissions cease on bankruptcy.

Income given to a man for life "until he should become bankrupt, or do or suffer something whereby the said income, "if belonging absolutely to him, or some part thereof, would become payable to or vested in some other person". is determined by the making of a receiving order [*In re Sartoris's Estate*, 1892, 1 Ch. 11], and when bankruptcy happens trustee takes income down to the act of bankruptcy. [*Montefiore v. Guedalla*, 1901, 1 Ch. 435.]

A man cannot make a settlement of his own property on himself defeasible in the event of his bankruptcy [*Merry v.*

Commission
earned by
bankrupt.

Income
ceasing
on bankruptcy.

Pownall, 1898, 1 Ch. 306]. but though such settlement would be set aside so far as necessary for payment of debts and costs, the debtor's life interest would be forfeited, and in event of a second bankruptcy trustee would have no title [*In re Johnson ex p. Matthews*, 1904, 1 K.B. 134]; and there is nothing against bankruptcy law in a company's articles which provide that a shareholder, on becoming bankrupt, must sell his shares to particular persons, at a particular price, fixed for all persons alike, and not shown to be less than the fair price which might otherwise be obtained. [*Borland's Trustees v. Steel Bros., Limited*, 1901, 1 Ch. 279.]

Contingent interests. The contingent interests of a bankrupt pass to the trustee, but a mere possibility of an interest will not pass. [*Carleton v. Leighton*, 3 Mer. 667.]

Where a fund is limited to one for a term with remainder to such persons as a certain person may appoint, and in default of appointment to certain named persons one of whom becomes bankrupt before either appointment or default in appointment, his interest, if it accrue afterwards by default in appointment, passes to trustee; but if it accrue by virtue of appointment it does not, because although the appointment for many purposes takes effect as if written in the original instrument, there is no relation back as to time of title accruing. [*Re Vizards Trusts*, L.R. 1 Ch. 588.]

Estate by the curtesy. Again where a bankrupt, after discharge, became entitled to an estate by the curtesy, but at date of discharge his wife was tenant in fee in remainder expectant upon a life interest it was held that until the estate subject to curtesy became vested in wife in possession, the husband had no inchoate or transmissible interest which could pass to the assignees. [*Gibbons v. Eyden*, L.R. 7 Eq. 371.]

Where a policy is taken out under the Married Women's Property Act, 1882, for the benefit of wife should she survive the assured, the latter has a contingent interest which passes to the trustee in the event of his bankruptcy; but an advance to pay premiums can be made to the assured as statutory trustee without the concurrence of the wife or the trustee in bankruptcy (*see* Mr. Barrand's paper, *J.I.A.*, xli, 189).

- (2) The capacity to exercise and to take proceedings for exercising all such powers in or over or in respect of property as might have been exercised by the

bankrupt for his own benefit at the commencement of his bankruptcy, or before his discharge, except the right of nomination to a vacant ecclesiastical benefice.

This will not include property subject to a general power of appointment by will, which the bankrupt, before his bankruptcy, exercises, and afterwards dies undischarged. Such property passes to his executor, who may deal with it for the benefit of subsequent creditors. [*In re Guedalla*, 1905, 2 Ch. 331.]

- (3) All goods being, at the commencement of the bankruptcy, in the possession, order, or disposition of the bankrupt, in his trade or business, by the consent and permission of the true owner, under such circumstances that he is the reputed owner thereof; provided that things in action other than debts due or growing due to the bankrupt in the course of his trade or business, shall not be deemed goods within the meaning of this section.

A Policy of insurance which, prior to Bankruptcy Act of 1869, was held to be subject to the doctrine of reputed ownership is now excluded from the operation of the section being "a chose in action other than debt due in course of trade." [*Ex p. Ibbetson*, 8 Ch. D. 519.]

BILLS OF SALE.

The principal Acts are those of 1878 and 1882.

The Act of 1878 was designed to prevent the rights of creditors from being affected by secret assurances of chattels which were permitted to remain in the ostensible possession of a person who had parted with his property in them, and by Sections 10 (1) and 20, which now only apply to absolute bills of sale, unless a bill of sale is duly attested by a solicitor and registered within 7 days, the chattels comprised therein will be deemed to be in the possession, order or disposition of the grantor within the meaning of the Bankruptcy Act.

The object of the Act of 1882, which only applies to bills of sale given by way of security for the payment of money by the grantor thereof, was to prevent needy persons from being entrapped into signing complicated documents which they might not understand. And so such a bill of sale is void, even as between the parties to it, unless drawn up in the prescribed form. Though such a bill of sale must be attested and registered within 7 days, such registration will not prevent any mortgaged goods

remaining in the mortgagor's possession, order or disposition, in his trade or business, from vesting in the trustee should he become bankrupt. But a bill of sale, even by way of security, duly registered before 1 November 1882, is not liable to be defeated by the possession, order and disposition section of the Bankruptcy Act, so long as the registration is not avoided by non-renewal or otherwise.

AVOIDANCE OF VOLUNTARY AND OTHER SETTLEMENTS (Sec. 47).

- (1) Any settlement of property not being a settlement made before and in consideration of marriage, or made in favour of a purchaser or incumbrancer in good faith, and for valuable consideration, or a settlement made on or for the wife or children of the settlor of property which has accrued to the settlor after marriage in right of his wife, shall, if the settlor becomes bankrupt within two years after the date of the settlement, be void against the trustee in the bankruptcy, and shall, if the settlor becomes bankrupt at any subsequent time within ten years after the date of the settlement, be void against the trustee in the bankruptcy, unless the parties claiming under the settlement can prove that the settlor was at the time of making the settlement able to pay all his debts without the aid of the property comprised in the settlement, and that the interest of the settlor in such property had passed to the trustee of such settlement on the execution thereof.
 - (2) Any covenant or contract made in consideration of marriage, for the future settlement on or for the settlor's wife or children of any money or property wherein he had not at the date of his marriage any estate or interest, whether vested or contingent in possession or remainder, and not being money or property of or in right of his wife, shall, on his becoming bankrupt before the property or money has been actually transferred or paid pursuant to the contract or covenant, be void against the trustee in the bankruptcy.
 - (3) "Settlement" shall for the purposes of this section include any conveyance or transfer of property.
- "Void" in this section does not mean void *ab initio*, but

“voidable”, and the section is aimed at donees under settlements, and not at *bonâ fide* purchasers or mortgagees from them.

In re Carter and Kenderdine's Contract, 1897, 1 Ch. 776, was taken to the Court of Appeal which unanimously decided that the trustees under a voluntary settlement could give a good title to a purchaser. The language of Lord Justice Rigby in his judgment was such that there can be no doubt that the assignee of an equitable interest would also be protected. The following is an extract from Lord Justice Rigby's judgment :

“I do not think it is possible that the Legislature . . .
 “should have left a purchaser taking a title under a perfectly
 “honest voluntary settlement entirely without protection, in
 “the event of the bankruptcy within 2 years or 10 years, as
 “the case may be, of the settlor. . . . I have not the
 “slightest doubt in my own mind that ‘*In re Holden*’ was
 “correctly decided; and it comes to this, that every equity
 “which has been acquired before the act of bankruptcy, which is
 “the beginning of the bankruptcy, is to be respected, and is not
 “in any way hit or interfered with by Section 47.”

In re Harrison v. Ingram, 1900, 2 Q.B. 710. Court of Appeal decided that a voluntary settlement of an insurance policy could not be avoided in whole or in part by reason of the settlor having paid premiums under the policy during the 13 years preceding bankruptcy, even though the settlement did not contain any covenant by the settlor for payment of future premiums on the settled policy.

In re Pope ex p. Dicksee, 1908, 2 K.B. 169. Court of Appeal upheld settlement in favour of wife made in consideration of her refraining from taking divorce proceedings.

In Shrager v. March, 1908, A.C. 402 P.C., a post-nuptial settlement of real estate was upheld, although no actual transfer of the real estate was made.

Where a settlement is avoided under Sec. 47, the trustees thereof are entitled to a lien on the trust property for expenses properly incurred by them as trustees in the performance of their duties; but they may be deprived of their costs of opposing the application to avoid the settlement. [*In re Holden*, 20 Q.B.D. 43]. And the trustee in bankruptcy is not entitled to stand in the place of the beneficiaries so as to get priority over incumbrancers subject to the settlement. [*Sanguinetti v. Stuckey's Banking Co.*, 1895, 1 Ch. 176.]

Also a settlement is only avoided so far as may be necessary

for payment of debts and costs of bankruptcy. [*In re Sims*, 3. Man. 340.]

Under the second paragraph of this section is included a settlement of specific property to be acquired in the future, *e.g.*, property which might be left to the settlor by his father; but a covenant in consideration of marriage to pay before a certain day, a certain sum to the settlement trustees, on the trusts thereof, is outside the section as the money is not earmarked and proof for the money, if not paid before bankruptcy, may be made. [*Ex p. Bishop*, L.R. 8 Ch. 718.]

Before making any payment to a voluntary assignee or purchasing the interest of a beneficiary under a voluntary settlement, it should be ascertained that the settlor is not already bankrupt; but it would seem that there would still remain the risk of avoidance should the settlor be made a bankrupt subsequently in respect of an act of bankruptcy committed before the transaction was concluded.

This does not seem equitable where the purchaser had no notice of the commission of an act of bankruptcy, but *In re Reis ex p. Clough*, 1904, 1 K.B. 451, it was decided that the settlor becomes bankrupt at the commencement of the bankruptcy, and that a case within Sec. 47 is outside Sec. 49, which protects certain transactions which take place after an available act of bankruptcy but before receiving order. These two points were approved by the Court of Appeal, 1904, 2 K.B. 769.

The case of *In re Reis ex p. Clough* referred to a case coming under Sub.-sec. (2) of Sec. 47, but as far as these two points are concerned, it seems to apply to Sub.-sec. (1).

Though the decision in this case was reversed by the Court of Appeal on the ground that the settlor had not, as a matter of fact, committed an act of bankruptcy before assigning the personal chattels in pursuance of his covenant in the marriage settlement, the law laid down by Wright, J., was confirmed. It seems, therefore, that a covenant in a marriage settlement to settle all after-acquired property, except business assets, is not fraudulent and void against creditors under 13 Eliz. c. 5, and is not a debt provable in the husband's bankruptcy, and, therefore, he is not released from the covenant by his discharge. Further, an assignment of personal chattels in pursuance of such a covenant does not require registration under Bills of Sale Act, 1878, being within the exception of marriage settlements contained in Sec. 4.

If it is considered unsafe to purchase property of any kind from a voluntary assignee in view of the possibility of decision "*In re Carter, &c.*" being over-ruled by House of Lords, it should be expressly stipulated, in every contract or agreement for purchase, that a title derived through a voluntary conveyance will not be accepted, for otherwise the vendor could force such a title on the purchaser.

In determining a settlor's ability to pay his debts without the aid of the settled property, the value of a life interest reserved to himself may be taken into account [*In re Lowndes* 18 Q.B.D. 677], and it is submitted (*see* William's Law of Bankruptcy) that proof that settlor was solvent some time after bankruptcy would be some evidence of his solvency at date of settlement.

Section 47 does not apply to the administration of the estates of deceased insolvents under Sec. 125. [*Hasluck v. Clark*, 1899, 1 Q.B. 699.]

In connection with settlements the Scottish case *In re Stewart and Stewart's Trustees*, 1901, O.H. 8 S.L.T. 436 may be mentioned.

A policy was taken out by assured for wife's benefit under Married Women's Policies of Assurance (Scotland) Act, 1880, and in the bankruptcy of assured, the trustee of sequestration alleged that the policy was effected in fraud of the creditors and proposed to retain the policy. He was, however, ordered to deliver up the policy to the assured as trustee for wife, his proper remedy being to recover from the trustee of the policy, out of the proceeds, any premiums which could be shown to have been paid in fraud of creditors.

DISCLAIMER OF ONEROUS PROPERTY. (Sec. 55.)

Where any part of the bankrupt's property is burdened with onerous covenants, is unsaleable or not readily saleable by reason of its binding the possessor to performance of any onerous act or payment of any sum of money, the trustee in bankruptcy may, in writing, and within 12 months after the first appointment of a trustee, or within 12 months of his becoming aware of it in cases where he did not know of it within one month of appointment, disclaim the property; but such disclaimer shall not, except so far as is necessary for the purpose of releasing the bankrupt and his property and the trustee from liability, affect the rights or liabilities of any other person.

Any person injured by the operation of a disclaimer under this section may prove in the bankruptcy to the extent of the injury.

Any person interested in the property may make written application to the trustee requiring him to decide whether he will disclaim or not and if the trustee does not disclaim within 28 days, or such longer period as Court may allow, he shall be deemed to have adopted it.

The trustee cannot disclaim a lease without leave of the Court, except where

(1) bankrupt has not sub-let whole or part of premises or created a charge on lease ; and

(a) Value of property is less than £20 p.a. ;

or

(b) Estate is being wound up under an order of summary administration ;

or

(c) Trustee has given lessor notice of his intention to disclaim, and lessor has not, within 7 days, given notice to trustee requiring the matter to be brought before the Court.

(2) Bankrupt has sub-let premises or created a charge on lease, and trustee has served lessor and sub-lessee or chargees with notice of intention to disclaim, and neither of them within 14 days requires the matter to be brought before the Court.

The Court in granting leave to trustee to disclaim a lease will impose such terms as it thinks just.

DEBTS PROVABLE AND NON-PROVABLE (Sec. 37).

“ All debts and liabilities, present or future, certain or contingent, to which the debtor is subject at the date of the receiving order, or to which he may become subject before his discharge by reason of any obligation incurred before the date of the receiving order, shall be deemed to be debts provable in bankruptcy.

“ An estimate shall be made by the trustee of the value of any debt or liability, provable as aforesaid, which by reason of its being subject to any contingency or contingencies, or for any other reason, does not bear a certain value.

“ Any person aggrieved by any estimate made by the trustee as aforesaid may appeal to the Court.”

The following three classes of debts or liabilities are not

provable in bankruptcy (or under a composition or scheme of arrangement by Sec. 3, Sub-sec. 17, of the Act of 1890).

- (1) Demands in the nature of unliquidated damages arising otherwise than by reason of a contract, promise or breach of trust.
- (2) Debts or liabilities contracted by the debtor with any person after that person has had notice of an available act of bankruptcy.

Where a debt, otherwise provable, is by reason of notice non-provable, the creditor cannot afterwards recover the debt in an action. [*Buckwell v. Norman*, 1898, 1 Q.B. 622].

- (3) Contingent debts and liabilities, the value of which cannot in the opinion of the Court be fairly estimated.

“Liability” as defined by the Act has a very wide application, and the principles governing the construction of the defining section may be gathered from *Hardy v. Fothergill*, 13 App. Cas. 351, where it was held that the assignee of a lease was released by an order of discharge, from his liability under a covenant to indemnify the lessees for a breach of their covenants to repair and yield up in repair the premises at the end of the term.

The section includes all liabilities which can fairly be estimated, excluding those specially excepted, and excluding, perhaps, those arising on contracts which have a different object from the payment of money in any contingency and those in which an injunction for specific performance would be the most proper remedy. The liability for calls in winding up of a Company is a debt provable in the bankruptcy of a contributory. [*Re Hallet ex p. National Insurance Corporation*, 1 Man. 380].

A surety who has not paid the debt for which he is contingently liable, is entitled to prove in respect of such liability in bankruptcy of principal debtor [*In re Paine ex p. Read*, 1897, 1 Q.B. 122], but not for voting purposes [*In re Parrett*, 39 W.R. 400].

Rights of
sureties.

In *ex p. Rushforth*, 10 Ves. 409, Lord Eldon says: “It is now the settled law that a surety in a bond may compel the principal creditor to go in and prove under the commission; and if the surety pays the whole debt the creditor will be a trustee of the dividends for him”, but “where a man, engaged for the whole debt, pays only part, he has no equity to stand in the place of the persons paid.” After a long series of cases the point was considered by the Court of Appeal in *Ellis v. Emmanuel*, 1 Ex.D.

157. The Court pointed out the distinction between a guarantee limited in amount to secure a floating balance and one limited in amount for a debt already ascertained which exceeds that limit. It is a question of construction whether intention was to guarantee whole debt with limit of liability or to guarantee part of debt only.

Blackburn, J., says, referring to *ex p. Rushforth, &c.*: "I think the class of cases referred to does not lay down any general doctrine that where there is a surety, with a limit on the amount of his liability, for the whole of a debt exceeding that limit, he is entitled to a rateable proportion of dividends paid on whole debt; but that only when a surety has given a continuing guarantee limited in amount, to secure a floating balance which may from time to time be due from the principal to the creditor, the guarantee is as between surety and creditor, to be construed as applicable to part only of debt, co-extensive with amount of guarantee, and this on the ground that it is inequitable in the creditor, who is at liberty to increase balance or not, to increase it at the expense of the surety."

If the guarantee is for the whole debt with limitation of liability and not for part of debt only, creditor can prove against estate of principal debtor for full amount, notwithstanding he has received from the surety the full sum for which latter was bound. [*Re Sass ex p. National Provincial Bank*, 1896, 2 Q.B. 12.]

A surety who has paid the debt is entitled to all the securities held by the creditor (*see* 19 and 20 Vict. c. 97 S. 5, and *Duncan Fox & Co. v. N. & S. Wales Bank*, 6 App. Cas. 1), and a surety who has paid the creditor in full and taken an assignment of the securities under the statute is entitled to prove as assignee of the creditor against his co-surety for the full amount of debt, but he can only recover the just proportion to which, as between the securities, he is entitled. [*Re Parker Morgan v. Hill*, 1894, 3 Ch. 400.]

In *Wolmerhausen v. Gullich*, 1893, 2 Ch. 514. Wright, J., held that the liability of a co-surety to contribution, though unascertained at the date of bankruptcy by reason that the proving surety has paid nothing, forms a provable debt though not one available for petition.

If the proving surety has paid more than his share he may apparently petition.

If he has paid his share he may take proceedings against the co-sureties to be indemnified against further payment.

Where a loan was guaranteed by several sureties, the liability of each surety being limited to a fixed amount, and certain sureties, having paid interest and premiums, claimed contribution from their co-sureties, it was held that, as they had not paid anything in excess of their proportion of the monies which were the subject of the surety-ship, they were not entitled to recover. [*Stirling v. Burdett*, 1911, 2 Ch. 418.]

From a proof on a guarantee must be deducted payments made by, or dividends declared on, the estate of principal debtor before proof was made; but such payments or dividends received after proof was made need not be deducted. [*Re Blakeley ex p. Aachener Disconto Gesellschaft*, 9 Mor. 173.]

A surety paying off his principal's debt after bankruptcy of the principal may set off any securities held by the creditor to which such payment may entitle him. [*Ex p. Barrett*, 34 L.J. 41.]

Generally a surety cannot receive anything until the creditor has been paid 20s. in the £. [*Ex p. Turquand v. Fothergill*, 3 Ch. D. 445].

The following case, which was referred to at greater length by Mr. Barrand, *J.I.A.*, xliv, 291, deals with the covenant of a surety to pay premiums and interest.

Moss mortgaged a policy *inter alia* to Hallet, and subsequently assigned all his interest in it to Cooke subject to Hallet's mortgage. Moss and Hallet jointly and severally covenanted to pay interest on sum advanced by Cooke, and also premiums on the policy. In Moss's bankruptcy Cooke proved for balance of principal less £100, at which he valued his equity of redemption in the policy, and Hallet lodged a proof for (1) balance of principal due under the first mortgage, (2) his estimated liability in respect of premiums, (3) his estimated liability for interest less the value of his interest in the policy. (2) and (3) were disallowed on grounds that as Cooke had valued his interest in the policy position is the same as if he had sold it, and Hallet's liability for payment of premiums is gone, and further that as the principal debtor is bankrupt, principal is no longer due from him to Cooke, the only liability being that of the trustee in bankruptcy to pay Cooke dividends in respect of debt, and therefore Hallet's liability is gone. [*In re Moss ex p. Hallet*, 1905, 2 K.B. 307.]

In the case of loans on personal security it is usual to insert a clause in the deed to the effect that as between the company and each of the sureties, each surety is to be considered as a principal debtor for all monies due under the deed.

In *Deering v. Bank of Ireland*, 12 App. Cas. 200, House of Lords held that mortgagee of policy is not entitled to prove for the value of a covenant to pay premiums in addition to proving for balance of debt after giving credit for the surrender-value. The reasoning of this decision seems applicable in England as well as in Ireland.

In *re Watson Turner v. Watson*, 1896, 1 Ch. 925, the testator, a surety for mortgagor, bequeathed to him a share of residue subject to a life interest. After the death of testator, the mortgagor became bankrupt, he never obtained his discharge, neither did mortgagees nor executors prove. The executors were, however, held to be entitled, upon the death of the life tenant, to retain out of bankrupt's share, the amount paid by them to the mortgagees after the bankruptcy in fulfilment of testator's liability as surety, with 4 per-cent interest.

Atkins v. Arcedeckne, 24 C.D. 709.

Here one of four sureties repaid the principal in full, took over policies and subsequently paid the premiums thereon. On policy becoming a claim the surety made a claim against one of his co-sureties for contribution of his share of principal. It was held that all the sureties were entitled to the benefit of the policies on payment of their share of premiums and other monies paid in keeping them in force.

Berridge v. Berridge, 44 C.D. 168.

Here five sureties gave joint and several guarantee to secure a bank balance not exceeding £2,000, and one of them deposited security for an overdraft, and debtor assigned to him two policies. On debtor's bankruptcy the five co-sureties paid the £2,000, and the one surety the balance. He also paid the subsequent premiums. On death of bankrupt, still undischarged, it was held that after the one surety had been repaid the premiums and costs he had paid, the joint claim for £2,000, and the one surety's further claim for the balance stood on the same footing, and the policy monies must consequently be divided among them rateably.

These two cases seem to show that even though one of several sureties takes it upon himself to keep the policy in force, the other sureties do not lose their interest in the policies. So that though the one surety took the risk of continuing to pay the premiums, if it should turn out to have been profitable by reason of the early death of the life assured, the other sureties can come in and claim the advantage on payment of their shares.

The following abstract from Bunyon's Law of Life Assurance,

dealing with the rights of a Company over a policy on the life of a surety is interesting (*see* page 450).

“Where a policy has been issued to a surety upon his own life, the Company will not have any lien upon it in the absence of an agreement to that effect, although it may be entitled to set off against any claim upon such policy, while in the hands of the representatives of the assured, any moneys actually recovered against them upon the bond or other security.”

The above does not come under the law of bankruptcy, but is mentioned because the circumstances might arise if the principal debtor became bankrupt.

Grace given to the principal debtor without the consent of the surety would discharge the latter in the absence of special provision in the bond.

PRIORITY OF DEBTS.

By the Friendly Societies Act, 1896, s. 35, the trustees of a friendly society have on the bankruptcy of an officer holding, by virtue of his office, money or property, a right to receive such money or property in preference to other claims and debts against the estate.

Trustees of Savings Banks have a somewhat similar priority. Miners in the Stannaries have a first charge on assets of mine for wages not exceeding three months.

Rights of an employer, in respect of insurances against accidents to his workmen, pass on his bankruptcy to his workmen.

Subject to the above the order in which debts are payable is regulated by the Preferential Payments in Bankruptcy Act, 1888, by which the following debts rank equally amongst themselves, and are paid in full, if assets are sufficient, in priority to all other debts.

Shortly expressed, they are as follows :

- (1) Rates and Taxes not exceeding 12 months.
- (2) Wages or salary (including commission *In re* Klein, 1906, 22 T.L.R. 664) of any clerk or servant for four months preceding receiving order and not exceeding £50.
- (3) Wages of any labourer or workman for two months preceding receiving order and not exceeding £25.

The above priorities also apply in the administration of a deceased insolvent, substituting date of death for date of receiving order, but they are subject to any claim by the legal personal

representative of the deceased debtor to payment of proper funeral and testamentary expenses incurred by him.

Joint and
separate
estates.

“In the case of partners the joint estate shall be applicable in the first instance in payment of their joint debts, and the separate estate of each partner shall be applicable in the first instance in payment of his separate debts. If there is a surplus of the separate estates it shall be dealt with as part of the joint estate. If there is a surplus of the joint estate it shall be dealt with as part of the respective separate estates in proportion to the right and interest of each partner in the joint estate.”

Deferred
debts.

Two classes of debts are deferred until other creditors have been paid in full.

(1) By Partnership Act, 1890, the lender or vendor to a person engaged or about to engage in business who is to receive a share of the profits cannot receive anything, in the event of the trader's bankruptcy, until the other creditors have been satisfied.

(2) By Married Women's Property Act, 1882, a wife's money or other estate, lent or entrusted to her husband for the purpose of any trade or business carried on by him or otherwise, will be treated as assets of his estate in case of his bankruptcy, but the wife can claim a dividend when other creditors have been satisfied. But where she has lent money to a firm of which her husband is a member, or lent money to her husband for private purposes, or being surety for him, has paid the debt, she may prove as an ordinary creditor.

INTEREST.

By Sec. 23 of B.A. 1890, a creditor cannot receive any higher rate of interest than 5 per-cent per annum until all debts proved have been paid in full. But this does not prevent a reserved creditor from allocating his security to the interest and then proving for the principal [*In re Fox v. Jacobs*, 1894, 1 Q.B. 438], though he cannot allocate his security to any interest accruing after the date of the receiving order. [*In re Bonacino*, 1 Man. 59].

A scheme of arrangement under Sec. 3 of B.A. 1890, excluding the operation of Sec. 23 may be approved. [*In re Nepean*, 1903, 1 K.B. 794].

As it is usual to charge a penal rate of 6 per-cent in connection with loans on personal security the above decisions are important.

Where a debt is overdue but interest is not reserved or

agreed for, creditor may prove for interest at 4 per-cent per annum up to the date of the receiving order.

A creditor may prove for and receive dividends on a debt payable at a future date, but from the dividend is deducted a rebate calculated at 5 per-cent per annum for the declaration of dividend to date at which debt is payable.

Where a debt is payable at a future time with interest in the meantime, the creditor proves for principal debt as a present debt and the rebate is deducted from the dividend; then he values and proves for interest accruing after receiving order dividend on which is not liable to rebate.

If interest at 5 per-cent is contracted for the principal sum may be proved for as a present debt, and no rebate deducted from the dividend. [*In re Browne v. Wingrove*, 1891, 2 Q.B. 574.]

POWER OF DISTRESS.

By Sec. 42, as amended by Sec. 28 of B.A. 1890, a landlord can distrain upon the goods or effects of bankrupt for rent, but if such distress be levied after commencement of bankruptcy, it is available only for 6 months' rent accrued due prior to date of order of adjudication or order of administration. Landlord can, however, prove for balance.

If landlord or other person distrains on goods of bankrupt within three months preceding date of receiving order, the debts to which priority is given (*see above*) shall be a first charge on goods distrained on or proceeds of sale. But in respect of any money paid under such charge, the landlord or other person shall have same rights of priority as person to whom such payment is made.

SET-OFF (Sec. 38).

Where there have been mutual debts, credits, or other dealings between debtor and person proving under a receiving order, only the balance of the account shall be claimed or paid, but a person cannot claim the benefit of set-off in any case where he had, at time of giving credit to the debtor, notice of an available act of bankruptcy. This section still applies where one party holds security for his debt. [*Ex p. Barnett*, L.R. 9 Ch. 293.]

Where a limited company is being wound up, a solvent contributory cannot set off against calls made by the liquidator money due to him from the Company, but if the contributory is

a bankrupt, his trustee may set-off such a debt. [*In re G. E. B.*, 1903, 2 K.B. 340.]

There can be no set-off between a debt and a claim for unliquidated damages (Parlby's case 19 W.R. 382), so that where an insurance company was being wound up and the policies were valued, it was held that a policyholder could not set-off a loan to him by the Company against the value of his policies. [*Ex p. Price re Lancaster*, L.R. 10 Ch. 648.]

Money borrowed from an insurance company by a policyholder cannot be set-off against their contingent liability to him in respect of the policy. But where liquidator of a Company sued a policyholder for amount of loan borrowed on security of his policy and policy matured before action was brought, set-off was allowed. [*Sovereign Life Company v. Dodd*, 1892, 2 Q.B. 574.] Liability of a company does not become a debt until claim is payable according to terms of policy; where, therefore, money was only payable three months after death, and winding-up order was made after death, but before expiration of the three months, set-off was not allowed. [*Delhi Bank Case*, 15 S.J. 923.]

Where a voluntary settlement by a bankrupt is set aside, the donee cannot set-off against it a debt due to him by the bankrupt, for the sum settled was never a debt due by him to the bankrupt. [*Lister v. Hooson* 1908, 1 K.B. 174.]

SECURED CREDITORS.

A secured creditor may either :

- (1) Realize his security and prove for balance,
- (2) Surrender his security and prove for whole debt,
- (3) Value his security and prove for balance.

The trustee may redeem or require to be sold a security so valued and allow a secured creditor under certain conditions to amend his valuation and proof if made on a mistaken estimate. But the creditor may at any time, by notice in writing, require the trustee to elect whether he will or will not exercise his power of redeeming the security or requiring it to be realized, and if the trustee does not, within six months after receiving the notice, signify in writing to the creditor his election to exercise the power, he shall not be entitled to exercise it : and the equity of redemption, or any other interest in the property comprised in the security which is vested in the trustee, shall vest in the creditor, and the amount of his debt shall be reduced

by the amount at which the security has been valued. (Schedule 2 Rule 12.)

Where a creditor has so valued his security he may at any time amend the valuation and proof, if made *bonâ fide* on a mistaken estimate, or if the security has diminished or increased in value since its previous valuation, upon such terms as the Court shall order, unless the trustee shall allow the amendment without application to the Court. (Schedule 2, Rule 13).

The mere fact that the trustee has told the creditor (no notice to elect having been given by the creditor) that he intends to purchase the security at the creditor's valuation, does not preclude amendment, even if the trustee tender the amount [*In re Newton*, 1896, 2 Q.B. 403], but if the trustee has been challenged, and has elected to redeem the security at the creditor's valuation probably the creditor cannot subsequently amend it. [*Ex p. Norris*, *In re Sadler*, 17 Q.B.D. 728.]

The judgment "*In re Newton*" laid down that where the security had largely increased in value the trustee could not tender the amount of creditor's valuation and redeem the security without the creditor having the opportunity of amending the valuation, and this seems to cover the case where the mortgagee having valued the policy and proved for the balance, the policy becomes a claim.

See also "*In re Fanshawe*, 1905, 1 K.B. 170", where a secured creditor was allowed to re-value his security which had increased in value by the death of a prior incumbrancer.

Rule 14 provides for the adjustment of dividends received by the creditor prior to the amendment of his valuation.

Rule 15.

"If a creditor after having valued his security subsequently
"realizes it, or if it is realized under the provisions of rule 12,
"the net amount realized shall be substituted for the amount of
"any valuation previously made by the creditor, and shall be
"treated in all respects as an amended valuation made by the
"creditor."

According to Bunyon, see "*Law of Life Assurance*", pages 390-1, "the value (of a current policy held as security for a debt) agreed on between himself (the creditor) and the trustee is binding on both parties, and any excess received in respect of the policy belongs to the estate; the creditor does not become a purchaser of the policy itself." And in the cases of *Ex p. King v. Palethorpe*, L.R. 20 Eq. 273, and *Bolton v. Ferro* 14 C. D. 171,

the mortgagee was only allowed to retain out of the policy monies the agreed value plus subsequent premiums with interest.

In re Pearce (1909) 2 Ch. 492.

Here Bullard & Co. valued their security under a deed dated 24 December 1898, at £2,481, and, at the trustee's request, stated that certain policies were valued at £205 and book debts and second charge on freeholds together at £2,276. The book debts realized more than was expected, and out of proceeds trustee paid them £2,276. He also paid them 12s. in the £ on the unsecured part of their debt. Bankruptcy was subsequently annulled and property revested in Pearce. In a subsequent action in the Chancery Division it was held that Bullard & Co. were only entitled to £2,481 plus premiums paid since the date of the receiving order with interest from that date, subject to an allowance in respect of the £2,276 received by them from the trustee.

It seems, therefore, that a secured creditor can, by amendment of his valuation, get the benefit of any increase in the value of his security during the bankruptcy, but not so after a composition has been agreed to and the bankruptcy annulled, or the bankrupt discharged, unless he has, in effect, foreclosed under Rule 12.

The existence of a subsequent incumbrancer does not affect the creditor's right to amend his valuation. [*Ex p. Arden re Arden*, 1884, 14 Q.B.D. 121.]

SUMMARY ADMINISTRATION (Sec. 121).

The Court will make an order for summary administration if it is satisfied by affidavit or the Official Receiver's report that the property is not likely to exceed £300.

As in the case of ordinary bankruptcies a petition by or against the debtor is the first step, and the provisions relating to the examination and discharge of the debtor are in no way affected.

Principal modifications.

- (1) If the debtor is adjudged bankrupt the O. R. is the trustee unless the creditors by special resolution appoint someone else, but this would have the effect of bringing the bankruptcy under the ordinary rules as if the order for summary administration had not been made.
- (2) Scale of Solicitor's costs is lower.

- (3) Six months are allowed for declaration and distribution of a dividend.
- (4) Estate must be realized with all despatch, and where practicable distributed in one dividend.

ADMINISTRATION ORDERS (SEC. 122).

Where a judgment has been obtained in a County Court and the debtor is unable to pay the amount forthwith and alleges that his debts, including judgment debt, do not exceed £50, the County Court may make an order for administration of his estate, payment of debts by instalments or otherwise, in full or not as appears practicable, and subject to conditions as to future earnings or income as the Court may think just.

DECEASED INSOLVENT DEBTORS (SEC. 125).

“Any creditor of a deceased debtor whose debt would have been sufficient to support a bankruptcy petition against such debtor, had he been alive, may present to the Court a petition in the prescribed form praying for an order for the administration of the estate of the deceased debtor, according to the law of bankruptcy.”

Such an order may be made within two months after grant of probate or letters of administration, without concurrence of legal personal representative and without proof of act of bankruptcy within three months before death. (*See* B.A. 1890, S.s. 21 (1) 29.)

The Official Receiver is the trustee unless creditors appoint someone else, and all provisions of the Act relating to the administration of a bankrupt's property, shall, as far as practicable, apply.

PROPOSED LEGISLATION.

It may be mentioned that a Bill was introduced into the House of Lords just before the close of last session for the amendment of bankruptcy law. The Bill proposes to give effect, with some modifications and additions, to the recommendations of a Departmental Committee on Bankruptcy Law and its administration, which reported in 1908. The more important amendments may be briefly described as follows :

1. Board of Trade are to be enabled to undertake prosecutions through their Solicitor or the Official Receiver's before Courts of Summary Jurisdiction.

2. Failure to keep accounts in case of a trader who has been previously insolvent, failure to explain disappearance of assets and insolvency brought about by gambling to be criminal offences in the case of bankrupts engaged in trade or business.
3. *Boni fide* purchasers of after-acquired real property to be protected, and if an undischarged bankrupt again becomes bankrupt, assets acquired since earlier bankruptcy to be distributed *pro rata* between new and old creditors.
4. All married women engaged in trade to be amenable to bankruptcy proceedings upon a bankruptcy notice following judgment.
5. Additional restrictions to be imposed on power to place property out of reach of creditors, in case of bankruptcy, by means of covenants in marriage settlements.
6. General Assignments of book debts to be void against trustee in bankruptcy unless registered.
7. Safeguards to secure honest administration by trustees under Deeds of Arrangement (outside bankruptcy).
8. Control of Bankruptcy Courts to be extended over foreigners trading in this country through agents or partners.
9. Power of Landlords to distrain after bankruptcy for rent payable in advance to be taken away.
10. Proposals to meet devices adopted by moneylenders to evade the provision of Bankruptcy Act, 1890, Sec. 23, by which claims for interest over 5 per-cent are postponed till all other claims have been paid in full.

SUMMARY.

It may facilitate discussion if I briefly summarize those points dealt with in the paper which are of practical interest, but as to which the legal position is difficult to define.

- (1) Would the purchaser for value of an interest under a voluntary settlement be protected in the event of the subsequent bankruptcy of the settlor in respect of an act of bankruptcy committed before the purchase but of which the purchaser had no notice?
- (2) Can the trustee in bankruptcy be ignored in dealing with the assured under a policy issued on the life of an undischarged bankrupt?

- (3) Where a person, who has a renewal commission account with an office, becomes bankrupt, who is entitled to the commission due after bankruptcy?
- (4) Would commission on new business introduced after bankruptcy be treated as personal earnings?
- (5) How far do the cases of "*In re Moss ex p. Hallet*" and "*In re Pearce*" apply to a loan on personal security as regards the covenant of a surety to pay premiums and the debtor's right to redeem the policy respectively?

ABSTRACT OF THE DISCUSSION.

Mr. R. C. SIMMONDS said that the present principles and practice of the law of bankruptcy might be summarised under five main headings. First, there was the Act of Bankruptcy; secondly, the Receiving Order; thirdly, the Investigation of Affairs (under which heading would be included the question of statements, lodgment of claims and public and private examination of the debtor, &c.); fourthly, the Decision upon the Case, which would lead to the consideration of adjudication (with its effect—trusteeship and winding-up), or compositions in bankruptcy, or—in a very few cases—annulment of the original receiving order; and fifthly, the Discharge, which covered such matters as suspension, offences, disabilities, and the conditions of restoration to full civil capacity. Proceeding to discuss various matters arising on the paper, the first point which he thought required special mention was that of after-acquired property. The leading case, as Mr. Barrand had pointed out, in that connection was *Cohen v. Mitchell*. In that case the wording of the judgment was very wide, so much so that many offices, he believed, were led to consider policies as being within its scope, and to think that they were protected in so doing. But since that time the position had been very considerably limited by various cases, and there was little doubt now that offices would not be safe in relying too much upon the particular decision mentioned.

The case (which Mr. Carter referred to in his paper, p. 223) in which the matter was submitted to counsel some years ago afforded a confirmation of the view that it was not safe to disregard the trustee in bankruptcy. In that connection he would like to refer to the suit of *In re Bennett*, which was discussed by Mr. Barrand in one of his valuable "Notes." In that instance the bankrupt had effected two policies, for which he paid the premiums out of current earnings. He died intestate and the estate was administered by his brother, who was quite ignorant of the previous bankruptcy and who distributed a large proportion of the money among various next-of-kin. The Official Receiver heard of the matter and claimed the money,

whereupon it was decided that the administrator should hand over the residue in his possession (which he had retained on behalf of one or two children involved in the case) and also his own share. Not only was this done, but also, on a subsequent action, one of the beneficiaries was obliged to refund the money which he had received, on the ground that although his receipt was *bonâ fide*, yet it was not for value. There was also the action of *Tapster v. Ward*, which was particularly apposite. In that instance a policy had no surrender value at the date on which the assured became bankrupt. Discharge was obtained in due course and the premiums were paid for a number of years until death. When that occurred the insurance company refused to pay the money without the concurrence of the Official Receiver, who on being approached refused to assent. When this case came before the Courts it was held that the Official Receiver's title must prevail and the decision was upheld on appeal. Therefore, he thought there was good reason for answering Mr. Carter's second query, as to whether the trustee in bankruptcy could be ignored, in the negative.

Adverting next to the question of agency commission, he had come across a case quite recently where the assured, who was also the agent, became bankrupt. The office suspended payment of commission, whereupon the solicitor for the assured requested that the allowance should be continued, and in doing so used words to this effect: "I think there is no doubt that you are safe in paying us commission. I refer you to the case of *Cohen v. Mitchell*, and if you have any doubt a word with your solicitors will put the matter right." The question, he believed, was submitted to the company's legal advisers and they agreed that the contention of the assured's solicitor was quite correct. So that, apparently, an office was safe in continuing to pay renewal commission to an agent after bankruptcy. With regard to the question of treating commission on new business introduced after bankruptcy as personal earnings, he could not say that he had any legal decision to rely upon, but he would suggest that it was reasonable to consider such commission as personal earnings if it could be held that the agent really canvassed the case. In reference to voluntary assignments, Mr. Carter raised a very interesting point. As he understood it, the risk to which Mr. Carter referred was that a volunteer might offer to deal with an insurance office after committing an act of bankruptcy, but before the three months which were allowed had expired, and, consequently, in view of the doctrine of the relation-back of the title of a trustee in bankruptcy, the interest of the office might be defeated, even though it were acquired *bonâ fide* and for value. Serious as the risk appeared at first sight, he thought that there were one or two considerations which minimised it. In the first place, offices were not disposed as a rule to lend upon personal security without being fairly well convinced of the status of the parties with whom they dealt, and, of course, there were the sureties, who were brought in as well, usually as principal debtors. Then, again, the system of searching for bankruptcy proceedings would reveal not only actual bank-

ruptcies but also one or two of the more important acts of bankruptcy. Therefore, he did not think that the risk was quite as serious as it appeared, although admittedly it was a risk; and, speaking entirely on his own responsibility, he should be disposed to think that an office would find itself called upon to defend its title if such circumstances actually arose.

At the close of his paper Mr. Carter touched upon the legislation that had been proposed by the Board of Trade. The Bill upon which the proposed legislation rested was founded upon the recommendations of the Departmental Committee of the Board of Trade, which reported in 1908. Many witnesses were called before the Committee, and, as usual, several advocated extreme measures in one direction, while others advocated extreme measures in the other direction. The Committee, in their Report, declined to proceed as far as many of the witnesses wished, and the Bill itself did not carry out the suggestions in the Report in their entirety. However, the general effect of the Bill was to strengthen the provisions of the existing law, and to that end certain rather important alterations were proposed, of which he thought the following five were the most essential. First of all, it was proposed that debtors should be prosecuted, in the case of suspected fraud and malpractice, before Courts of Summary Jurisdiction. The idea was to secure not only greater simplicity and economy, but also greater expedition, which would give a certain type of debtor less time to abscond. He thought himself that that reform would tend, in practice, to be of considerable assistance. The second important alteration was that in cases where there was a proved failure to keep accounts (the trader having been insolvent previously), or where there was a failure to explain the disappearance of assets, or where insolvency was due to gambling, there should be power to institute a criminal prosecution. With regard to the first two—failure to keep accounts and failure to explain the disappearance of assets—there could be no doubt that such a provision would be extremely salutary, because really there could be no excuse for a debtor's failing to conform to either of those elementary safeguards. With regard to insolvency due to gambling, that was rather a larger question, inasmuch as in many cases the gambling was not primarily a fault of the debtor himself, but was due to the unscrupulous manipulation of other people who had an influence over him. Nevertheless, it was well to make the provision, and, perhaps, later on they would be able to trace and to punish the persons really responsible. The third point was that married women were all to be subject to bankruptcy proceedings. At the present time, as the author said, there were many loopholes, and, therefore, the new provision would be very important.

In that connection he might mention a case which appeared in the last Annual Report of the Solicitor to the Board of Trade—the case of a bankrupt at Kingston-on-Thames. There were two brothers, of whom one was an undischarged bankrupt, while the other had proceedings pending against him. They established a

building and contracting business on the basis of the partnership of their wives. The deeds and documents affecting the matter appeared to be in a mixed state, and when the firm came to an end through insolvency the Official Receiver attempted to ascertain the real relations of the parties, whereupon it appeared that it was quite impossible to reach the assets of the only person who had any money at all, namely, the wife of one of the men in question. The result was that after a good deal of investigation and expense the creditors got nothing and the persons who were responsible escaped with their own resources untouched. That was a crying evil which the new legislation should remedy. Similar remarks applied to the fourth provision, regarding foreigners who traded in this country through their agents or partners; it was most important that such persons should be brought within the scope of the Bankruptcy Law. The last reform to which he would call attention was in the direction of preventing debtors still further from placing their property out of the reach of creditors, and in that connection he might refer to a little point which he had noticed in actual office practice. It seemed to him that in quite a number of cases where an assured became bankrupt it was found that after the lapse of a few weeks the policy passed, *via* the trustee in bankruptcy, either to a friend or to the wife, and although one could not prove the point it appeared probable that the destination of a good proportion of the assured's vanished assets would be found to be his wife's pocket, the transfer having been made with a view to her obtaining the policy when the actual financial crash came. Such proceedings should certainly be stopped. The remaining provisions of the Bill were mainly matters of improved procedure, into the discussion of which he did not think he need enter.

In closing he would like to raise two supplementary matters for consideration. The first was the question of expenses in bankruptcy proceedings. In the last Annual Report of the Inspector-General in Bankruptcy there was a very interesting statistical table giving the percentage of costs to gross assets for estates of various sizes. It appeared that in very small cases the ratio of expenses to assets was largely in excess of 100 per-cent. In cases of £500 to £1,000, the expenses with official trustees reached 22 per-cent, and with non-official trustees 36 per-cent; while in cases of £1,000 to £2,000, the expenses were 19 per-cent in the former group, and 26 per-cent in the latter. He could hardly think that such a heavy expense ratio was justified. He was aware, of course, that persons did not usually become bankrupt until their affairs were in a hopeless condition, and that a great deal of investigation had to be carried out, but at the same time it did seem that in the interests both of debtors and creditors something should be done to diminish such very heavy charges. When it was remembered that there were between three hundred and four hundred different rules laid down for administration in bankruptcy, it tempted one to suggest that there was considerable scope for simplification and improvement.

Lastly, he would refer to the question of further stringency.

The ideal of any bankruptcy system—if there could be said to be any ideal about such a thing—was that it should operate to relieve the unfortunate debtor, while at the same time it should mete out justice to those who preyed upon others. He was afraid that the present system could not be said to do that. Everyone was aware of flagrant cases in which it had proved possible to run through the mesh of the law with comparative impunity. He suggested that an endeavour should be made to press for reforms which would make the existing law stronger in dealing with offenders and more lenient in the case of unfortunates. On the first page of his paper Mr. Carter said that bankruptcy was formerly a crime. He trusted that steps would be taken by the community to see that, in the case of a certain type of person, it did not become a luxury also.

Mr. J. R. HART said that members of the Institute were mostly concerned with bankruptcy in connection with titles to policies, but occasionally came across the question in connection with loans on personal security. At the beginning of the paper Mr. Carter mentioned that a creditor could not present a petition unless the debtor were domiciled in England, and it seemed to be of importance to know how far those proceedings applied to Scotland, because there were a good many Scotsmen who were sureties. The question also related to sureties resident abroad. Companies were often asked to lend on personal security with sureties abroad, and he believed that, as a rule, most offices would refuse, although he had heard of occasions when they had been taken. With regard to Mr. Carter's remarks about married women, he believed it was the custom to refuse to take married women as sureties, principally on the ground that they might not understand the contract, and difficulties might be raised and disputes arise on that score, but the fact that married women could not be made bankrupt seemed to add force to the objection to taking them as sureties. With regard to mortgages, he thought the question of bankruptcy would seldom arise if careful enquiries were made as to the borrower. If that were done it would not be often that an office found itself with a security where the borrower was bankrupt. In view of the difficulties shown to arise in connection with such complicated procedure as that involved in the bankruptcy law it was well to make the most searching enquiries so as to avoid trouble.

Mr. S. G. DUNN said Mr. Carter had written an extremely interesting and exhaustive paper, and one that was likely to become an abbreviated text-book for those who were preparing for examinations, and probably, also for those who had to deal with matters in office practice, and in that connection he thought it would be desirable if a warning could be embodied in the paper pointing out the necessity of seeking professional advice on all doubtful questions. One was apt to rely too much on these non-professional papers. There was one point, which might be considered hypercritical, but which he thought it well to mention. Mr. Carter set out some of the disabilities which a bankrupt incurred before his discharge. It might, of course, be very unpleasant not to be able to sit in

Parliament or act as a Justice of the Peace, but he thought that what would appear most important to the bankrupt would be the penalties which attached to obtaining credit without revealing his unfortunate position, and also the questions which might arise in respect of his after-acquired property beyond certain limits. In the latter part of the paper there was a reference to proposed further legislation, and he thought the members would be grateful to Mr. Carter if he would not only state what legislation was proposed, but what legislation he considered ought to be proposed. It would be of interest also to know the opinion of the meeting on a question mentioned in the paper, relating to voluntary settlements. In the case of the death of a settlor, insolvent but not bankrupt, a voluntary settlement, he believed, was good against the trustee administering the insolvent estate, and that seemed anomalous. He saw no reason why the insolvent estate of a deceased settlor should stand in a different position from the estate of a living bankrupt. The present position was extremely convenient for life offices, but it would be interesting to know the opinion of the meeting whether it was or was not desirable on general grounds.

With regard to Mr. Simmonds's remarks on the proposed legislation involving greater stringency in the case of certain classes of debtors, it should not be forgotten that the creditors had themselves to blame to a certain extent. Proper enquiries ought to be made and reasonable precautions taken before credit was allowed, and if a man's business capacity was such that he did not keep proper books and so on, a creditor should not complain if he suffered loss.

Mr. ARTHUR J. HICKS was afraid he could not agree with the last speaker in expressions which might appear to show that he held a brief for the bankrupt. He himself thought that in any new legislation the present stringency of the law of bankruptcy with regard to the bankrupt himself should be maintained, if not increased, or the commercial community of this country would be found to suffer much more in the future than it had in the past from people who made a practice of preying upon the ordinary business man. It had to be remembered that our commercial system was built up to a very large extent on credit, and credit involved good faith. If a man made up his mind that he would deceive and defraud people with whom he was doing business he would do it, but at the same time his course should be made as difficult as the law could possibly make it. He wished to refer to one case mentioned in the paper to which Mr. Carter gave a very brief reference, but which in his judgment was a most important case—that of *In re Johnson*—where a bankrupt effected a voluntary settlement of his property on terms which gave him an immediate life interest, the income to go over to the trustees, in the event of his bankruptcy, with a discretionary trust empowering them to apply it to the settlor's benefit. He shortly after became a bankrupt, and on the trustee in bankruptcy applying to have the settlement set aside the trustees were ordered to raise out of the monies which the bankrupt had settled a sufficient sum to pay his debts in full. Soon after, however, he became bankrupt again,

but this time the creditors had no remedy. The point being taken to the Court of Appeal, there was a decision that the first bankruptcy had operated to bring his life interest in the settlement to an end, and his creditors were left with nothing. It seemed to him that that was a decision which was unfair, and one upon which it would not be safe to rely. He might illustrate his point by supposing that instead of a discretionary trust in favour of the bankrupt there had been a direct trust that on his bankruptcy some other person was to take the income. Even on the strength of such a judgment as was given in that case he thought no lender would be well advised to make an advance to a person who succeeded to the income on the bankruptcy of the settlor. It certainly appeared to him that that was a case which must be looked at far more carefully than the case which Mr. Carter had referred to, where he spoke of the possibility of an appeal to the House of Lords, despite the very strong terms of Mr. Justice Rigby's judgment in deciding that a beneficiary under a voluntary settlement which had not been in force for ten years could give a good title to an incumbrancer. There was no doubt that the very long period of ten years, during which under the present law of bankruptcy a voluntary settlement was liable to be upset, sometimes operated to cause serious inconvenience, if not injustice, to parties who were interested in such settlements, and it seemed to him that some provision might be made under which a settlor acting in perfect good faith could consent to the registration of such a deed, or the advertisement of it at the time it was made, so that in some shorter term than ten years the deed would have a more absolute effect than it would appear to have under the present law.

He would venture to refer just for a moment to another case mentioned by Mr. Carter, and which was mentioned by Mr. Barrand in an earlier paper in the *Journal*. It was the interesting but complicated case of *Moss ex p. Hallet*. There was one small point which required consideration. It was stated that Cooke proved for balance of principal less £100, at which he valued his equity of redemption in the policy. He could not understand how it was that the second mortgagee (Cooke) could have an equity of redemption in the policy of any value when the security was insufficient to satisfy the claim of the first mortgagee. He turned up Mr. Barrand's paper and found that he used the same term, and the original King's Bench Report to which he referred had the same wording. He thought it must be a misprint in the first instance, and perhaps Mr. Carter would consider the point. That case, however, was differentiated entirely from the personal loans with which life assurance societies occasionally concerned themselves, because it would be noticed that the surety gave a covenant for interest and premiums only; there was no covenant for principal. Although Mr. Carter pointed out that the rule was for a surety to be considered as a principal debtor, yet in that case the surety had given no covenant in respect of principal, and for that reason it was probably not of great interest to insurance societies.

Mr. W. PENMAN, Junr., said the only point to which he wished

to refer was the question : Can the trustee in bankruptcy be ignored in dealing with the assured under a policy issued on the life of an undischarged bankrupt ? He took it that if the office had received no notice of the bankruptcy, payment to an assignee or to the personal representative of the deceased policyholder would render the office quite safe ; but the difficulty appeared to be that the office might very well have a notice in respect of a prior policy, that policy possibly being out of force, and under such circumstances it would be very easy for a notice of that description to be overlooked. Under those circumstances searches in bankruptcy were not much good, *e.g.*, in the case of a man who went bankrupt in 1897 and was undischarged when he took out a policy in 1907, as when making a search in bankruptcy the presumption was that the search would be made no further back than 1907. The important point appeared to be, supposing an office took a discharge from an assignee or a personal representative ignoring the trustee in bankruptcy, whether the remedy of the trustee would lie against the office or against the recipients of the money ? If the office had a good discharge it was of secondary importance to the office that the trustee in bankruptcy had a remedy against those people to whom it had paid the money.

Mr. R. R. TILT congratulated the author upon the paper, which he said conveyed a large amount of information, first in connection with the general law of bankruptcy, and then on various decisions of interest to insurance companies. In connection with the general subject, it might be of interest to consider former methods of administration of insolvent estates and then endeavour to foreshadow amendments of the present system. The legislation now in force was comparatively modern. The last thirty years had seen great changes, and it could not be said that the present procedure had yet stood the test of experience. Looking back, it was curious to see the oscillations between undue leniency to a debtor and undue severity, the swing of the pendulum being now, he thought, in the direction of leniency. In the early part of last century every debtor came immediately under the control of the Court, and his affairs were carefully investigated ; the debts were scrutinised, the list of creditors was carefully examined, and nothing could be done except under the decision of the Court. The system secured substantial justice, but was found to be cumbrous and expensive. Then there gradually sprang up a system of "arrangements" between the debtor and his creditors, free from the control of the Court. These arrangements, favoured by Parliament (probably by accident rather than design), increased rapidly, and it was found that debtors, by collusion with favoured, and sometimes fictitious, creditors could get rid of their liabilities very easily and defraud many of the genuine creditors. A list of creditors was furnished by the debtor, and many claims were admitted which ought not to have been. For instance, a bill of exchange holder could prove on the bankrupt estate of an endorser, although the bill was not due and the acceptor was perfectly solvent. It was also legal, under those arrangements, for the majority of creditors to bind the minority. That dissenting creditors should be

bound by the arrangement without a full disclosure of the debtor's circumstances and without judicial discussion was a wrong principle, aggravated by the fact that the creditors' claims were subject to no independent investigation. Ten years after the Act of 1869 was passed, it was reported that out of 13,000 failures only 1,000 came under the more important provisions of the Bankruptcy Act for preventing abuses by insolvent debtors and by the agents employed to carry out the compositions with creditors. Then came the Act of 1883. Voluntary arrangements with creditors could still be made outside the Bankruptcy Acts, but a majority could not bind the minority and any creditor within three months of the date of the arrangement could petition for a receiving order. An arrangement with creditors under the Act of 1883 did not involve making the debtor a bankrupt, and the majority of creditors could bind the minority only after an examination of the debtor by the Official Receiver in private and in public in Court, and when the Court had sanctioned the arrangement.

It was under the voluntary arrangements (outside the Bankruptcy Acts) that the doctrine of the relation-back of the bankruptcy trustee's title came into operation. A debtor who had committed an act of bankruptcy ceased to be qualified to deal with his property, and if (for example) a policyholder assigned his policy to a trustee for the benefit of his creditors, any creditor who had not concurred in the arrangement could (within three months) have the assignee adjudicated a bankrupt. The opener of the discussion had already mentioned the Report of 1908 on the Bankruptcy Laws, by a very strong Committee appointed by the Board of Trade. As was said by the author, the Bill recently introduced was largely founded on the Report, but some of the recommendations of the Committee were not adopted. The Committee, in dealing with the subject of the relation-back of the trustee's title, said: "If a person who owes money to a debtor, or has property of the debtor in his hands, pays the debt, or delivers the property, in fulfilment of his legal obligation, with knowledge of an act of bankruptcy committed by the debtor, and within three months of the date of the act of bankruptcy, a bankruptcy petition is presented by or against the debtor on which he is adjudged bankrupt, the person who has thus paid his debt or properly handed over the property in his hands, will have to pay the debt over again to the bankruptcy trustee, or account to the bankruptcy trustee for the value of the property." Then the Committee goes on to say: "In practice an almost intolerable situation may be created; for if payment of a debt is refused or property is withheld on the ground that the person to whom the money is payable, or who owns the property, has apparently committed an act of bankruptcy, and a bankruptcy petition is not presented against him, or he turns out not to have committed an act of bankruptcy, the person who owed the money or withheld the property might become liable to damages and law costs, although he was always willing to discharge his legal obligation, and the situation was not created by him in any way."

It is a special incident of the hardship which the law inflicts on honest innocent persons that under the present law it is not a plain or easy matter to determine what is an act of bankruptcy. In a late case the Judge of first instance held that what the debtor did was an act of bankruptcy; the Court of Appeal held that it was not, and the members of the House of Lords differed in opinion. Mr. Carter, in the paper, gave a long list of acts of bankruptcy, but it was necessary to remember that acts of bankruptcy could not be always clearly defined. The Committee made various suggestions. Their recommendation for the alteration of the law as to the relation-back of the title of a trustee was as follows: "That it is desirable that the present law be altered by providing that nothing in the Bankruptcy Acts shall invalidate a payment or delivery to the bankrupt, or to a person claiming by assignment from him, if the payment is made before the receiving order and before notice of the presentation of a bankruptcy petition, and is made pursuant to the ordinary course of business or is otherwise *bonâ fide*." It would be noted that under the existing law knowledge of the act of bankruptcy was necessary, and the question arose as to what constituted such knowledge. There were acts of bankruptcy of which a creditor would have no knowledge, but it was the custom of solicitors in important transactions to search for bankruptcy notices, and also, he believed, to search for deeds of arrangement under the Deeds of Arrangement Act, 1887. As to policies, he thought there was protection by the fact that, under the 1867 Act, notice had to be given to the company.

With regard to arrangements with the sanction of the Court, there the doctrine of relationship-back would hardly apply, as regarded payment to the estate of the debtor, because the Official Receiver was the assignee of the debtor's property, and there could be no risk in paying the Official Receiver. They might next consider the procedure, when the debtor had been adjudicated a bankrupt, from the point of view of the discharge. It had been stated that "The discharge was the key to the efficiency of every bankruptcy system," and the Report of 1908 pointed out that if the debtor was made a bankrupt many of his offences were not punishable and not even cognisable by the Court, unless the debtor applied for his discharge. In former times the debtor was punished because he would not be released from prison until he had accounted for his actions or done what was required by the Court. The Committee reported that a majority of persons who became bankrupt did not apply for their discharge but carried on business, sometimes under assumed names, to the detriment of tradesmen who dealt with them, and that the suspension of discharge was not considered a punishment at all by the debtor. He believed there were now no fewer than 70,000 undischarged bankrupts in the United Kingdom, a manifest danger to the trading community. The number of adjudications was from 4,000 to 5,000 a year, so that it would be seen that undischarged bankrupts constituted a good many years' purchase of the annual supply of bankrupts. The Committee recommended, as had been stated by the opener of the debate, that various offences

should be punished, such as failing to keep proper books—which was already punished in Scotland and on the Continent—and unjustifiable speculation, which should be punished by imprisonment. They also recommended it should be punishable for an undischarged bankrupt to obtain credit for more than £10 (the existing limit being £20) or to trade under an assumed name ; also—and that was the strongest point of their Report—that after the bankrupt's conduct and affairs had been investigated by private and public examination he should be compelled to attend the Court on a definite date and required to give his address from time to time until that date arrived, and then he should have his conduct considered with a report from the Official Receiver, and the Committee recommended that the case should be dealt with definitely and the bankrupt punished once and for all, and that the Court should have power to make an Order fixing the date on which the bankruptcy should be closed, after which date the trustee's right to property should be at an end. The method by which a man was discharged or imprisoned would not make a black sheep white, but it would get rid to a great extent of the various difficulties which arose in connection with the property of undischarged bankrupts. It seemed really that it would be an improvement in the law to whitewash a man and give him power to resume his rights of citizenship and to trade. At present a man might trade in an assumed name and people might do business with him without knowledge of his bankruptcy, which was a very serious matter, especially in regard to matters where the group of decisions headed by *Cohen v. Mitchell* did not apply. It might be noted that in some of the Colonies a bankrupt was bound to apply for his discharge within a fixed period, otherwise he was guilty of contempt of Court.

Dealing with the special point Mr. Carter had brought forward as to after-acquired property, according to the wording of the Bankruptcy Act all property, real as well as personal, which devolved on, or was acquired by, an undischarged bankrupt, vested in the trustee in bankruptcy and the bankrupt could not make a good title to it in favour of a purchaser or mortgagee. Like other things in the Bankruptcy Act, that provision seemed to the layman perfectly clear, but the decisions showed that an honest sale or a mortgage by a bankrupt of after-acquired personal property, even when the purchaser or mortgagee had knowledge of the bankruptcy, was protected from a subsequent claim by the bankruptcy trustee ; transactions in real property were not, however, so protected. The 1908 Committee reported that there seemed no reason for the distinction between the two classes, and that the rule of law operated to create hardship on mortgagees and purchasers in respect of dealings with the real estate acquired by an undischarged bankrupt, possibly in an assumed name. This might be a serious thing. An undischarged bankrupt could obtain real property in an assumed name and sell it, and the purchaser might lose his title through no fault of his own. Here was another instance of the advantage which would accrue if the number of undischarged bankrupts were compulsorily diminished.

In the case of a policy on the life of an undischarged bankrupt, he supposed there was protection by the notice that had to be given under the Policies of Assurance Act, 1867. The Committee recommended that a person who took a mortgage from an undischarged bankrupt, whether real or personal, acquired by or developing on the bankrupt during the continuance of his bankruptcy, should be protected from any claims by the trustee in bankruptcy; and they also recommended that the debts incurred after bankruptcy should rank first on after-acquired property. With regard to this latter recommendation, under the present rule a man who had not received his discharge might go on dealing with after-acquired property, and any money he made would go to his creditors and the people who had dealt with him when he was trading would lose their money. With regard to a voluntary settlement, the Bankruptcy Act appeared to a layman absolutely clear that it would be void against a trustee, but it was interpreted that it was only void against the trustee if the donee had not dealt with it for value before an act of bankruptcy on the part of the settlor. That was the decision in the "*Carter and Kenderdine*" case. With regard to Section 47, Sub-section (2), he might be permitted to again quote from the 1908 Report. It was there pointed out that "A covenant to settle future property is generally void as against the creditors of the settlor, unless the property has been actually transferred before the bankruptcy of the settlor; but it has been held that this last-mentioned provision does not apply where the covenant is to settle a large sum of money without specifying the source from which it is to come. The evidence before the Committee does not disclose many complaints that the law is not sufficiently protective of creditors' interests, but it has been pointed out to the Committee that by means of a covenant by a settlor to settle property, the creditors of the settlor, if he becomes bankrupt, may be defeated if on the eve of bankruptcy he succeeds in transferring the property, and that a wholly unreasonable covenant in a settlement to pay a large sum of money may give to settlement trustees, or to the wife of the settlor, a large right of proof for voting and dividend, which may be used to the prejudice of trade creditors." The Committee went on to recommend certain provisions counteracting the effect of these provisions of the Act. It seemed a serious thing that a wife or trustee should be admitted to proof under a general covenant. There were various other recommendations of the Committee, amongst them questions affecting married women, who seemed very troublesome people when they became traders. The case of *Moss ex p. Hallet* (already referred by to Mr. Hicks) was interesting, although he doubted whether it bore much relation to the insurance companies' business of loans upon personal security. It appeared that Hallet was the first mortgagee of the policy and Cooke the second mortgagee, Hallet having guaranteed payments of premium and interest in the case of Cooke's mortgage. Cooke appeared to consider that the policy was worth £100 after satisfaction of Hallet's mortgage, but as Hallet threw up his security and proved for the debt he appears to have thought that his security was worth

less. Perhaps some further explanation of the circumstances could be furnished.

The PRESIDENT, in moving a hearty vote of thanks to the author for his paper, said the amount of time which Mr. Carter had bestowed upon the subject, and the clearness with which he had marshalled his references, entitled him to the gratitude of the members. It had occurred to him, when glancing at the paper and listening to the discussion, that the paper might be regarded in two aspects. In its first, Mr. Carter's paper was a sort of compressed and compendious essay for the instruction of students who were preparing themselves for Part IV. of the examination, and also, perhaps, for the convenience of examiners who wanted to find questions by which to test the student's knowledge. The second aspect of the paper was that it might be, to some extent, a guide and aid in their practical dealing with affairs. From the first point of view the paper deserved nothing but praise. It was undoubtedly very convenient that students should be able, in the course of an hour or two, to see pretty much what would be required of them on the subject of bankruptcy, which, after all, was not the main or one of the main streams of their subject, but was a sort of back-water in which they could not be expected to spend much time. He thought the students would be very much indebted to Mr. Carter and also to Mr. Barrand, and no doubt to those who had taken part in the discussion that evening, for having placed in a convenient form practically all they needed on the subject for the purpose of their examination. But passing to the second aspect of the paper, and looking at it as a practical guide to the decisions which had to be come to in actual work, as a sort of partial substitute for "Every Man his own Lawyer," it was quite right that a note of warning should be sounded, as it had been sounded by Mr. Dunn. The fact was that such papers could only provide interesting light-reading and could not be resorted to in solving difficulties. They gave a useful amount of general, but not of expert, knowledge, and those who referred to such a paper in order to solve a difficult and complicated case involving any considerable sum of money would obviously be going to the wrong source and would not be putting Mr. Carter's paper to the use which he would claim for it. When it came to a question of advising companies as to the risks they should run and as to the steps they should take, it was not the actuary but the legal adviser who should be responsible and who should study up the matter with the thoroughness which his calling in life required him to give to the subject.

The vote of thanks having been cordially agreed to,

Mr. CARTER, in reply, said he was pleased to see there had been so much discussion, as it had enabled a good many views to be put forward. Mr. Simmonds had referred to the fourth question in the summary as to whether commission on new business introduced after bankruptcy would be treated as personal earnings. He was doubtful on the point, but would mention that while Mr. Simmonds said that he had not found a case to support his view, viz., that

such commission would be treated as personal earnings, the case referred to in the paper, *Mercer v. Vans Colina*, was much to the point the other way. It was not a life assurance case, but referred to a man who habitually introduced purchasers of houses to vendors ; he had no office or clerks and carried on nothing in the nature of a business. It was not like the case of *In re Rogers ex p. Collins*, where a man put his personal time into the work of a dental business in partnership with someone else. It seemed to be very much on all fours with a man who introduced life business, and it was therefore difficult to say that new commissions would be personal earnings. He was unable to follow Mr. Simmonds' references to voluntary settlements. His own point was whether a purchase of an interest under a voluntary settlement made in ignorance that an act of bankruptcy had been committed by the settlor came under Section 49, which protected certain transactions taking place after an available act of bankruptcy, but in ignorance of it, and before the receiving order.

He was very much obliged to Mr. Dunn for calling attention to the fact that he had omitted to mention the penalty attaching to an undischarged bankrupt obtaining credit. As several subsequent speakers had pointed out, the penalty did not altogether prevent bankrupts doing so, and probably the reason why so many people who broke the law in that respect got off scot free was that it was very often not worth while to prosecute them owing to the cost of the prosecution. Mr. Hicks had raised a question on the case of *Moss ex p. Hallet* and Mr. Tilt had subsequently referred to it. He had taken the same view as Mr. Tilt, viz., that Hallet had the first charge on the policy and that Cooke was proving for what was left after the satisfaction of Hallet's charge ; but he could not quite see why Hallet proved at all. Hallet seemed to be trying to prove for everything, and several of his proofs were disallowed. With regard to the possibility of an office having had notice of a bankruptcy in respect of a prior policy which had lapsed, in the point brought out by the opinion of counsel there was no question of the office having had notice which had passed out of sight in some such way as Mr. Penman had suggested. There was no doubt they knew the man was an undischarged bankrupt and the question was, with that knowledge could money be lent to him ? With regard to what would have happened if the opinion had been asked in respect of paying a claim instead of making a loan, it was really very difficult to say, especially as the case *In re Bennett* rather supported Mr. Penman's view. In that case the office was not proceeded against, but the administrator.

Mr. Tilt had referred to the first question in the summary, and he was bound to say he was rather disappointed that someone had not been able to settle that question. As Mr. Tilt observed, it was very hard if, in a case like that, the purchaser for value of an interest could not be protected, but he could not see that he was protected. With regard to the point raised in connection with a decision of Mr. Justice Wright, he himself had taken it the

distinction was that in the case of *ex p. Bishop* the money was not earmarked and therefore proof could be made ; but in the case of *In re Reis* there was a covenant in a marriage settlement to settle all after-acquired property, except business assets, and such a covenant would come under Sub-section (2) of Section 47, and therefore would not be provable in bankruptcy. The Court of Appeal, in the case of *In re Reis*, was not called upon to decide that question. The case was really decided by the Court of Appeal agreeing with Mr. Justice Wright on a question of fact, as to whether the man had committed an act of bankruptcy or not, and they only more or less incidentally said that they agreed with Mr. Justice Wright's statements on the question of law.

With regard to the President's remarks, he fully concurred as to the undesirability of settling any difficult points in practice by referring to his own or any other paper of a similar character. He had tried not to be dogmatic and to state the case for and against as completely as possible without committing himself to a definite opinion. He did not actually say so in as many words, but he hoped that the way in which he had referred to doubtful points, and especially to the fact that he put the summary in an interrogative form, would show that, as far as the paper was concerned, those points were undecided and that no proper solution had been laid down.

Notes on the Construction of Mortality Tables. By W. PALIN
ELBERTON, F.I.A., *Actuary of The Star Assurance Society,*
and RICHARD C. FIPPARD, F.I.A., *of the Prudential Assurance*
Company.

[Read before the Institute, 25 March 1912.]

I.—INTRODUCTORY.

THE following notes are an attempt to show

- (1) How select and aggregate rates of mortality can be found from insurance office data by the methods at present used to find rates of mortality from censuses and the records of deaths among the general population, and
- (2) How the census method can be modified in the case of a prolonged investigation to give a continuous mortality investigation.

We have also tried to deal with a few of the objections to the method, and some of the practical points that arise in its application.

II.—METHOD OF FINDING SELECT AND AGGREGATE RATES BY CENSUS METHOD.

If we know (1) the population at age x last birthday on a given date, say 1 January 1912, (2) the population at age x last birthday on 1 January 1913, and (3) the deaths at age x last birthday during the calendar year 1912, then the rate of mortality is :

Deaths \div (Mean population plus one-half deaths) :

and the interval between the two censuses being only one year, the error in using the arithmetical mean of the two populations would be small, especially if the population had not shown much variation during the year.

A priori this method would apply equally well to the whole community or to any town, or to some other subdivision of the population, and would give accurate results.

The method could, therefore, be applied to that part of the community which has been assured in the various insurance offices for more than ten years, and the result would give a table similar to the O^M table. It could also be applied to those persons who have been assured between n and $n+1$ years.

Let us take this last case as an example and see to what conclusion we are led :

No. of persons who have been assured for more than n years but less than $n+1$ years on 1 January 1912	10,252
Do. do. 1 January 1913	10,538
Mean Population	10,395

No. of deaths among persons who have been assured for more than n years but less than $n+1$ years during 1912	106
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$$\text{Rate of mortality for duration } n = \frac{106}{10,395 + 53} = \cdot 0101$$

Let us take this a little further and assume that the figures given above relate to lives aged 30 last birthday only, then we have found $q_{\overline{30-n}+n}$.

It will be observed that in this work we require to be furnished with the particulars at the ages last birthday, and the result then gives the rate of mortality for exact age 30. If particulars

were only conveniently available for the nearest age (x) the final result would be the rate of mortality at an age six months younger ($x - \frac{1}{2}$), since lives of nearest age x are between $x - \frac{1}{2}$ and $x + \frac{1}{2}$.

III.—MODIFICATION TO SUIT SPECIAL CIRCUMSTANCES.

If combined investigations were made on these lines for a number of offices there would be little difficulty in the offices furnishing particulars to the investigating body of the number of policies at risk in the various valuation age groups for whole life assurances, but it would be more difficult to furnish particulars for Endowment Assurances as these are generally grouped according to unexpired term, and in order to supply the information, the cards would have to be sorted out of the order in which they are kept. This could be done once without much difficulty, but it would be troublesome to do such work yearly.

A modification could be made, however, in the following way :

The numbers could be furnished in age groups on, say, 1 January 1912. Short particulars of the new entrants and exits could then be sent to the investigating body from time to time, The investigating body would then use this information much in the same way as it is used in the office class books for whole life assurances and so form appropriate groups for various ages at the end of successive calendar years. This is simply a matter of book-keeping and is not laborious.

IV.—SUGGESTED METHOD OF OBTAINING PARTICULARS.

As at 1 January 1912 (say) the contributing office or offices might return to the investigating body a schedule (Table I) giving the number of all single life policies according to their attained age (see subsequent remarks, however), the following cases being excluded :

- (1) Cases having extra premiums or debts.
- (2) Cases in which there is no medical examination, *e.g.*, Children's Deferred Assurances, which, with other non-medical cases, such as Double Endowments, might be returned separately.
- (3) Female lives, if practicable.

TABLE I.

Number of Policies in force on 1 January 1912.

Age last birthday on 1st Jan. 1912	DURATION										Over 10 years in force
	Less than 1 year in force	More than 1 and less than 2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	
...											
54	500	396	320	12,250
55	512	401	385	318	10,260
56	501	485	400	325	9,127

* Or valuation age if preferred, provided method of finding this age is given: this might save offices the trouble of re-grouping their valuation cards for the particular purpose of the investigation: allowance could be made for different groupings during the investigation.

During the next 10 (or 5) years a card in a form provided could be sent in for each new entrant and for each claim, withdrawal, revival or maturity. Such a card should give (a) date of entry, (b) date of birth, (c) date of exit, (d) cause of exit, (e) policy number; (e) would only be required for new business if a specialized investigation for family history, etc., were being made concurrently with the ordinary investigation, and in these circumstances a fuller card would be necessary for new entrants. These cards could be sent in monthly, quarterly, half-yearly or yearly. The investigating body could, by means of books similar to ordinary continuous class books, keep a record of the "exposed to risk."

V.—SUGGESTED METHOD OF TABULATING EXPERIENCE.

The tables required are—

- (1) An ultimate table (say, excluding 10 years).
- (2) A select table for, say, the first 10 years of assurance; or what is, of course, the same thing, 10 separate tables of mortality—one for the first year of assurance, one for the second year, and so on.
- (3) An aggregate table—which can be found from (1) and (2), so that it need not be dealt with further.

It should be borne in mind in considering the schedules which follow, and are the same as class books in principle, that they are simply used to find the actual number of policies in force in each "duration" and "year of age" on 1 January in each year; i.e.,

the Population at that date. They are, in this respect, exactly similar in principle to the valuation class books of almost every office. We attach no importance to the particular form; it can doubtless be improved; but it may be useful to show at the present juncture that some form can easily be devised.

The numerical figures inserted in the table may assist the reader in following the method, and the particulars for two consecutive ages have been inserted so as to indicate the continuity of the process (*see* Tables II and III). The following table (Table IV) gives the tabulation of claims; the numbers would be obtained by sorting the cards sent in each year.

TABLE II.—*Showing Populations each year. Age last birthday 55.*

	DURATION									
	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	Over 10
Population* 1 January 1912 (see Table I.), age 55 last birthday ...	512	401	385	10,260
Carried forward from previous duration, age 51 last b.d. on 1 Jan. 1912	506†	500	396	12,250\$ 320
Loss—										12,570
Withdrawals in 1912‡ out of carried forward ...	20	60	50	18	57
Claims in 1912‡ out of carried forward ...	6	15	10	12	180
Carried forward ...	—	26	—	60	30	237
Population 1 January 1913, age 55 last birthday ...	480	425	336	12,333
Carried forward from previous duration, age 51 last b.d. on 1 Jan. 1913	515†	470	440	11,195 360
Loss—										11,555
Withdrawals in 1913 out of carried forward ...	18	52	40	15	48
Claims in 1913 out of carried forward ...	7	8	8	11	170
Carried forward ...	—	25	—	48	26	218
Population 1 January 1914, age 55 last birthday ...	490	410	392	11,337
&c.	...	&c.	&c.	&c.

* For sake of clearness we have used the word "Population"; alternatively, "existing" might be used.

† Number of policies effected.

‡ For the purpose of finding the population aged 55 last birthday at the end of the year from the population aged 51 at the previous duration, we must deduct only those who died or withdrew among the number in the population carried forward. This is easy to follow if it is borne in mind that we are merely using the figures as book-keeping entries to enable us to find the population at the end of the year. The claims, &c., so deducted may be among some people aged 54 last birthday, and some 55 last birthday at death, and will be different from the claims appearing in the fraction giving the rate of mortality; these latter are all the claims during the calendar year for the duration and age last birthday under consideration (see Table IV). The withdrawals are throughout net, *i.e.*, withdrawals *less* revivals.

\$ The first entry, 12,250, represents the carried forward from 54 durations over 10; and the second entry, 320, the carried forward from 54, duration 9-10.

TABLE IV.

Claims arising at Age 55 last birthday at death.

Calendar year of Death.	DURATION AT DEATH.										
	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	Over 10
1912	12	11	155
1913	10	13	170
1914 &c.	8	12	162
Total	C_0	C_1	C

The figures for "populations" (existing) and claims can then be used for finding the rates of mortality. Thus, for example, taking age 55 we could abstract the populations as follows:

TABLE V.

Populations abstracted for finding rate of mortality. Age 55 last birthday.

Date	DURATION.									
	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
1 January										Over 10
1912	512	401	385	10,260
1913	480	425	336	12,333
1914	490	410	392	11,337
&c.										
1921	436	510	500	13,000
Total.	P'_0	P'_1	P'_2	P'
Less $\frac{1}{2}$ first term	256	200.5	192.5	159	5,130
Less $\frac{1}{2}$ last term	218	255	250	180	6,500
	— 474	— 455.5	— 442.5						— 339	— 11,630
Mean Population	P_0	P_1	P_2		P

The rates of mortality required are :

$$q_{[35-r]+r} = \frac{C_r}{P_r + \frac{1}{2}C_r}$$

$$q_{55} \text{ (ultimate)} = \frac{C}{P + \frac{1}{2}C}$$

We have suggested in the footnote to Table I. that the method can be applied when particulars are not furnished at ages last birthday, and we have worked out approximations suitable to various circumstances. As, however, we are concerned here with the principle of the method, it is unnecessary to go into the details of these modifications, and we have confined our attention to the case in which the facts are given in a form that most nearly corresponds with that of census data.

VI.—CRITICISM.

It will not unnaturally be asked why we suggest the adoption of a "census" method when methods that are considered more exact have been used in the past. The answer to this criticism is that the methods formerly adopted are open to some objections which it is, perhaps, advisable to indicate here.

When investigations have been made in the past the procedure has been to obtain as full particulars as possible, from one or more offices, of policies that have been on the books at any time during the previous twenty or thirty years.

The objections to this course are—

- (1) The information takes some years to collect and investigate, so that the results are more or less out of date when they are ready for use ; *e.g.*, O^M tables, when published, were about 10 years old if reckoned from the end of the period of observation, or 25 years if reckoned from the middle of the observation period.
- (2) The work is discontinuous and has to be begun again for each investigation.
- (3) The data, so far as they relate to "select" material, can hardly be considered entirely satisfactory as methods of selection change considerably in 25 years.
- (4) The method means an amount of work both to the contributing offices and the investigating body, which can, we think, be decreased.

These objections can be summarized in the one word, "delay"; and nearly all the delay is due to delving into the past which is a lengthy and troublesome, and, consequently, expensive proceeding to almost every office.

The argument in favour of the present suggestion is that by fixing a date for the "start of observations" instead of for the "close of observations," the contributing offices would be put to less trouble and the investigating body would be enabled to get out its final results as easily as and more rapidly than under the present usual conditions.

It will be noticed in Section IV that the only cases which we suggest omitting are those relating to extra premiums and unexamined lives, and it has been assumed throughout that the number of policies will be given. In our view this is the most satisfactory basis, but quite apart from this opinion there is a gain in simplification and the small difference likely to arise if "lives" or "amounts" are used instead of policies makes the additional labour unnecessary.

It may seem that policies by way of reinsurance should be excluded: for the sake of simplicity we do not think this need be done. The arguments for and against their exclusion are similar to those for and against the elimination of duplicates.

Another point against which criticism of our suggestion might be directed is that we have implied that selection need not be traced beyond 10 years. Of course, our method could be modified to meet this criticism, but we feel that even if select tables did not run into ultimate tables at the end of 10 years, or earlier, the resulting difference could be overcome comparatively easily by a graduation process.

VIII.—SEPARATION OF CLASSES.

When the last experience of the British Offices was taken out the various classes were separated, and while this was of value and interest then, we are rather inclined to feel, for the following reasons, that it might be more useful to amalgamate all classes in obtaining a general experience. This is, however, independent of the particular method suggested as there would be no inherent obstacle in the way of dealing with all classes separately.* The separation of policies under various classes is presumably based

* In the same way the experience of each of the contributing offices could be kept separate if desired. If the various contributing offices gave their data on different methods this would probably be necessary.

on the theory that the mortality under certain tables will always differ, and that the business of each table is more homogeneous than the business of all tables combined. The answer to this is :

- (1) When the experiences of various offices are amalgamated some heterogeneity must be introduced, whatever course we follow.
- (2) There probably is considerable change in the incidence of business being done in various classes; *e.g.*, there may be a tendency for Endowment Assurances which were previously "provident" policies to be required in certain loan business.
- (3) The difference between the mortality at the last investigation under Endowment Assurances and under Whole-Life New Assurances is about the same as the difference between the Whole-Life New Assurances, and Whole-Life Old Assurances, even although the Endowment Assurances were on the average selected at a later date than the Whole-Life New Assurances. It does not seem to us that greater homogeneity was secured by amalgamating old and new policies than would result from the amalgamation of Whole-Life and Endowment Assurances.

The separation of tables opens up the further criticism that offices are now valuing endowment assurances, non-profit business, &c., by a table from which such business was particularly excluded and they are calculating endowment assurance premiums, &c., in a similar way.

VIII.—CONCLUDING REMARKS.

The methods given above have been suggested with the following practical points* in mind :

- (1) That mortality investigation should be up to date and therefore continuous.
- (2) That contributing offices should not be asked to undertake a large amount of preliminary work on expired policies.
- (3) That by arranging that practically no past information is furnished the labour to contributing offices would be spread over the time covered by the investigation.

* Though our methods are different, some at least of these points were clearly in Mr. G. King's mind when he gave his Inter-valuation Formula for the Exposed to Risk, *J.I.A.*, vol. xxvii, p. 218.

This would be more satisfactory both to the contributing offices and the investigating body.

(4) That refinement in details should be avoided.

(5) That the work should be so arranged that the investigation can be joined on to an investigation of particular classes of experience (impaired lives, &c.).

The methods we suggest may be open to many objections, but they would enable us to know the rates of mortality that have been experienced during any period within six months of the moment when that period ended.

ABSTRACT OF THE DISCUSSION.

MR. GEORGE GREEN said the object of the paper was to describe a process of keeping a continuous record of mortality for contributing offices in the form of Select Tables. There were certain difficulties of detail in the method adopted, one of which would arise in dealing with the commencement of the period of observation. Apparently they would have to take the business in force and analyse it, so as to obtain exactly the numbers which, at a given date, had been insured for a given number of years, and many offices, he believed, would find it not very easy to do that without considerable examination into records. Before considering the details, the broad question of principle might be considered, whether it would be useful for the companies to agree to have an investigating body continuously at work, and on that he thought the authors made a very good case. There was very little doubt that some system could be adopted and easily maintained without a very great expenditure of time or money, and it would, as the authors said, give a continuous record of mortality available for use without the necessity for ten years' preparation at any time such as was experienced in the case of the Oⁿ Tables.

It might be possibly of interest if he were to put forward an alternative method which, he thought, could be easily applied to the keeping of a continuous record. The method as applied to Select Tables had actually been in use for a time.

Considering only policies effected in a given year a list of their numbers arranged under the nearest age at entry was made, thus :—

Year of Entry 1908.

[25]	[26]	[27]	[28]
2,053	2,051	2,058	2,056
2,076	2,062 W 2	2,059	2,057
2,087	2,063	2,060 D 5	2,086
⋮	⋮	⋮	⋮

The lists were corrected year by year for lapses and surrenders which were ruled out and the nearest duration of exit recorded thus: W 2 where 2 is the nearest duration and also for deaths in a similar manner, except that in this case the curtate duration was recorded. From these lists the data for working out the exposed to risk were obtained year by year, and thus gradually a set of Select Tables in respect of each year of entry was derived.

From these tables a summary would have to be made, bringing together the experience of the successive years of entry. That was an alternative method of getting out Select Tables by a continuous process, and it seemed to him that in some respects it was rather better than the method set out in the paper. The Population Table method adopted in the paper seemed to him to introduce a certain amount of confusion between durations and ages, and he believed it would give results not quite in accordance with a strict year of duration method. The deaths would be scheduled under years of duration correctly, but he thought the exposed-to-risk would not be quite so accurately obtained. It might be found possible to combine in one and the same operation, the getting out of the expected claims by an office for its own use and the keeping of a continuous record by combined offices, and if a few of the offices would assent to that course, and would agree upon a simple form of card to enable them to do it, he thought it would accomplish the objects of the authors with very little trouble. It had been the practice of his office to get out the expected claims year by year by means of the valuation cards, and if duplicates of those cards were sent to the investigating body it would enable them to get out practically any records they wished. The cards were simple and the writing of duplicates would be a matter of very little trouble.

There would be some points of detail with regard to which care would have to be exercised. There was sometimes considerable difficulty in determining the date of exit in the case of lapsed policies, as most policies now were subject to non-forfeiture regulations and it was not always an easy matter to say whether an office was definitely off the risk or not. Endowment assurances also offered difficulties because of the way in which many offices grouped them for the purposes of valuation. In connection with that it would seem that the best and simplest way would be to start afresh from the present time, and to furnish cards to the central body in respect of endowment assurances as they were issued. Those could be then arranged under the dates of birth, and the records could be obtained in the form required for mortality purposes. On the question of confining the investigation to numbers of policies only, and taking no account of amounts, he thought that experience in watching the expected amounts as compared with the actual amounts of claims would confirm the authors' view. He had looked at some records in regard to the matter, and had found that, taking a series of thirteen years, in every one of them, the expected number of claims was considerably above the actual

number, but that that was by no means the case with the amounts of the claims. In no fewer than five out of the thirteen years the reverse was true, and yet, on the whole, the ratios of the expected to the actual number and the expected to the actual amounts in the whole period of thirteen years were practically identical. That showed that the amounts were in themselves much more liable to violent fluctuations, and were therefore less suitable as a basis for rates of premiums or valuation factors. The main difficulties of the scheme, once the principle was settled, would be the working out of the details, and that would be really better done by a committee in consultation than by general discussion, because many of the arrangements would depend on the form in which the various offices could supply the data required.

Mr. C. W. KENCHINGTON thought that the advantages to the individual offices themselves of tabulating their own experiences year by year and being able to compare their experiences with the current standard of the combined offices were inestimable. It appeared to him that it would be desirable to separate the assurances into their respective classes. The authors suggested that endowment assurances might be combined with whole-life assurances, and some evidence was given indicating that the combination of the old and the new assurances in the whole-life with-profit section of the British Offices Experience introduced as great an error as the combination of whole-life assurances and endowment assurances would have done. While that might be so, he was not at all certain that it would not be advisable to have an entirely up-to-date investigation of the mortality of endowment assurances. It was generally thought that the rate of mortality, as shown by endowment assurances in the British Offices Experience, was abnormally low, and that the experience which had since been obtained in that class would tend to show rates more nearly approaching those of the whole-life assurance business. As to that it was impossible to speak definitely at the moment, but he had recently seen a large body of experience of endowment assurances, which, although he was unable to give details, entirely confirmed the rates as brought out by the British Offices Endowment Assurance Experience. That alone, he thought, would be a reason for separating the endowment assurances from the whole-life assurances. The authors also suggested that female lives should be excluded if practicable. He himself thought it was absolutely essential that female lives should be separated from male lives for the purpose of mortality investigation.

One point to be borne in mind in the suggested method was that, as the particulars were to be entered in books in the form of class books, an error introduced would not be eliminated until extreme old age, and, in consequence, great care would have to be exercised in the tabulation, and also every effort would have to be made by the offices themselves to give absolutely correct data. It was suggested that the experience should be obtained on the basis of policies, not lives, and the question of re-assurances therefore

arose. He thought it was perfectly justifiable to include all policies issued at different dates on the same life, but where a number of policies were effected at the same time, as was the case where there was a number of re-assurances, it would be desirable that the re-assurances should be eliminated. He did not think there would be any serious difficulty in carrying that out practically.

Mr. ERNEST WOODS said that a special debt of gratitude was owing to the authors, for at the last meeting the President had been obliged to announce that, owing to unforeseen circumstances, the paper which was to have been read that evening could not be produced, and in these conditions the authors had hurriedly written what they had heard. He did not propose to criticise the paper, but, like Mr. Kenchington, he had been struck by the proposal to treat policies as lives, and he would ask the authors whether they had made any investigations as to the effect of this method of procedure? He had taken two pages at random from the Index of his company, and on one he found sixty-two policies on forty-four lives, and on another fifty-eight policies on forty-one lives, and if these were fair samples of the whole business, it pointed to an average of, say, 1.5 policies to each life. He found on one page ninety-nine policies on forty-one lives. What would be the probable variation from the truth if their plan was adopted? He noted that for the particular purpose in view the plan of grouping endowment assurance policies according to year of maturity was unsuitable. Would not this be another argument in favour of the use of Altenburger's method?

The paper was most interesting, as it brought within the bounds of possibility the power of compiling at any time a new Table of Mortality without that long delay which had preceded the publication of the last experience of assured lives, and, from the speaker's point of view he was specially interested in the use to which its methods might be put for the purpose of that Specialised Mortality Investigation which had been foreshadowed. He would like to emphasize his own personal view that, if such an investigation were to be taken in hand, it would be better to collect statistics hereafter, and put off for awhile the preparation of tables, than to attempt to collect statistics from past experience with a view to more or less immediate results.

In the Specialised Mortality Investigation, published by the Actuarial Society of America in 1903, there were ninety-eight classes of risks—made up of twenty-two groups for place of residence in America, sixteen for weight and height, thirty-five for occupations, sixteen for medical features, two only for family history, and seven for other reasons—such as Black or White, Amount Assured, and so on. In their more recent investigation, of which the results are not yet public, they have, in addition to the necessary groups to provide a Standard Table of Heights, Weights and Build, 154 other groups—sixty-eight of occupations involving hazard, seventy-six of medical impairment, two of Coloured risks, four of Joint-Life Policies, and four of Women. Now the labour of preparing the cards for so

minute an investigation into the records of the past would in this country be enormous, since our medical reports do not, as a rule, contain such multitudinous and minute enquiries as those of our American friends. He had grave doubts as to the value of the information when elicited, having regard to the character of the records which had been open to his inspection, and he had no reason to think that the records of other offices were any better. Take, for example, the disease "cancer." How many reports gave any real information which would be useful—such as the variety or the part affected, and so on? Take, again, height and weight. How often was the proposer actually weighed and measured? Was not his mere statement often accepted as sufficient evidence? Then, again, the practice of offices varied as to the verification of the statements of family history—some were very much more particular than others. As an example, when the cause of the mother's death is given as "child-birth," are further enquiries made? In an experience of thirty years he had found that the true cause was, in ninety-nine cases out of a hundred, "phthisis."

Of course, if offices decided to examine their statistics of lives to be hereafter assured, there would be no difficulty in arranging the reports in such a form as would bring to the medical examiner's attention the special features which it might be desired to record exactly. Then, again, apart from the accuracy or fullness of the reports, other questions arose as to the value of statistics of recent years; for example, the risks of foreign residence—say, in Panama, as an extreme case. How could the experience of the years previous to the discovery of the yellow fever mosquito be any guide to future probabilities? In conclusion, he desired to add that it must not be taken that he was absolutely opposed to any examination of the statistics of previous years, as there were, no doubt, special features even in the existing records of offices which might be usefully investigated, but on the whole he was inclined to repeat that the experience of the next few years, even though not now available, was more likely to repay the work spent on it than the experience of the past, which, in any case, it would take time to collect.

Mr. GEORGE KING wished, first of all, to express his gratification at being asked by the Council to close the discussion, because it enabled him to welcome the paper as one heralding a new departure in the matter of mortality experience investigations. The paper, as had been said, was prepared in a great hurry, but evidently the authors had already the ideas in their minds and had had only to throw them into shape. They themselves, however, would probably admit that if they had had more time they could have developed the paper further and made it even more interesting than it was. The new departure which was heralded by the paper was a continuous collective mortality investigation, and he cordially agreed that if that could be set on foot it would be most useful. To be able to make a start at a given point and year by year, or quinquennium by quinquennium, get in select form the mortality experience for each of those periods of the contributing offices, and in such form

that the whole of any part could be brought together to make one table, would be a very fine achievement. He thought the authors were not wrong in saying that it could be done with very little trouble and without great expense, and he would return to that again after saying a word or two on the paper itself. In the first place, he was rather puzzled to find that the authors claimed that the method they proposed was a census method. When he read, at the beginning of the paper, that it was intended to show how select and aggregate rates of mortality could be found from insurance office data by the methods at present used to find rates of mortality from census returns, and how the census method could be modified, he thought something entirely novel in that respect was about to be put forward, but he was afraid he could not see it. It was a very remote resemblance at the best. That, however, was a trifling point, which he mentioned in passing.

With regard to the details, he was afraid the companies would find considerable difficulty in starting as proposed by the authors. To get the age last birthday of the lives on their books, or, perhaps, some modification of it, would be very difficult, and the authors had themselves pointed out the difficulty with regard to such a class as endowment assurances. Whether it would not be possible to have a more complete record in another way without more trouble he thought was worthy of enquiry. He was pleased to see that the authors were quite content to adopt certain approximations. They said, near the close of their paper, that refinement in details should be avoided, and with that he cordially agreed. If he might be excused for bringing up a subject not strictly germane to the present discussion, he should like to illustrate the point by referring to the modified nearest duration method followed in the British Offices Experience. Since that Experience was brought out he had been thinking constantly of writing a note upon it for the *Journal*, but had never found time to do so, and, therefore, might be permitted in a few words to give the pith of his views on that point. It was said in the volume of "Principles and Methods" that the object of the modification of the nearest duration method was to show for each age at entry or group of ages at entry the actual incidence of the cases withdrawing in each year of assurance. He would point out, however, that that object was not achieved. It was only by the Summary Tables at the end of the volume of "Unadjusted Data," and by one or two illustrative examples in the volume of "Principles and Methods," that it was possible to find any real indication of the incidence of withdrawals, and that indication was incomplete and defective. If there were no such modification it would be quite easy, with a very small addition to the tabulation, by the nearest duration method, accurately to allocate to each policy-year the withdrawals. All the trouble involved in the modification would, perhaps, have been justified if there had been any effect upon the rate of mortality, but he had taken the trouble years ago to work out pretty well every example that was available to see what effect the modification had upon the rate of mortality, and he found that the modification

never altered the rate of mortality by more than 3 in the fifth place of decimals. Why all that trouble should have been taken for so minute a result when the graduation sometimes involved a difference of as much as 2 in the third place of decimals he could not imagine. They had worked hard to correct 3 in the fifth place of decimals, and they thought nothing of changing by graduation the third place by 2; it was surely straining at a gnat and swallowing a camel. It should be remembered in that connection that there were over 260,000 withdrawal cards, each of which had had to be examined and then marked with the appropriate *W*. Then for each entry age and against the proper duration those withdrawals had had to be scheduled in their four columns, and then they had to be modified, and the results were those given in the final tables as published. He believed the Experience would have been got out years sooner had the nearest-duration method pure and simple been followed, and that would not have altered the ungraduated *q* by more than 3 in the fifth place of decimals in any case.

He had taken that opportunity, he hoped not improperly, just to make those two or three remarks. Without criticizing the paper, there was no reason why he should not throw out some other ideas, and he would suggest that if the companies would prepare for existing business cards in the simplest form—the form mentioned by the authors for new business—and send these cards in to the central authority, one for each policy on their books at a particular valuation, that would cause really no more trouble than getting the particulars the authors asked for, and it would be done once, for all, without the necessity of going into past records. It simply required the writing of a card for each existing policy. Then it would be very simple, by getting similar cards for new policies and for those that went off the books, to have tables in select form year by year prepared by the central office, and those tables could be kept up just as the authors proposed by their method. The tables could be prepared by policy years, and they could fit in exactly with the British Offices Tables, the only difference being that the British Offices Tables had the modification in respect of withdrawals, which he put aside as quite unnecessary. The question of what central body should take the matter up was one that would require consideration. The first start would involve some little trouble on account of getting in the returns of existing business from the contributing offices. Any company might, later on, join in by simply sending in the cards for its then existing business. Once, however, the thing had been started there would be very little trouble, and a short time in each year would be sufficient to get the Experience ready and tabulated in schedules. He looked forward to the time, not many years hence, when there would be a large number of consulting actuaries—under the National Insurance Act they were bound to come—and it could be easily arranged with one of them to take up the work for a moderate fee, depending on the number of cards each year. So far as he could see, with very little trouble and very little cost, a continuous mortality experience in a select

form could be obtained, fitting in almost exactly with the British Offices Tables.

There were one or two minor points in the paper he thought of interest, one of which was quite new to him, namely, that the proportionate difference between the mortality of endowment assurances and new whole-life assurances was almost the same as that between new whole-life assurances and old whole-life assurances. He had not known that before, but he had now looked up the point and found it was very remarkably so. It was a curious thing, and he was glad to see it noted. With regard to Mr. Woods' remarks as to the American experience, he was in New York last October and heard a good deal about the very elaborate experience the Americans were taking out. He did not know whether Mr. Woods was aware of the real reason for the course they were following. They had a system in America of taking defective lives by a kind of average rule. The idea was to pick out the unfavourable and the favourable points in the medical examiner's report, and refer to the same points in the mortality experience, and thus to obtain a kind of index figure. The Experience would give figures for each of the points, and some of the figures would be positive and some negative, and by adding algebraically the figures thus derived, it would be possible to get a figure measuring the combined weight of the various features, some adverse and some favourable, and that would give at once the rated-up age at which the life could be taken for insurance. American offices did such business wholesale, and while the method did not actually, in every case, give the proper loading, yet, on the average, he had no doubt whatever it answered fairly well. It would not suit Great Britain, where, often, under-average lives had to be assured for large amounts and the policy had to be re-insured, and consequently the rating had to be carefully adjusted for each particular case. In America they had no reversionary transactions, and the American method, while it did not give the exact rating suitable to each individual case, yet answered very well on the average. He hoped a good deal would be heard about that Experience when it came out. He was surprised and gratified to find how very active and enterprising and original the Actuarial Society of America was proving to be.

The VICE-PRESIDENT (Mr. Geoffrey Marks), in moving a vote of thanks to the authors for the instructive paper they had read, said he was only called upon during the day to occupy the chair and had not had an opportunity of reading the paper, but from Mr. Fippard's summary of it, he gathered it was a practical attempt to devise means by which the staff of the Research Bureau, which it was hoped and believed would be established, could be kept supplied with information and material, without casting too great a strain on the resources of the Bureau, either as regarded the amount of work possible to the staff, or as regarded its finance. He was sure every member welcomed the paper, because they looked to Mr. Elderton to provide suggestions for carrying out his ideas of a Research Bureau, and this his skill and experience were eminently

adapted to enable him to do. One or two points had arisen in the discussion to which he might refer. With regard to the rates of mortality in endowment assurances, the belief when he was a student—and it was a belief which held for a very long time—was that the rates of mortality amongst endowment assurance policies were very much lighter than they were in any other class of insurance, and a suggestion had been made that those rates were changing. If that was so—although Mr. Kenchington said that so far as his observations went it was not the case—he might suggest that possibly it was due to the fact that the convenient but unscientific practice of granting an endowment assurance policy to an under-average life had been used as a method of loading the life. It was felt, perhaps, that the proposer could not be taken at ordinary rates under a whole-life policy, but that the risk of his dying during the endowment assurance period was very little, if any, above the average, and that he would have paid rather more, if he happened to die, than would be the case if he were insured under a whole-life policy. The remarks of Mr. King as to the effect of the modification in the Institute Mortality Experience confirmed an idea which he had always had, that for such purposes as the authors indicated it was futile to go into minute differences, having regard to the fact that the whole structure of their tables was founded on approximations. To attempt very exact methods which hardly altered the results at all, and yet involved an enormous expenditure in work and great delay, was open to serious objection from the practical point of view, since, as had been pointed out, while the investigations were being made, the whole basis on which the superstructure was erected might change.

There was a point which had occurred to him when he had the honour of presiding at one of the Students' Debates the other evening. Ever since 1848 there had hardly been a speech made in the Institute which had not opened with a more or less graceful compliment to the author of the paper. He really thought the time had come when the practice might be adopted which was enforced by rule at the Students' Society, that no speaker should be allowed to pay any compliment to the author of the paper. It went without saying that the members were extremely grateful to every gentleman who read a paper, not only for the trouble he had taken in preparing it, but for the information he was good enough to place at the disposal of his fellow members. He did not know that the Institute was in a position to pass a definite rule on the subject, but it was a point that was worth considering by those gentlemen who frequently took part in the debates.

Mr. W. PALIN ELDERTON, in reply to the discussion, said that Mr. Green had asked how it could be conveniently arranged that the particulars of the various offices could be obtained in the calendar years in which the policies were issued? He did not think there was any difficulty about that. Most offices kept their entries in their Year-of-Birth Classes according to policy number, so that the information could be easily obtained from the policy

numbers themselves. Mr. Green's point about the difficulty with regard to the date of exit in case of lapses would be common to every method that was adopted, whether a continuous method or discontinuous method.

With reference to Mr. Kenchington's remarks on endowment assurances, they left him unconvinced, because Mr. Kenchington did not say that he had compared the mortality of endowment assurances with the mortality of corresponding life policies effected in the same years. That was really the point of the question in the paper as to the separation of the endowment assurances from whole-life assurances, and Mr. Kenchington left that point untouched. Mr. Kenchington also asked whether in using a continuous method there was not the probability of having an error carried on year after year? The answer to that was that there would be a balancing at the end of one, two or five years. With regard to re-assurances, he did not see there was much reason for taking them out, his own feeling being that it would save trouble to leave them in, and there would not be much danger or harm in doing so. He did not think it was wrong to count one man n times who insured in n different offices at the same time, and re-assurances implied a similar problem.

With regard to Mr. Woods' point as to a number of lives corresponding to a larger number of policies, the only way in which that could affect the mortality was if the people who took out more policies than one suffered from either a heavier or lighter mortality. Some evidence had been produced, he believed, which tended to show that not very much difference resulted whether lives or policies or amounts were used, and if that was so he was inclined to say the less trouble taken the better, because it enabled results to be obtained with greater rapidity.

Mr. King had said that he did not think the method ought to be called a census method, but he could not agree with that, although he agreed with Mr. King in nearly everything else he said. It seemed to him that they suggested (1) a "census" of policies on a certain date; (2) the calculation of subsequent "censuses"; and (3) the enumeration of the corresponding deaths. Although he agreed with Mr. King's remarks as to the British Offices Tables, he thought they were entitled to the defence that such investigations had to be made once before it could be discovered that they were not worth making. He hoped the indirect result of the paper would be that the construction of mortality tables in future would be somewhat simplified and shortened, so that the thought and time and expense that would have gone to details might be spent in extending knowledge in what were really more useful and practical directions.

THE INDIAN LIFE ASSURANCE COMPANIES ACT, 1912.

THE full text of the above Act is given below, but for the convenience of readers of the *Journal* Mr. A. T. Winter, F.I.A., has kindly contributed the following notes dealing with the provisions of the Act :—

This is the first legislative measure passed by the Indian Government for the better control of Life Assurance Companies, and, with a few exceptions, it adheres closely to the provisions relating to Life Assurance contained in our Assurance Companies Act, 1909. It also follows, to a certain extent, the Ordinances of the Legislative Councils of Hong Kong and Ceylon in 1907 and 1911, respectively, in exempting from its operation those Companies which have made Deposits for Life Assurance under the British Acts. The only requirements in the case of such a Company are :—

At the Outset (Secs. 19 and 32).

(a) To file with the Registrar—

(1) The Instrument defining its constitution.

(2) List of Directors.

(3) The name of an Agent authorized to accept service of process or notices.

(b) To make application to the Governor General in Council for a declaration that it carries on Life Assurance business in the United Kingdom.

(c) To furnish satisfactory evidence in support of such application.

In future (Sec. 33).

To deposit copies of every Account, Balance Sheet, Abstract, Statement, or other document required to be deposited with the Board of Trade under our 1909 Act.

There are other Sections of the Act from which British Companies are not exempted, but these are unimportant to such Companies, or contain provisions with which they already have to comply under our 1909 Act.

The following Sections constitute the chief departures from our 1909 Act :

Winding up (Sec. 22b) empowers the Government to make application to the Court to order the winding-up of an insolvent Company.

The Clause apparently applies to British as well as Native Companies, although, of course, it is of no practical importance to the former Companies.

Deposit (Sec. 4) (from which British Companies are exempted).

A Deposit in Government Securities varying from Rs. 25,000 (£1,667) to Rs. 200,000 (£13,333), under circumstances explained in the Section, is required to be maintained with the Comptroller General. Apparently 3 per-cent Rupee Paper at face value will

answer the requirements of the Act, and, as the value of this is at present only about 82 per cent., the cash value of the deposit may be considerably lower than the above amounts.

Inspectors (Sec. 37) (from which British Companies are exempted).

The Government may appoint Inspectors to examine and report upon the affairs of any Life Assurance Company.

To western ideas the minimum Deposit mentioned above appears very small for the purpose required. It should be borne in mind, however, that most of the Native Companies are of quite recent formation, with small capital and funds, whose position at present does not enable them to make a deposit of any considerable amount.

An Act to provide for the regulation of Life Assurance Companies.

WHEREAS it is expedient to provide for the regulation of life assurance companies ; It is hereby enacted as follows :—

Preliminary.

Short title and extent.

1. (1) This Act may be called the Indian Life Assurance Companies Act, 1912.

(2) It extends to the whole of British India, inclusive of British Baluchistan, the Santhal Parganas and the Pargana of Spiti.

Definitions.

2. In this Act, unless there is anything repugnant in the subject or context—

(1) “ Actuary ” means an actuary possessing such qualifications as may be prescribed by rules made by the Governor General in Council :

(2) “ Chairman ” means the person for the time being presiding over the board of directors or other governing body of a life assurance company :

(3) “ Court ” means the principal Civil Court of original jurisdiction in a district, and includes the High Court in the exercise of its ordinary original civil jurisdiction :

(4) “ Financial year ” means each period of twelve months at the end of which the balance of the accounts of the life assurance company is struck, or, if no such balance is struck, then the calendar year :

(5) “ Life assurance business ” means the issue of, or the under-taking of liability under, policies of assurance upon human life, or the granting of annuities upon human life :

(6) “ Policy of assurance on human life ” means any instrument by which the payment of money is assured on death (except death by accident only) or the happening of any contingency dependent on human life, or any instrument evidencing a contract which is subject to payment of premiums for a term dependent on human life :

(7) "Policyholder" means the person who for the time being is the legal holder of the policy for securing the contract with the life assurance company :

(8) Where a company grants annuities upon human life, "policy" includes the instrument evidencing the contract to pay such an annuity, and "policyholder" includes annuitant : and

(9) "Registrar" means any person who may be appointed by the Local Government to perform the duties of the Registrar under this Act.

3. Save as hereafter expressly provided, this Act shall apply to all persons or bodies of persons, whether corporate or unincorporate (which persons and bodies of persons are hereafter referred to as life assurance companies) whether established before or after the commencement of this Act and whether established within or without British India, who carry on life assurance business within British India.

Explanation.—A company registered under the Indian companies Act, 1882, which carries on life assurance business in any part of the world shall for the purposes of this section be deemed to be a company carrying on such business within British India.

Exception.—Nothing in this Act shall apply to any society to which the Provident Insurance Societies Act, 1912, applies, or to any Fund which the Governor General in Council may, by notification in the *Gazette* of India, exempt from the operation of this Act.

Deposits.

4. (1) Every life assurance company shall, if established before the commencement of this Act, within one year from such commencement, or if established after such commencement, before it commences to carry on the business of life assurance, deposit and keep deposited with the Comptroller General, for and on behalf of the Governor General in Council, Government securities, as defined by the Indian Securities Act, 1886, of the face value of twenty-five thousand rupees or of a face value equal to one-third of the income derived from life assurance business as shown in the revenue account for the last financial year, whichever is greater ; and, until the company keeps deposited securities of the face value of two hundred thousand rupees, shall annually deposit and keep deposited in like manner like securities of a face value—

(a) equal to one-third of the income derived from life assurance business as shown in the revenue account for the last financial year, until the face value of the securities deposited exceeds one hundred thousand rupees ;

(b) and thereafter equal in amount to one-third of the increase to the life assurance fund as shown in the revenue account for the last financial year :

Provided that a company may at any time deposit securities of a face value of two hundred thousand rupees or make up its deposit of securities to that value.

(2) The interest accruing due on the securities deposited under sub-section (1) shall be paid to the company.

(3) The deposit may be made by the subscribers of the memorandum of association of a company or any of them, in the name of a proposed company, and, upon the incorporation of the company, shall be deemed to have been made by, and to be part of the assets of, the company, and the Registrar of Joint Stock Companies shall not issue a certificate of incorporation of the company under the Indian Companies Act, 1882, until the deposit has been made.

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(4) The deposit shall be deemed to form part of the life assurance fund of the company.

Accounts and Documents.

5. In the case of a life assurance company transacting other business besides that of life assurance, a separate account shall be kept of all receipts in respect of the life assurance business, and the said receipts shall be carried to and form a separate fund to be called the life assurance fund.

Explanation.—Nothing in this section shall be deemed to require any life assurance fund to be invested in separate investments from any other fund, but a separate balance-sheet as prescribed under section 7 shall be kept in respect of the life assurance fund.

Exception.—Nothing in this section shall apply to a life assurance company established before the commencement of this Act, by the terms of whose deed of settlement the whole of the profits of all the business carried on by the company are paid exclusively to the life policy-holders, and on the face of whose life policies the liability of the life assurance fund in respect of the other business distinctly appears.

6. The life assurance fund shall be as absolutely the security of the life policy-holders as though it belonged to a company carrying on no other business than life assurance business, and shall not be liable for any contracts of the company for which it would not have been liable had the business of the company been only that of life assurance, and shall not be applied, directly or indirectly, for any purposes other than those of life assurance.

Exception.—Nothing in this section shall affect the liability of the life assurance fund, in the case of a company established before the commencement of this Act, for contracts entered into by the company before such commencement.

7. Every life assurance company shall, at the expiration of each financial year, prepare—

- (a) A revenue account for the year in the form or forms set forth in the First Schedule and applicable to the class or classes of business carried on by the company ;
- (b) A profit and loss account in the form set forth in the Second Schedule, except where the company carries on life assurance business only and no other business ;
- (c) a balance-sheet or balance-sheets in the form or forms, set forth in the Third Schedule ;

- (d) A statement containing the name of every person who during the year was a member of the board of directors or other governing body or was manager or secretary or held any similar office by whatever name called.

8 (1) Every life assurance company shall once in every five years, or at such shorter intervals as may be prescribed by the instrument constituting the company, or by its regulations or bye-laws, cause an investigation to be made into its financial condition, including a valuation of its liabilities, by an actuary, and shall cause an abstract of the report of such actuary to be made in the form set forth in the Fourth Schedule.

(2) The provision of sub-section (1) regarding the making of an abstract shall also apply whenever at any other time an investigation into the financial condition of a life assurance company is made with a view to the distribution of profits, or whenever the results of any such investigation are made public.

9. In the case of a mutual life assurance company whose profits are allocated to members wholly or mainly by annual abatements of premium, the abstract of the report of the actuary on the financial condition of the company, prepared in accordance with the Fourth Schedule, may, notwithstanding anything in section 8, be made and returned at intervals not exceeding five years : Provided that, where such return is not made annually, it shall include particulars as to the rates of abatement of premiums applicable to different classes or series of assurances allowed in each year during the period which has elapsed since the previous return under the Fourth Schedule.

10. Every life assurance company shall, within three years from the commencement of this Act, and thereafter at the date to which the accounts of the company are made up for the purpose of the investigation prescribed by section 8, prepare a statement of its assurance business in the form set forth in the Fifth Schedule : Provided that, if the investigation is made annually by any company, the company may prepare such a statement at any time, so that it be made at least once in every five years.

11. (1) Every account, balance-sheet, abstract or statement hereinbefore required to be made shall be printed, and four copies thereof, one of which shall be signed by the chairman and two directors of the company, and by the principal officer of the company, and if the company has a managing director, by the managing director, shall be deposited with the Governor General in Council within six months in the case of accounts and balance-sheets required by section 7, and within one year in other cases after the close of the period to which the account, balance-sheet, abstract or statement relates : Provided that, if in any case it is made to appear to the Governor General in Council that the circumstances are such that a longer period should be allowed, he may extend that period by such period as he may think fit.

Actuarial report
and abstract.

Actuarial
abstract in case
of mutual
company.

Statement of
life assurance
business.

Deposit of
accounts, &c.,
with Governor
General in
Council.

(2) The Governor General in Council shall consider any document deposited in accordance with the provisions of sub-section (1) and, if any such document appears to the Governor General in Council to be inaccurate or defective in any respect, the Governor General in Council may call upon the company to furnish a further statement correcting any such inaccuracies or supplying any such deficiencies.

12. There shall be deposited with every revenue account and balance sheet of a life assurance company every report on the affairs of the company submitted to the shareholders or policy-holders of the company in respect of the financial year to which the account and balance-sheet relate.

13. Where a life assurance company registered under the Indian Companies Act, 1882, in any year deposits its accounts and balance-sheet in accordance with the provisions of section 11, the company may, at the same time, send to the Registrar of Joint Stock Companies a copy of such accounts and balance-sheet; and, where such copy is so sent, it shall not be necessary for the company to file a balance-sheet with the Registrar of Joint Stock Companies as required by section 74 of the Indian Companies Act, 1882, and the copy of the accounts and balance-sheet so sent shall be dealt with in all respects as if it were a balance-sheet filed in accordance with that section.

14. A printed copy of the accounts, balance-sheet, abstract or statement last deposited shall, on the application of any shareholder or policy-holder of the company, be forwarded to him by the company by post or otherwise.

15. The accounts of every life assurance company shall be audited annually in such manner as the Governor General in Council may prescribe.

16. Every life assurance company which is not registered under the Indian Companies Act, 1882, shall keep a list of the names and addresses of its shareholders, and shall, on the application of any shareholder or policy-holder of the company, furnish to him a copy of such list on payment of a sum not exceeding two annas for every hundred words required to be copied.

17. Every life assurance company which is not registered under the Indian Companies Act, 1882, shall cause a sufficient number of copies of its deed of settlement or other instrument constituting the company to be printed, and shall, on the application of any shareholder or policy-holder of the company, furnish to him a copy of such deed of settlement or other instrument on payment of a sum not exceeding one rupee.

18. Where any notice, advertisement or other official publication of a life assurance company contains a statement of the amount of the authorized capital of the company, the publication shall also contain a statement of the amount of the capital which has been subscribed and the amount paid up.

Deposit of
report.

Exemption from
certain provi-
sions of Act VI
of 1882.

Right of share-
holders, &c., to
copies of
accounts, &c.

Audit of
accounts.

List of share-
holders.

Deed of settle-
ment.

Publication of
authorized, as
well as sub-
scribed and paid-
up capital.

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19. (1) Every life assurance company, constituted outside British India, which establishes a place of business within British India, or appoints an agent in British India with the object of obtaining life assurance business shall, within three months from the establishment of the place of business or the appointment of such agent, file with the Registrar—

Requirements as to companies established outside British India.

- (a) A certified copy of the charter, statutes or memorandum and articles of the company, or other instrument constituting or defining the constitution of the company, and, if the instrument is not written in the English language, a certified translation thereof ;
- (b) A list of the directors of the company ;
- (c) The names and addresses of some one or more persons resident in British India authorized to accept on behalf of the company service of process and any notices required to be served on the company ;

and, in the event of any alteration being made in any such instrument or in the list of directors or in the names and addresses of such persons as aforesaid, the company shall, within such time as the Governor General in Council may prescribe, file with the Registrar a notice of the alteration.

(2) Any process or notice required to be served on the company shall be sufficiently served if addressed to any person whose name has been so filed as aforesaid and left at or sent by post to the address which has been so filed.

(3) There shall be paid to the Registrar for registering any document, required by this section to be filed, a fee of five rupees or such smaller fee as the Governor General in Council may prescribe.

Amalgamation or Transfer.

20. (1) Where it is intended to amalgamate two or more life assurance companies, or to transfer the life assurance business of one company to another, the directors of any one or more of such companies may apply to the Court, by petition, to sanction the proposed arrangement.

Amalgamation or transfer.

(2) Before any such application is made to the Court—

- (a) Notice of the intention to make the application shall be published in the *Gazette* of India and in the local official *Gazette* of the Province in which the principal place of business of the company is situate at least two months before the application is made ;
- (b) A statement of the nature of the amalgamation or transfer, as the case may be together with an abstract containing the material facts embodied in the agreement or deed under which the amalgamation or transfer is proposed to be effected, and copies of the actuarial or other reports upon which the agreement or deed is founded, including a report by an independent actuary, shall, unless the Court otherwise directs, be transmitted to each policyholder of each company ; and

- (c) The agreement or deed under which the amalgamation or transfer is effected shall be open for the inspection of the policy-holders and shareholders at the offices of the companies for a period of fifteen days after the last publication of the notice.

(3) The Court, after hearing the directors and other persons whom it considers entitled to be heard upon the petition, may sanction the arrangement if it is satisfied that no sufficient objection to the arrangement has been established.

(4) The Court shall not sanction the amalgamation or transfer in any case in which it appears to the Court that the life policy-holders representing one-tenth or more of the total amount assured in any company which it is proposed to amalgamate, or in any company the business of which it is proposed to transfer, dissent from the amalgamation or transfer.

(5) No life assurance company shall amalgamate with another, or transfer its business to another, unless the amalgamation or transfer is sanctioned by the Court in accordance with this section.

21. Where an amalgamation takes place between any life assurance companies, or where any life assurance business of one such company is transferred to another company, the combined company or the purchasing company, as the case may be, shall, within one month from the date of the completion of the amalgamation or transfer, deposit with the Governor General in Council—

Statement in case of amalgamation or transfer.

- (a) Certified copies of statements of the assets and liabilities of the companies concerned in such amalgamation or transfer, together with a statement of the nature and terms of the amalgamation or transfer ; and
- (b) A certified copy of the agreement or deed under which the amalgamation or transfer is effected ; and
- (c) Certified copies of the actuarial or other reports upon which that agreement or deed is founded ; and
- (d) A declaration under the hand of the chairman of each company, and the principal officer of each company, that to the best of their belief every payment made or to be made to any person whatsoever on account of the amalgamation or transfer is therein fully set forth, and that no other payments beyond those set forth have been made or are to be made either in money, policies, bonds, valuable securities or other property by or with the knowledge of any parties to the amalgamation or transfer.

Winding up.

22. The Court may order the winding up of a life assurance company, in accordance with the Indian Companies Act, 1882, and the provisions of that Act shall apply accordingly, subject, however, to the modification that the company may be ordered to be wound up—

Special provisions as to winding up of assurance companies.

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(a) On the petition of ten or more policy-holders :

Provided that such a petition shall not be presented except by the leave of the Court, and leave shall not be granted until a *prima facie* case has been established to the satisfaction of the Court, and until security for costs for such amount as the Court may think reasonable has been given ; or

(b) On application made on behalf of the Governor General in Council, showing that from a consideration of the documents deposited with him under the provisions of this Act it appears to him that the company is insolvent.

23. (1) Where a life assurance business or any part of the life assurance business of a life assurance company has been transferred to another company under an arrangement in pursuance of which the first-mentioned company (in this section called the subsidiary company) or the creditors thereof has or have claims against the company to which such transfer was made (in this section called the principal company), then, if the principal company is being wound up by or under the supervision of the Court, the Court shall (subject as hereinafter mentioned) order the subsidiary company to be wound up in conjunction with the principal company, and may by the same or any subsequent order appoint the same person to be liquidator for the two companies, and make provision for such other matters as may seem to the Court necessary, with a view to the companies being wound up as if they were one company.

(2) The commencement of winding up of the principal company shall, save as otherwise ordered by the Court, be the commencement of the winding up of the subsidiary company.

(3) In adjusting the rights and liabilities of the members of the several companies between themselves, the Court shall have regard to the constitution of the companies, and to the arrangements entered into between the companies, in the same manner as the Court has regard to the rights and liabilities of different classes of contributories in the case of the winding up of a single company, or as near thereto as circumstances admit.

(4) Where any company alleged to be subsidiary is not in process of being wound up at the same time as the principal company to which it is subsidiary, the Court shall not direct the subsidiary company to be wound up unless, after hearing all objections (if any) that may be urged by or on behalf of the company against its being wound up, the Court is of opinion that the company is subsidiary to the principal company, and that the winding up of the company in conjunction with the principal company is just and equitable.

(5) An application may be made in relation to the winding up of any subsidiary company in conjunction with a principal company by any creditor of, or person interested in, the principal or subsidiary company.

(6) Where a company stands in the relation of a principal company to one company, and in the relation of a subsidiary company to some

other company, or where there are several companies standing in the relation of subsidiary companies to one principal company, the Court may deal with any number of such companies together or in separate groups as it thinks most expedient upon the principles laid down in this section.

24. Where a life assurance company is being wound up by the Court, or subject to the supervision of the Court, or voluntarily, the value of a policy or of a liability under a policy requiring to be valued in such winding up shall be estimated in manner applicable to policies and liabilities provided by the Sixth Schedule.

25. The rules in the Sixth Schedule shall be of the same force, and may be repealed, altered or amended as if they were rules made in pursuance of section 254 of the Indian Companies Act, 1882, and rules may be made under that section for the purpose of carrying into effect the provisions of this Act with respect to the winding up of life assurance companies.

26. The Court, in the case of a life assurance company which has been proved to be unable to pay its debts, may, if it thinks fit, reduce the amount of the contracts of the company upon such terms and subject to such conditions as it thinks just, in place of making a winding-up order.

Special Provisions relating to Accounts and Documents.

27. The Governor General in Council may direct any documents deposited with him under this Act, or certified copies thereof, to be kept by the Registrar or by any other officer appointed in this behalf, and any such documents and copies shall be open to inspection, and copies thereof may be procured by any person on payment of such fees as the Governor General in Council may direct.

28. The Governor General in Council shall annually publish in the *Gazette* of India and cause to be published in the local official *Gazette* of the Province in which the life assurance company has its principal place of business the accounts, balance-sheets, abstracts, statements and other documents under this Act, or purporting to be under this Act, deposited with him during the preceding year, except reports on the affairs of life assurance companies submitted to the shareholders or policy-holders thereof, and may append to such accounts, balance-sheets, abstracts, statements or other documents any note of the Governor General in Council thereon, and any correspondence in relation thereto.

29. Every document deposited under this Act with the Governor General in Council, and certified by the Registrar or by any person appointed in that behalf by the Governor General in Council to be a document so deposited, shall be deemed to be a document so deposited.

30. Every document purporting to be certified by the Registrar, or by any person appointed in that behalf by the Governor General in Council, to be a copy of a document so deposited, shall be deemed to be a copy of that

Valuation of annuities and policies.

Rules of valuation.

Power to Court to reduce contracts.

Custody and inspection of documents deposited with Governor General in Council.

Accounts, &c., to be published.

Evidence of documents.

Evidence of copies of documents.

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document, and shall be received in evidence as if it were the original document unless some variation between it and the original document be proved.

31. The Governor General in Council may, on the application or with the consent of a life assurance company, alter the forms contained in the Schedules to this Act as respects that company, for the purpose of adapting them to the circumstances of that company.

Alteration of forms.

Companies carrying on business in the United Kingdom.

Edw.VII
cap. 49.

32. (1) An assurance company which carries on life assurance business in the United Kingdom in accordance with the Assurance Companies Act, 1909, may, if carrying on life assurance business in British India before the commencement of this Act, within three months of such commencement, or, in any other case, before it commences to carry on life assurance business in British India, apply to the Governor General in Council for a declaration that it so carries on such business in the United Kingdom.

(2) A company applying under the provisions of sub-section (1) shall furnish, at the time of its application or at such further time as the Governor General in Council may prescribe, such evidence as he may direct of the facts alleged in its application.

Edw.VII
cap. 49.

(3) Where the Governor General in Council is satisfied that a life assurance company applying as aforesaid is a life assurance company which carries on business in the United Kingdom in accordance with the Assurance Companies Act, 1909, he shall, by notification in the *Gazette* of India, make a declaration to that effect, and shall cause such notification to be republished in the local official *Gazette* of the Province where the Company has or proposes to have its principal place of business.

33. Where the Governor General in Council has notified a declaration in accordance with the provisions of section 32 in respect of a life assurance company, nothing in section 4, section 5, sections 7 to 12, sections 15, 20, 21 or 37 shall apply to the company :

Application of the Act to companies which carry on life assurance business in the United Kingdom.

Provided that—

Edw.VII
cap. 49.

(1) The company shall deposit with the Governor General in Council, in the manner prescribed in section 11, copies of every account, balance-sheet, abstract, statement or other document which the company is required by the Assurance Companies Act, 1909, to deposit at the Board of Trade ;

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cap. 49.

(2) If, at any time, a company in respect of which a declaration has been notified under section 32 ceases to carry on life assurance business in the United Kingdom in accordance with the provisions of the Assurance Companies Act, 1909, it shall, if it continues to carry on life assurance business in British India, be subject to all the provisions of this Act from the date it ceased to carry on such business in the United Kingdom in accordance with the said Act

Penalties and Procedure.

34. Any life assurance company which makes default in complying with any of the requirements of this Act, and every director, manager or secretary, or other officer or agent of the company who is knowingly a party to the default, shall be punishable with fine which may extend to one thousand rupees, or, in the case of a continuing default, with fine which may extend to five hundred rupees for every day during which the default continues; and, if default continues for a period of three months after notice of default by the Governor General in Council (which notice shall be published in one or more newspapers as the Governor General in Council may, upon the application of one or more policy-holders or shareholders, direct), the default shall be a ground on which the Court may order the winding up of the company, in accordance with the Indian Companies Act, 1882.

35. If any account, balance-sheet, abstract, statement or other document required by this Act is false in any particular to the knowledge of any person who signs it, such person shall be punishable with imprisonment for a term which may extend to two years, or with fine, or with both.

36. No Court inferior to that of a Presidency Magistrate or a Magistrate of the first class shall try any offence against this Act.

Miscellaneous.

37. (1) The Governor General in Council may appoint one or more inspectors to examine into the affairs of any life assurance company, and to report thereon in such manner as he may direct—

(i) In the case of a life assurance company which is not registered under the Indian Companies Act, 1882, upon the application—

(a) Of shareholders being in number not less than one-fifth of the whole number of persons for the time being entered on the list of shareholders kept in accordance with the provisions of section 16; or

(b) Of twenty or more policyholders owning policies of an aggregate value of not less than twenty thousand rupees;

(ii) In any case where a life assurance company has failed to furnish a further statement when required to do so under the provisions of section 11, sub-section (2), or where the Governor General in Council is of opinion that any such further statement is insufficient or unsatisfactory.

(2) On an appointment being made under sub-section (1), the provisions of section 84 of the Indian Companies Act, 1882, shall apply to the examination made by such inspectors.

38. Any notice or other document which is by this Act required to be sent to any policy-holder may be addressed and sent to the person to whom notices respecting such policy

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Appointment of inspectors.

Cognizance of offences.

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are usually sent, and any notice so addressed and sent shall be deemed and taken to be notice to the holder of such policy :

Provided that where any person claiming to be interested in a policy has given to the company notice in writing of his interest, any notice which is by this Act required to be sent to policyholders shall also be sent to such person at the address specified by him in his notice.

39. (1) The Governor General in Council may make rules to carry out the purposes of this Act.

(2) In particular and without prejudice to the generality of the foregoing power, such rules may—

- (a) Prescribe the qualifications to be possessed by actuaries, auditors and inspectors under this Act, and the manner in which the accounts of life assurance companies shall be audited ;
- (b) Prescribe the time within and the form in which notice of alteration of the particulars specified in section 19 of the Act shall be filed with the Registrar :
- (c) Subject to the provisions of this Act, prescribe the fees payable thereunder.

(3) All rules made under this Act shall be published in the *Gazette* of India, and on such publication, shall have effect as if enacted in this Act.

40. The Governor General in Council may, by notification in the *Gazette* of India, and subject to such conditions and restrictions as he thinks fit, delegate to any Local Government all or any of the powers (other than the power to make rules under section 39) conferred on him by this Act.

41. The Governor General in Council may, by notification in the *Gazette* of India, and subject to such restrictions and conditions as he thinks fit, exempt any life assurance company from all or any of the provisions of this Act.

42. In section 131 of the Indian Companies Act, 1882, the words from “ In the case of a life assurance company ” to “ unable to pay its debts ” are hereby repealed.

Power of Governor General in Council to delegate to Local Governments the powers conferred by this Act.

Amendment of Act VI, 1882, Section 131.

THE FIRST SCHEDULE.

(See section 7.)

REVENUE ACCOUNTS OF THE.....FOR THE YEAR ENDING.....

(1)—Life Assurance Account.

Rs.	Rs.
Amount of life assurance fund at the beginning of the year.	Dividends payable on 19 for the year ending 19 (This is only to be stated here by companies not supplying a Profit and Loss account).
Premiums	Claims under policies paid and outstanding—
	By death
	By maturity... ..
	Surrenders, including surrenders of bonus additions.
	Annuities
	Bonuses in cash
	Bonuses in reduction of premiums
Consideration for annuities granted *(see Note 1).	Expenses of management :—
	Commission
	Agents' and Canvassers' allowances
	Salaries, etc. (other than to Agents and Canvassers).
	Travelling expenses
Interests, dividends and rents Rs.	Directors' fees
	Auditors' fees
	Medical fees... ..
	Rents for offices belonging to and occupied by the company.
	Rents of other offices occupied by the company.
Less income-tax thereon	Law charges... ..
	Advertising
	Printing and stationery
	Other expenses of management (accounts to be specified).
Other receipts (accounts to be specified)	Other payments (accounts to be specified).
	Amount of life assurance fund at the end of the year, as per Third Schedule.
Rs.	Rs.

* NOTE 1.—Companies having a separate annuity fund with investments separate from those of the life assurance fund to return the particulars of their annuity business in a separate statement, in Form B of this Schedule.

NOTE 2.—Items in this account to be net amounts after deduction of the amounts paid and received in respect of reassurances of the company's risks.

NOTE 3.—If any sum has been deducted from the expenses of management account, and taken credit for in the balance-sheet as an asset, the sum so deducted to be separately shown in the above account.

(B) Revenue Account applicable to annuity business of those companies having a separate annuity fund, the investments of which are kept separate from those of the life assurance fund.

	Rs.		Rs.
Amount of annuity fund at the beginning of the year		Annuities	
Consideration for annuities granted		Surrenders	
		Expenses of management :—	
		Commission	
		Other expenses (to be specified)	
Interest, dividends and rents	Rs.	Other payments (accounts to be specified)	
Less income tax thereon		Amount of annuity fund at the end of the year as per Balance-sheet.	
Other receipts			
	Rs.		Rs.

NOTE.—Items in this account to be net amounts after deduction of the amounts paid and received in respect of reassurances of the company's risks.

(C) General Revenue Account applicable to all classes of business other than life assurance and annuity transactions.

	Rs.		Rs.
Amount of funds at the beginning of the year ...		Claims less reassurances (accounts to be specified).	
Premiums (accounts to be specified)		Expenses of management :—	
		Commission	
		Other expenses (to be specified)	
Interests, dividends and rents	Rs.	Losses (accounts to be specified)	
Less income tax thereon		Other payments (accounts to be specified)	
Profits (accounts to be specified)		Amount of funds at the end of the year as per Balance-sheet.	
Other receipts (to be specified)			
	Rs.		Rs.

NOTE 1.—All the items in the above account to be exclusive of life assurance and annuity transactions.

NOTE 2.—Items in this account to be net amounts after deduction of the amounts paid and received in respect of reassurances of the company's risks.

(D) Statement to be submitted along with the Revenue Account by all life assurance companies.

Class of Policy.	TOTAL NEW LIFE ASSURANCES COMPLETED IN INDIA DURING THE YEAR 19 .			PORTION THEREOF REASSURED.		
	Sum Assured	Annual Premium	Single Premium	Sum Assured	Annual Premium	Single Premium
	Rs.	Rs.	Rs.	Rs.	Rs.	Rs.
Whole Life						
Whole life by limited payments						
Endowment assurances ...						
Pure endowments... ..						
Term assurances						
Other classes						
TOTAL						

State also:—

New annuities (state number and annual amount).

Total sums assured and bonuses (less reassurances) remaining in force at end of year 19 on lives of residents in India.

Number and amount of annuities (less reassurances) remaining in force at end of year 19 on lives of residents in India.

Largest sum for which the company has granted an assurance on any one life during the year, after deduction of any portion reassured.

Statement of the total investments in India of the life assurance and annuity funds.

THE SECOND SCHEDULE.

(See section 7.)

PROFIT AND LOSS ACCOUNT OF THE.....FOR THE YEAR ENDING
19 .

	Rs.		Rs.
Balance of last year's account		Dividends and bonuses to shareholders payable on 19 , for the year ending 19	
Interest and dividends not carried to other accounts	Rs.	Expenses not charged to other accounts	
Less income-tax thereon		Loss realized (accounts to be specified)	
Profit realized (accounts to be specified)		Other payments (accounts to be specified)	
Other receipts (accounts to be specified)		Balance as per Third Schedule	
	Rs.		Rs.

THE THIRD SCHEDULE

(See section 7.)

(A) BALANCE-SHEET.....OF THE.....ON THE.....19 ..

LIABILITIES,	Rs.	ASSETS,	Rs.
Life assurance fund—			
Outstanding liabilities of life assurance fund.	_____	Assets of life assurance fund as per separate balance-sheet (if any)
Annuity fund (if any) as per separate balance-sheet.	_____	Assets of annuity fund as per separate balance-sheet (if any).	...
Outstanding liabilities of annuity fund.	_____	Assets of funds other than those shown in the above mentioned balance-sheets.	...
Shareholders' capital paid up (if any)	Mortgages on property within India	...
Profit and Loss account (if any)	do. out of India	...
Funds contained in General Revenue Account	...	Loans on public rates	...
(if any) [Schedule I (c).]	_____	do. life interests and reversions	...
Other sums owing by the Company...	do. stocks and shares...	...
(Accounts to be specified and stated separately under each class of business).	_____	do. company's policies within their surrender values	...
		do. personal security
		Investments—	...
		Deposit with the Comptroller General (securities to be specified).	...
		Indian Government securities
		British and Colonial Government securities
		Foreign Government securities...
		Indian Municipal and Provincial securities
		British and Colonial	do.
		Foreign	do.
		Bonds, debentures, stocks and other securities whereon interest is guaranteed by the Indian Government.	...
		Bonds, debentures, stocks and other securities whereon interest is guaranteed by the British or any Colonial Government.	...
		Bonds, debentures, stocks and other securities whereon interest is guaranteed by any Foreign Government.	...
		Ordinary stocks and shares of any Indian Presidency Bank.	...
		Debentures of any Railway in India.	...

Debentures of any Railway out of India.	
Preference or guaranteed shares of any Railway in India.	
Preference or guaranteed shares of any Railway out of India.	
Ordinary stocks and shares of any Railway in India.	
Ordinary stocks and shares of any Railway out of India.	
House property in India.	
House property out of India.	
Freehold and leasehold ground rents and rent charges in India	...
Life interests and reversions in India	...
Do.	do. out of India
Other investments in India (to be specified)	...
Other investments out of India (to be specified)	...
Agents' balances	...
Outstanding premiums*	...
Do. interests, dividends and rents*	...
Interest accrued but not payable*	...
Bills receivable	...
Cash :—	...
On deposit	...
In hand and on current account	...
Other assets (to be specified)	...

Rs.

Rs.

* These items are or have been included in the corresponding items in the First Schedule.

NOTE 1.—When part of the assets of the company are specifically deposited under local laws, in various places out of India, as security to holders of life assurance policies there issued, each such place and the amount compulsorily lodged therein must be specified.

NOTE 2.—The balance-sheet must state how the values of the Stock Exchange securities are arrived at, and on the occasions when a statement respecting valuation under the Fourth Schedule is made a certificate must be appended, signed by the same persons as signed the balance-sheet, to the effect that in their belief the assets set forth in the balance-sheet are in the aggregate fully of the value stated therein, less any investment reserve fund taken into account.

NOTE 3.—Companies having investments with any uncalled liability shall state separately the full amount thereof.

NOTE 4.—Particulars must be given of all loans, including temporary advances, except loans on policies within their surrender values, made at any time during the year to any director or officer of a company or to any other company in which any of the said directors or officers may hold the position either of director or of officer.

(B) BALANCE-SHEET OF THE LIFE ASSURANCE FUND.....ON THE.....19....., TO BE COMPLETED
BY COMPANIES DOING BUSINESS OTHER THAN LIFE ASSURANCE FOR WHICH THEY HAVE SEPARATE FUNDS.

LIABILITIES.	Rs.	ASSETS.	Rs.
Life assurance fund	Mortgages on property within India	...
Claims admitted or intimated* but not paid	...	Do. do. out of India	...
Other sums owing by the company* (under this class of business)	...	Loans on public rates
...	...	Do. life interests and reversions	...
...	...	Do. stocks and shares...	...
...	...	Do. company's policies within their surrender values	...
...	...	Do. personal security...	...
...	...	Investments—	...
...	...	Deposit with the Comptroller General (securities to be specified)...	...
...	...	Indian Government securities
...	...	British and Colonial Government securities	...
...	...	Foreign Government securities
...	...	Indian Municipal and Provincial securities	...
...	...	British and Colonial do. do.	...
...	...	Foreign do. do.	...
...	...	Bonds, debentures, stocks and other securities whereon interest is guaranteed by the Indian Government	...
...	...	Bonds, debentures, stocks and other securities whereon interest is guaranteed by the British or any Colonial Government.	...
...	...	Bonds, debentures, stocks and other securities, whereon interest is guaranteed by any Foreign Government...	...
...	...	Ordinary stocks and shares of any Indian Presidency Bank.	...
...	...	Debentures of any Railway in India.	...
...	...	Debentures of any Railway out of India.	...
...	...	Preference or guaranteed shares of any Railway in India.	...
...	...	Preference or guaranteed shares of any Railway out of India.	...
...	...	Ordinary stocks and shares of any Railway in India.	...
...	...	Ditto ditto out of India	...

	Rs.		Rs.
House property in India.			
Ditto out of India	
Freehold and leasehold ground rents and rent-charges in India.		...	
Life interests and reversions in India.		...	
Ditto ditto out of India.		...	
Other investments in India (to be specified)		...	
Ditto out of India (to be specified)		...	
Agents' balances	
Outstanding premiums*	
Do. interests, dividends and rents*	
Interest accrued but not payable*...	
Bills receivable	
Cash :—		...	
On deposit	
In hand and on current account...	
Other assets (to be specified)	
			Rs.

* These items are or have been included in the corresponding items in the First Schedule.

NOTE 1.—When part of the assets of the company are specifically deposited under local laws, in various places out of India, as security to holders of life assurance policies there issued, each such place and the amount compulsorily lodged therein must be specified.

NOTE 2.—A balance-sheet in the above form must be rendered in respect of the annuity fund if the investments of that fund are distinct from those of the life assurance fund.

NOTE 3.—The balance-sheet must state how the values of the Stock Exchange securities are arrived at, and on the occasions when a statement respecting valuation under the Fourth Schedule is made, a certificate must be appended, signed by the same persons as signed the balance-sheet, to the effect that in their belief the assets set forth in the balance-sheet are in the aggregate fully of the value stated therein, less any investment reserve fund taken into account.

NOTE 4.—A certificate must be appended hereto, signed by the same persons as signed the balance-sheet (Form A), and by the auditor, to the effect that no part of any such fund has been applied, directly or indirectly, for any purpose other than the class of business to which it is applicable.

NOTE 5.—Companies having investments with any uncalled liability shall state separately the full amount thereof.

NOTE 6.—Particulars must be given of all loans, including temporary advances, except loans on policies within their surrender values, made at any time during the year to any director or officer of a company, or to any other company in which any of the said directors or officers may hold the position either of director or of officer.

THE FOURTH SCHEDULE.

(See sections 8 and 9.)

STATEMENT RESPECTING THE VALUATION OF THE LIABILITIES UNDER LIFE POLICIES AND ANNUITIES OF THE....., TO BE MADE AND SIGNED BY THE ACTUARY.

(The answers should be numbered to accord with the numbers of the corresponding questions.)

1. The date up to which the valuation is made.

2. The general principles adopted in the valuation, and the method followed in the valuation of particular classes of assurances, including a statement of the method by which the net premiums have been arrived at, and whether these principles were determined by the instrument constituting the company or by its regulations or bye-laws, or how otherwise; together with a statement of the manner in which policies on under average lives are dealt with.

3. The table or tables of mortality used in the valuation. In cases where the tables employed are not published, specimen policy values are to be given, at the rate of interest employed in the valuation, in respect of whole-life assurance policies effected at the respective ages of 20, 30, 40 and 50, and having been respectively in force for 5 years, 10 years, and upwards at intervals of five years, respectively; with similar specimen policy values in respect of endowment assurance policies, according to age at entry, original term of policy and duration.

4. The rate or rates of interest assumed in the calculations.

5. The actual proportion of the annual premium income (if any), reserved as a provision for future expenses and profits, separately specified in respect of assurances with immediate profits, with deferred profits, and without profits. (If none, state how this provision is made.)

6. The consolidated revenue-account since the last valuation, or, in case of a company which has made no valuation, since the commencement of the business. (This return should be made in the form annexed. No return under this heading will be required where a statement under this schedule is deposited annually.)

7. The liabilities of the company under life policies and annuities at the date of the valuation, showing the number of policies, the amount assured and the amount of premiums payable annually under each class of policies, both with and without participation in profits; and also the net liabilities and assets of the company with the amount of surplus or deficiency. (These returns to be made in the forms annexed.)

8. The principles upon which the distribution of profits among the shareholders and policy-holders is made, and whether these principles were determined by the instrument constituting the company or by its regulations or bye-laws, or how otherwise, and the number of years' premiums to be paid before a bonus (*a*) is allotted, and (*b*) vests.

9. The results of the valuation, showing—

(1) The total amount of profit made by the company, allocated as follows :—

(a) Among the policy-holders with immediate participation, and the number and amount of the policies which participated ;

(b) Among policy-holders with deferred participation, and the number and amount of the policies which participated ;

(c) Among the shareholders ;

(d) To reserve funds, or other accounts ;

(e) Carried forward unappropriated ;

(2) Specimens of bonuses allotted to whole-life assurance policies for Rs. 1,000 effected at the respective ages of 20, 30, 40 and 50, and having been respectively in force for 5 years, 10 years, and upwards at intervals of 5 years respectively, together with the amounts apportioned under the various modes in which the bonus might be received ; with similar specimen bonuses and particulars in respect of endowment assurance policies, according to age at entry, original term of policy, and duration.

(Form referred to under Heading No. 6 in Fourth Schedule.)

Consolidated Revenue Account of the.....for.....years commencing.....and ending.....

	Rs.		Rs.
Amount of life assurance fund at the beginning of the period		Claims under policies paid and outstanding :—	
Premiums		By death	Rs.
Consideration of annuities granted		By maturity	
	Rs.		
Interest, dividends and rents		Surrenders	
Less income-tax thereon... ..		Annuities	
		Bonuses in cash	
Other receipts (accounts to be specified)		„ „ reduction of premium-Commission	
		Expenses of management... ..	
		Other payments (accounts to be specified)	
		Amount of life assurance fund at the end of the period as per Third Schedule	
	Rs.		Rs.

NOTE.—If any sum has been deducted from the expenses of management account and taken credit for in the balance-sheet as an asset, the sum so deducted to be separately shown in the above statement.

(Form referred to under Heading No. 7 in Fourth Schedule.)

Summary and valuation of the policies of the.....as at.....19.....

	PARTICULARS OF THE POLICIES FOR VALUATION				VALUATION		
	Number of Policies	Sums assured and bonuses	Office yearly premiums	Net yearly premiums	Value by the Sums assured and bonuses	Office yearly premiums	Table, interest per-cent
ASSURANCES.							
I.— <i>With immediate participation in profits.</i>							
For whole term of life...
Other classes (to be specified)...
Extra premiums payable
II.— <i>With deferred participation in profits.</i>							
For whole term of life
Other classes (to be specified)...
Extra premiums payable
Total assurances with profits
III.— <i>Without participation in profits.</i>							
For whole term of life
Other classes (to be specified)...

Extra premiums
Total assurances without profits ...			
Total assurances
Deduct re-assurances (to be specified according to class in a separate statement).			
Net amount of assurances
Adjustments, if any (to be separately specified)
ANNUITIES ON LIVES.			
Immediate
Other classes (to be specified)...
Total of the results

NOTE 1.—The term “extra premium” in this Act shall be taken to mean the charge for any risk not provided for in the minimum contract premium. If policies are issued in or for any country at rates of premium deduced from tables other than the European mortality tables adopted by the company, separate schedules similar in form to the above must be furnished.

NOTE 2.—Separate returns and valuation results must be furnished in respect of classes of policies valued by different tables of mortality, or at different rates of interest, also for business at other than European rates.

NOTE 3.—In cases also where separate valuations of any portion of the business are required under local laws in places outside British India, a summary statement must be furnished in respect of the business so valued in each such place showing the total number of policies, the total sums assured and bonuses, the total office yearly premiums and the total net liability on the bases as to mortality and interest adopted in each such place, with a statement as to such bases respectively.

(Form referred to under Heading No. 7 in Fourth Schedule.)

Valuation Balance-Sheet of.....as at.....19 ..

Dr.	Rs.	Cr.	Rs.
To net liability under life assurance and annuity transactions (as per summary statement provided in Fourth Schedule).		By life assurance and annuity funds (as per Balance-sheet under Third Schedule) ...	
To surplus, if any... ..		By deficiency, if any...	

THE FIFTH SCHEDULE.

*(See section 10.)*STATEMENT OF THE LIFE ASSURANCE AND ANNUITY BUSINESS OF
THE ON THE 19 , TO BE SIGNED BY THE
ACTUARY.

(The answers should be numbered to accord with the numbers of the corresponding questions. Statements of reassurances corresponding to the statements in respect of assurances are to be given throughout.) Separate statements are to be furnished in the replies to all the headings under this Schedule for business at other than European rates.

1. The published table or tables of premiums for assurances for the whole term of life and for endowment assurances which are in use at the date above-mentioned.

2. The total amount assured on lives for the whole term of life which are in existence at the date above-mentioned, distinguishing the portions assured with immediate profits, with deferred profits, and without profits, stating separately the total reversionary bonuses and specifying the sums assured for each year of life from the youngest to the oldest ages, the basis of division as to immediate and deferred profits being stated.

3. The amount of premiums receivable annually for each year of life, after deducting the abatements made by the application of bonuses in respect of the respective assurances mentioned under Heading No. 2, distinguishing ordinary from extra premiums. A separate statement is to be given of premiums payable for a limited number of years, classified according to the number of years' payments remaining to be made.

4. The total amount assured under endowment assurances, specifying sums assured and office premiums separately in respect

of each year in which such assurances will mature for payment. The reversionary bonuses must also be separately specified, and the sums assured with immediate profits, with deferred profits, and without profits separately returned.

5. The total amount assured under classes of assurance business, other than assurances dealt with under questions 2 and 4, distinguishing the sums assured under each class and stating separately the amount assured with immediate profits, with deferred profits, and without profits, and the total amount of reversionary bonuses.

6. The amount of premiums receivable annually in respect of each such special class of assurances mentioned under Heading No. 5, distinguishing ordinary from extra premiums.

7. The total amount of premiums which has been received from the commencement upon pure endowment policies which are in force at the date above-mentioned.

8. The total amount of immediate annuities on lives, distinguishing the amounts for each year of life, and distinguishing male and female lives.

9. The amount of all annuities on lives other than those specified under Heading No. 8, distinguishing the amount of annuities payable under each class, and the amount of premiums annually receivable.

10. The average rate of interest yielded by the assets, whether invested or uninvested, constituting the life assurance fund of the company, calculated upon the mean fund of each year during the period since the last investigation, without deduction of income-tax.

It must be stated whether or not the mean fund upon which the average rate of interest is calculated includes reversionary investments.

11. A table of minimum values, if any, allowed for the surrender of policies for the whole term of life and for endowments and endowment assurances, or a statement of the method pursued in calculating such surrender values, with instances of the application of such method to policies of different standing and taken out at various interval ages from the youngest to the oldest.

THE SIXTH SCHEDULE.

(See sections 24 and 25.)

RULES FOR VALUING ANNUITIES, LIFE POLICIES AND LIABILITIES.

Rule for valuing an annuity.

An annuity shall be valued according to the tables used by the company which granted such annuity at the time of granting the same, and, where such tables cannot be ascertained or adopted to the satisfaction of the Court, then, according to such rate of interest and table of mortality as the Court may direct.

Rule for valuing a policy.

The value of the policy is to be the difference between the present value of the reversion in the sum assured according to the contingency upon which it is payable, including any bonus or addition thereto made before the commencement of the winding up, and the present value of the future annual premiums.

In calculating such present values interest is to be assumed at such rate, and the rate of mortality according to such tables, as the Court may direct.

The premium to be calculated is to be such premium as according to said rate of interest and rate of mortality is sufficient to provide for the risk incurred by the office in issuing the policy, exclusive of any addition thereto for office expenses and other charges.

Rule for valuing a liability.

The liquidator, in the case of all persons appearing by the books of the company to be entitled to or interested in policies granted by such company, is to ascertain the value of the liability of the company to each such person, and give notice of such value to such persons in such manner as the Court may direct, and any person to whom notice is so given shall be bound by the value so ascertained unless he gives notice of his intention to dispute such value in manner and within a time to be prescribed by a rule or order of the Court.

W. H. VINCENT,

Secretary to the Government of India.

Note on Mortality on the West Coast of Africa.

By J. R. HART, F.I.A.

IN the following Returns are given statistics distributed according to various age groups and for separate Colonies of West Africa, showing :

- (1) The number of non-native officials in the British Service on 1 January 1911.
- (2) The number of deaths and invalidings during the year 1911.
- (3) The ratio of deaths and invalidings to the number in the service on 1 January 1911.
- (4) The *average* number in the service during the year 1911, the number of deaths, pensioned, and invalidings, and their ratio to the average number in service, and
- (5) Summary of the last mentioned returns for the nine years 1903–11.

It will be remarked in the first place, that Table 3 does not give the rate of mortality as ordinarily understood by actuaries. To obtain this rate, it would be necessary to add to the number in the service on 1 January 1911, half the entrants and deduct half the number of those who left the service, omitting the deaths. But the number of entrants is not furnished for each age group, and it is, therefore, impossible to say to what extent the ratios according to age distribution as shown in Table 3 would be affected. It may be assumed, however, that the rate of mortality would be less at young ages and greater at the older ages than the ratios given.

In Tables 4 and 5 the ratios given correspond to central death rates. After making the necessary adjustment, the rates of mortality would be found to be slightly less than these ratios.

Taking all Colonies of the West Coast together and all ages, we find that the returns showed for the year 1911 a central death rate of 1.39 per-cent. In the remarks appended to the Report reference is made to the satisfactory fall in this rate as compared with the previous year, but it will be noticed that in that year the rate was higher than in the previous three years, and a consideration of the summary in Table 5, and of the graphs shows that about 1.7 per-cent may be taken as the average rate for the last five years. This compares favourably with the previous four years, 1903-6, when the average was 2.43 per-cent. Comparing further these results with those previously obtained and summarized in Dr. A. E. Sprague's paper on "the Rates of Mortality in certain parts of Africa" (*J.I.A.*, xxxiii, p. 285) we find a remarkable diminution in the rate, the difference no doubt being due to the improvement in sanitation, water supply, and other conditions in the colonies.

		Rate per-cent.
Non-native	Officials (1907-11)	.. 1.82
Do.	do. (1881-90)	.. 5.2
Dr. Lyon	.. (1878-90)	.. 5.1
Dr. A. E. Sprague	(1804-)	.. 4.7

Taking the Colonies separately, we find that the death rate is furnished for one year only, namely, 1911, and that the figures are too small to allow of any trustworthy conclusion being drawn from them, but the following comparisons may be made for what they are worth:

1. <i>Gambia.</i>		Rate per-cent.
Non-native Officials (1911)	..	7.1
Dr. Lyon (1878-90)	..	5.3
2. <i>Sierra Leone.</i>		
Non-native Officials (1911)	..	1.19
Dr. Lyon (1879-88)	..	2.8
Dr. A. E. Sprague (1804-)	..	7.0
3. <i>Gold Coast.</i>		
Non-native Officials (1911)	..	1.06
Dr. Lyon (1879-88)	..	5.1
4. <i>Southern Nigeria.</i>		
Non-native Officials (1911)	..	1.26
Dr. Lyon (Lagos) 1879-88	..	7.0

As regards Northern Nigeria, I cannot find that there are any former results with which the present figures may be compared.

A general consideration of the results shown above would lead one to the conclusion that the extra premium charged by most insurance companies for residence on the West Coast of Africa might be reduced. The average rate of mortality for assured lives brought out from statistics for the years 1856-1896, as appearing in the *Journal (J.I.A., xxxiii, 518)* was 2.76 per-cent; and it may be assumed that the experience of assured lives would show the effect of the improved conditions during the last 20 years as well as that of Government Officials. It must, however, not be overlooked that the rate of mortality in the annexed report will probably not give the full extra mortality experienced by the class of lives in question, as those pensioned for ill-health or invalided would probably show a higher death rate than the ordinary class. It will be seen from Table 5 that the rate of "invaliding", though only 2.52 per-cent in 1911 was over 5 per-cent for the five years 1903-7. In the writer's note on *Statistics, &c., relating to Government Officials (J.I.A., xxxiii, 309)*, a rate of invaliding of 3.6 per-cent was brought out, and although too much importance must not be attached to this rate owing to the smallness of the data, yet it might be conjectured that the lower death rate now prevailing is to some extent due to the higher rate of "invaliding." As against this supposition it will be noticed from Table 5 that the higher death rates in the first four years are accompanied by a higher rate of "invaliding."

Returns for 1911 as to the Vital Statistics of Non-notice Officials in the West African Civil Service.

[Issued by the Colonial Office—Cd. 6089, Price 3½d.—and reprinted by permission of the Controller of H.M. Stationery Office.]

No. 1.

Return showing Age Distribution of Officials in the Service on 1st January 1911.

Colony or Protectorate	Central Age 25	Central Age 30	Central Age 35	Central Age 40	Central Age 45	Central Age 50	Age not known	Totals
Gambia	6	7	12	7	3	6 <i>b</i>	...	41
Sierra Leone	26	41	37	29	20	11 <i>c</i>	...	164
Gold Coast	47	136	127	66	48	15 <i>d</i>	...	439
Northern Nigeria	100	204	201	99	42	13 <i>e</i>	1	630
Southern Nigeria	146 <i>a</i>	240	259	160	74	31 <i>f</i>	...	910
West Africa	325	628	636	361	187	76	1	2,214

a. 1 aged 22. *b.* 1 aged over 53, 1 aged 61. *c.* 1 aged over 53, 1 aged 58. *d.* 1 aged 56.
e. 2 aged over 53, 1 aged 54, 3 aged 55, 2 aged 56. *f.* 3 aged over 53, 2 aged 54.

No. 2.

Return showing Deaths and Invalidings.

Colony or Protectorate	DEATHS										INVALIDINGS									
	Central Age 25	Central Age 30	Central Age 35	Central Age 40	Central Age 45	Central Age 50	In 1st year	In 2nd year	In 3rd year and over	Totals	Central Age 25	Central Age 30	Central Age 35	Central Age 40	Central Age 45	Central Age 50	In 1st year	In 2nd year	In 3rd year and over	Totals
Gambia	1	1	1	2	...	1	3
Sierra Leone	1	...	1	1	1	2	...	2	1	1	3
Gold Coast	2	1	12	5	5	1	7	6	1	...	1	3	4	9	16
Northern Nigeria	3	2	5	1	1	8	10	3	7	5	4	1	...	2	3	15	20
Southern Nigeria	5	3	3	1	...	2	1	9	12	3	3	8	3	2	...	4	2	13	19
West Africa	2	11	8	10	1	...	3	4	25	32	7	19	20	8	3	1	8	7	43	58

No. 3.

Return showing rates per 1,000 of Deaths and Invalidings at various ages.

Year	DEATHS						INVALIDINGS					
	Central Age 25	Central Age 30	Central Age 35	Central Age 40	Central Age 45	Central Age 50 & over	Central Age 25	Central Age 30	Central Age 35	Central Age 40	Central Age 45	Central Age 50 & over
1911 ...	6.1	17.5	12.5	27.7	5.3	...	21.5	30.2	31.4	22.1	16.0	13.1

No. 4.

Return giving Particulars of Officials who left the Service during the year.

(1) Colony or Protectorate	(2) Average No. in Service	(3) DEATHS			(4) PENSIONED		(5) INVALIDINGS			(6)	(7)	(8)	(9)
		<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	Total No. who left Service	Average age on termination	Average length of Service on termination	No. in Service on 31st December
		Total	Rate per 1,000	Average length of Service	Ill-health	Total	Total	Rate per 1,000	Average length of Service				
				Y. M.					Y. M.		Y.	Y. M.	
Gambia ...	42	3	71.4	1 2	1	2	13	35	2 11	42
Sierra Leone ...	168	2	11.9	1 8	2	3	3	17.8	4 10	32	36	4 7	173
Gold Coast ...	470	5	10.6	6 7	12	12	16	34.0	2 10	75	37	4 10	501
Northern Nigeria	663	10	15.0	4 7	5	8	20	30.1	3 6	132	34	4 1	666
Southern Nigeria	950	12	12.6	4 3	8	12	19	20.0	3 0	135	35	3 11	989
West Africa	2,293	32	13.9	4 7	28	37	58	25.2	3 4	355	35	4 4	2,371

No. 5.

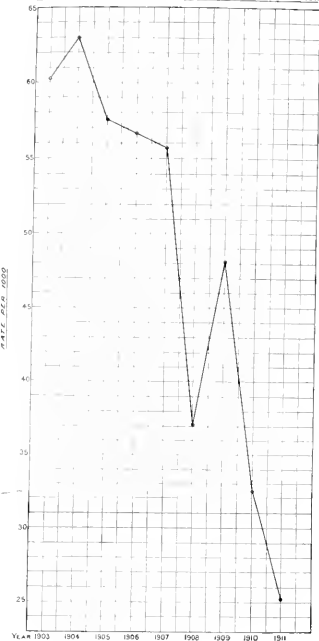
Summary of Returns from 1903 to 1911.

Year	Average No. in Service	DEATHS			PENSIONED		INVALIDINGS			Total No. who left Service	Average age on termination	Average length of Service on termination	No. in Service on 31st December
		Total	Rate per 1,000	Average length of Service	Ill-health	Total	Total	Rate per 1,000	Average length of Service				
				Y. M.					Y. M.		Y.	M. Y.	
1903 ...	1,259	26	20.6	1 9	5	8	76	60.3	2 0	235	...	2 4	1,398
1904 ...	1,428	39	27.3	3 3	6	8	90	63.0	2 0	327	...	2 6	1,461
1905 ...	1,491	42	28.1	3 6	7	8	86	57.6	2 7	300	...	3 1	1,524
1906 ...	1,553	33	21.2	3 10	7	10	88	56.6	2 4	285	...	3 2	1,586
1907 ...	1,628	29	17.8	4 5	9	13	91	55.8	2 10	267	33	3 5	1,670
1908 ...	1,803	32	17.7	4 0	16	21	67	37.1	3 5	245	35	3 11	1,932
1909 ...	2,015	35	17.3	4 6	17	22	97	48.1	3 1	330	34	3 9	2,091
1910 ...	2,153	44	20.4	5 1	15	21	70	32.5	3 5	350	35	3 9	2,212
1911 ...	2,293	32	13.9	4 7	28	37	58	25.2	3 4	355	35	4 4	2,371

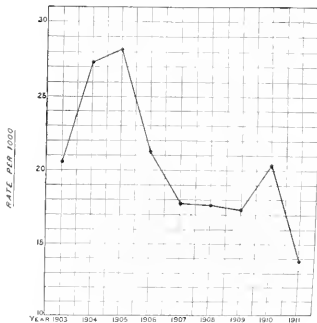
THE PERIOD OF A YEAR OR SO IN THIS COUNTRY. THE CHINESE pensioned generally enjoy their pensions for many years : in one or two cases the period on pension has been as much as 40 years.

12^a
NON-NATIVE OFFICIALS IN WEST AFRICA

GRAPH SHOWING INVALIDING RATE FROM 1903 TO 1911



GRAPH SHOWING DEATH RATE FROM 1903 TO 1911



REMARKS.

In 1911 the total death rate was 13·9 per 1,000, a decrease of 6·5 per 1,000 on the previous year; excluding 5 deaths due to causes other than disease the rate per 1,000 was 11·7, or a decrease of 5·0 per 1,000 on the corresponding rate for 1910. The considerable fall in the death rate for 1911 must be regarded as very satisfactory, especially in view of the fact that during the year there were serious outbreaks of yellow fever in the Gold Coast and the Gambia. These outbreaks accounted for the deaths of 13 non-native unofficials in the two Colonies, in addition to the 4 deaths of officials included in the returns.

Taking the period from 1903 to 1911 the number of deaths was 312, of which 41 were due to causes other than disease. The total death-rate for the period works out at 19·9 per 1,000 and the rate due to disease only, climatic and non-climatic, at 17·3 per 1,000. The rates corresponding to the latter rate were 19·3 for the period 1903–1908, 18·6 for the period 1903–1909 and 18·3 for the period 1903–1910.

The causes of deaths during the year 1911 were as follows:— 3 blackwater fever, 4 blackwater fever and heart failure, 1 subtertian malarial fever of cerebral type causing meningitis, 1 malaria and cardiac asthenia due to fatty degeneration of the heart, 4 yellow fever, 1 liver abscess, 1 dysentery and hepatic abscess (operation), 1 acute hepatitis and syncope, 1 cirrhosis and cardiac failure, 1 iliac abscess and peritonitis, 1 valvular heart disease, 1 dilatation of heart and valvular disease with aortic dilatation, 1 nephritis, 1 aneurism of arch of aorta, 1 acute pneumonia, 1 acute pneumonia and meningitis, 1 pulmonary tuberculosis, 1 erysipelas of head and hyperpyrexia, 1 œdema of glottis, asphyxia, and heart failure. 1 accident, 1 blasting accident (explosion), 1 gunshot accident, 1 killed by natives, and one accidentally drowned. 22 deaths occurred in West Africa, 8 in this country, and 2 at sea.

In addition to the deaths included in the returns there was one death amongst the “construction” officials, numbering approximately 67, employed in the Gold Coast. The cause was acute yellow atrophy of the liver and exhaustion. There was also one death from blackwater fever at Jebba in Northern Nigeria amongst the Railway construction staff employed on the Lagos Railway Northern Extension. The number of officials employed on this extension during the year was approximately 81.

A further decrease of 7·3 per 1,000 on the invaliding rate should be noted, but it will also be observed that the number pensioned on account of ill-health has very nearly doubled, though the proportion of these to the total has not materially altered. It may be mentioned here, as supporting the view that a man quickly recovers from the effects of the West African climate, that officials invalidated from the service are not infrequently found fit for re-employment after residence of a year or so in this country. Also officials pensioned generally enjoy their pensions for many years: in one or two cases the period on pension has been as much as 40 years.

As modifying the figures given in column 6 of return No. 4, it should be noted that about 18 per-cent of the officials left the service owing to the expiration of their agreements or the completion of the work for which they were engaged, and about 7 per-cent at the expiration of the five years for which they were seconded by the War Office.

Under "invalidings" are included not only officials who were invalidated home before completing their tours of service and whose appointments were terminated, but also all officials who, though they completed their tours, did not return to West Africa for health reasons, and also West African Frontier Force officers and non-commissioned officers who were unable, on account of ill-health, to rejoin their regiments at the expiration of their vacation leave. In practically every case the official had less than seven years' service. Pensioned officers are not included in this column.

In reckoning the length of service of an official the periods spent on the voyage to and from West Africa and during leave in this country are included. The actual period of residence in West Africa may be reckoned, roughly, as ranging from two-thirds to four-fifths of the total service.

Officials pensioned (column 4) invariably have over seven years' service in West Africa, except in a few cases of transfer from a pensionable appointment in another Colony or the United Kingdom.

With regard to column (6) of Return No. 4, it will be noticed that the total for West Africa as given in the return is less than the actual total for the five separate Colonies; the difference is accounted for by transfers from one Colony to another. This column includes death. Again, with regard to Return No. 2, the totals given for West Africa of the columns headed "In 1st year", "In 2nd year", "In 3rd year and over", are not, as will be seen, in all cases the actual totals of these columns. This is explained by the fact that certain officials, though they died or were invalidated from a particular Colony in their first or second year of service, had, owing to previous service in another Colony, two years' service or three years' service and over in West Africa as a whole, at the time of their death or invaliding.

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., *Barrister-at-Law.*

Income tax in
respect of interest
on foreign
investments.

THE case of *The Scottish Provident Institution v. Inland Revenue* (Farmer, Collector of Taxes) 1912, 49 S.L.R. 435, is one of some interest and importance to life assurance companies in view of the widespread and increasing tendency for such companies to invest their funds abroad.

In this case the Scottish Provident Institution appealed to

the Income Tax Commissioners at Edinburgh against an assessment to tax of two sums of £129,019 and £26,557 respectively, in respect of the year ending 5 April 1909. The sums in question represented interest which had accrued to the Institution in the United States and Canada, and had been received there and invested during the year ending 31 December 1907 in the purchase of bearer bonds which were thereafter transmitted to this country. The greater part of this claim was not sustained by the Commissioners, in view of the decision in the case of the *Scottish Widows' Fund v. Surveyor of Taxes*, 1909, 46 S.L.R. 993, *J.I.A.*, xliv, 97, in which it was held that the transmission of such securities was not equivalent to a remittance of the interest to the United Kingdom, but they held that the Institution was assessable to the extent of £15,681, which represented the proceeds of certain of the bonds in question which were sold in 1908, these proceeds having been received by the Institution at its head office in Edinburgh. The Commissioners were of opinion that the sum of £15,681 in question was interest received in the United Kingdom within the meaning of the fourth case of Schedule D, and as such chargeable with income tax.

The Institution appealed against this decision, and a case was stated for the opinion of the Court of Session as the Court of Exchequer in Scotland, but the Court refused the appeal. In delivering judgment to this effect the Lord President said: "The whole of the argument before your Lordships turned upon this fact . . . that the actual earning of the interest upon which the charge is now proposed to be made was not in the year of assessment with which we are dealing, but the realization of the earnings by means of the sale of the bonds in this country was in the year of assessment. . . .

" . . . Now the argument for the Crown is that the interest in question was received in Great Britain in the year of assessment, and therefore must be charged. The argument for the Institution is that inasmuch as it was not earned in that year it does not fall within the Income Tax Acts at all.

"I am of opinion that the determination of the Commissioners is right. There is nothing said in the Acts about profits or gains being necessarily earned within the year of assessment. No doubt that will be the natural result of the way in which the whole matter is worked; but I would like to make, first of all, this observation, that although the Act is full of the expression 'annual profits and gains', in every section

“ which deals with this matter, the presence of that word
 “ ‘annual’ does not seem to me to connote necessarily the idea
 “ of nothing being chargeable which is not earned within the year
 “ of assessment, but it is there for the purpose of showing that
 “ the tax which is being levied is a tax upon income, or, in other
 “ words, upon annual profits and not upon capital. When a
 “ ‘profit’ or an interest is earned in this country, the question
 “ really cannot arise, because the profit which is earned in this
 “ country is necessarily received in this country. I use the
 “ word ‘received’, because you may quite well have a profit
 “ which has not been paid to you in hard cash. Many partner-
 “ ships do not pay profits in hard cash, or a partner does not take
 “ his profits in cash, but nevertheless the profits are earned,
 “ and being earned they are necessarily received by the
 “ partner at the time they are earned. But when the profit is
 “ earned abroad, it is not necessarily received at the same time
 “ in this country. It is, of course, received in the sense of there
 “ being a right to it there, but it is not received in this country,
 “ and accordingly this fourth case provides that the duty shall
 “ only be computed on sums ‘ which have been or will be received
 “ ‘in Great Britain in the current year.’ As soon as they are
 “ received I think they become chargeable. . . . On the whole
 “ matter I think the determination of the Commissioners is right.”

Income tax in
 respect of double
 endowment
 premiums.

In the last number of the *Journal* reference was made to the case of *Gould v. Curtis* (*J.I.A.*, xlv, 192), and it was there stated that the case would be dealt with more fully when it was reported in the Law Reports. It has now been so reported ([1912] 1 K.B. 635), but as the Inland Revenue authorities have lodged an appeal against the decision of Mr. Justice Hamilton, it is thought better that any further notes on the case should be deferred until the appeal has been heard.

Income tax in
 respect of interest
 received abroad
 when company is
 taxed on profits.

The case of *The Liverpool and London and Globe Insurance Company v. Bennett*, which deals with the liability of an insurance company to income tax in respect of interest received abroad and reinvested there, when the company is taxed on profits, has already been reported (*J.I.A.*, xlv, 387). The company appealed against the decision there reported, but that decision has now been affirmed by the

Court of Appeal ([1912] 2 K.B. 41). Two other cases in which the same point was involved, namely, *Brice* (Surveyor of Taxes) v. *Northern Assurance Company* and *Brice* (Surveyor of Taxes) v. *Ocean Accident and Guarantee Corporation*, were dealt with at the same time in a similar manner.

Nomination under the Industrial and Provident Societies Act, 1893. The case of *Eccles Provident Industrial Co-operative Society v. Griffiths*, deals with the question as to whether the limit of amount for a valid nomination depends on the amount standing to a member's credit at the date of the nomination or at the date of his death, and has already been reported in the *Journal* (*J.I.A.* xlv, 390). Mr. Justice Avory and a majority of the Court of Appeal decided that the question of validity depended on the amount standing to the member's credit at the date of the nomination, and this decision has now been affirmed by the House of Lords (1912, 28 T.L.R. 299).

An enquiry into the existence of Historical Analogies to Life Assurance. By T. E. YOUNG, B.A., F.R.A.S.

A FEW remarks upon this interesting question may not be unwelcome to the readers of the *Journal*: and, in order to secure some continuity of observation, I venture to embody them in the form of criticisms upon an instructive book which has recently been published.*

THE adequate historian of the lineage of social institutions is a man of special endowments. He must, obviously, *inter alia*, be possessed intensely of the dramatic faculty, and the power of detachment of imagination, so that he may decipher the significance of ancient forms, however diversified in appearance and surroundings, which may be regarded as primitive stages in the development of institutions; while, on the other hand, he must punctually refrain from introducing into those forms the exacter meanings and uses which elaborated institutions may now explicitly reveal. He must also, as a scientific explorer, be the master of Analogies instead of being, as too constantly occurs, their unthinking slave: for, generally speaking, analogies possess no inferential implication but simply afford a pictorial view (as an aid to vividness of conception and in a more concrete and familiar form) of the object which has already been discovered *aliunde*. The discovery having been completed,

* An Introduction to the History of Life Assurance. By A. FINGLAND JACK, sometime Gartside Research Scholar of the University of Manchester.

analogies from other departments of knowledge frequently help to clearer perception. The havoc is incalculable which was produced in commercial life (and in the consequent arrest of development of a nation) by the Aristotelian objection against interest that money does not "breed," from false analogy with living creatures; while history has frequently been distorted by minute attempts to apply the process of growth of an animal organism to a collection of people, and by the assumption of the necessary decay of a nation on the foolish analogical basis of the stages and termination of individual life. It may be justly laid down that all social institutions possess an historic course of development (usually obscure) since they are dependent upon needs and feelings which are inherent in human nature, and to which they form the external and intelligent response. For all effort towards an end is the product of feeling; the function of intellect is to find the way.

When Man is regarded in relation to his environment (attaching a large but obvious meaning to the latter term), it is perceived that the element of Risk or Danger (with the instinct of self-preservation) forms the origin of associated effort or union. In Life Assurance, however, the author justly indicates a distinction. In other activities it is the risk that is guarded against—the event which may, or may not, occur, or which may, or may not, happen frequently. But in Life Assurance there exists no risk in respect of death; that event is certain and universal; the term "risk" is irrelevant, and the event provided against is simply the uncertain date of its individual arrival. For, as the author rightly says, if there were some definite point of time at which life always ceased, "personal saving" would meet "every need." But even here, I remark, assurance would still be demanded, since the personal saving might be restricted or extinguished by the occurrence of ruined fortunes or the advent of ill-health or accident which prevented work.

Thus, we start with principles of human nature—permanent as to quality, variable only as to relative intensity; and our ultimate object (if it be attainable) is to discover some ancient forms of association which dealt with the uncertainty of the advent of death, and which, if possible, may be traced directly and genetically downwards into the finished mode of our modern system of assurance.

As a first stage the author furnishes a very interesting chapter (Chapter II.) upon the Roman Collegia, and particularly directs attention to special Collegia for the provision of a fitting burial for the members—the Collegia Tenuiorum, founded under the Empire. I interpose a word of explanation. The term "Collegium" simply means (as its etymology shows) an association of, at least, three persons (as prescribed by the Digest) for some permanent and lawful purpose recognised by the State as possessing the attributes of a Corporation. Rights and obligations appertain properly to persons, and the Collegia were invested with these rights and obligations by legally constructing an artificial or juristic person under the form of a Corporation. It is interesting and helpful to consider the title "Tenuiorum" of these Collegia which the author

particularly describes, and hence I adduce a further explanation. "Tenuis" (sometimes anciently written "Tenuius") is derived from a root "to be stretched out": hence it came to signify the qualities of *slim* or *fine*; then *poor* or *mean* or *insignificant*; and finally, as used by Lucretius and Cicero, the term was applied to persons of inferior rank and connoted summarily the "lower orders." Originally, and almost exclusively, pervaded, as these Collegia were, by a religious feeling, their secondary object of defraying the cost of members' funerals gradually assumed the primary care; and in respect of one of these poor Colleges, the author furnishes many instructive particulars regarding the entrance fees—the monthly contributions of about 3*d.* each, regardless of the duration of life or difference of age—the cessation of contributions after 50 years as some compensation to those who lived long, and the mutual constitution of the society with its mixture of good risks and bad. The result of our enquiry at this stage simply reveals the existence, under the Republic and Empire, of Burial Societies on a mutual foundation, without any trace of relationship to Life Assurance.

The Gild organisations are then considered in Chapter III (and sequent chapters), and as the author has suggested no definitions, it will be useful to perform this duty myself. The term "gild" (or "guild") is derived from the Low Latin (*see* Dufresne's Glossarium) and originally signified a "tax" or "tribute"; it was then also applied to the fine exacted in redress of a wrong (such as the Wergild), and even denoted the price of an article; in course of time, the term became transferred to any confraternity, brotherhood, or association established for the mutual aid and protection of its members, or for the prosecution of some common purpose, *e.g.*, the protection and advance of the common trade. The extension of the term, it will be noticed, as the title of a Society, is founded on the "tax," or contribution paid by the members for the realisation of the purpose of the union. It is of interest to observe—as I find from general reading—that the gilds mentioned in old English pre-Conquest documents present an affinity with the Collegia of Roman days, since the objects of the former included the providing of masses for the souls of dead members, besides the payment of Wergild in cases of justifiable homicide. With this digression, I return to our author. He justly points out that there is no continuity of development from the Roman Collegia, and hence a definite hiatus appears, so that we cannot affiliate these gild associations with the Latin Bodies as progenitors, except on the general basis of response to a common need.

Chapter IV treats instructively of the early history of gilds in England. In Chapter V, the Frith Gilds (Anglo-Saxon Frith = "peace, security and protection") are described, with their essential character of attaching responsibility to the members, in their corporate capacity, for the good conduct of each, and the provision of the cost of burial. Originated through the stress of defective Law and Civil order, these Associations gradually vanished when a firmer Central Government had accomplished the purpose which

they were established to fulfil. Here again we discover no trace of any community of form with Life Assurance.

With the Merchant Gilds (discussed in Chapter VI) our special interest is even more remote. While these Gilds provided monetary and social assistance to impoverished and feeble members, their primary object consisted in the acquisition and retention of the privilege of solely conducting particular trades; the possession of a trading monopoly against the other inhabitants of a town—and here it is to be remembered that the term “merchant” did not then possess the present meaning of an important dealer in goods, but practically included all who traded, whether on an inferior or extensive scale.

In Chapters VII, VIII, IX, X, and XI, Mr. Jack, with great knowledge and research, discusses the Craft Gilds, and rightly admits that no evolutionary connection can be discerned between them and the Collegia. These bodies provided their specific handicrafts with the protection which insecurity of government failed to afford, and continually advanced in importance through the minuter specialization of industry. A primary element of the gilds was the exclusion of all competition in the modern sense of the term; and the author indicates their additional functions of providing services of prayer and devotion for the members, compelling attendance (under fine) at members’ funerals, and the distribution of alms among the poor on such occasions for the soul’s sake of the dead. It is at once obvious, as the writer says, that these gilds thus exercised a narrowing effect upon industry and tended to destroy individual enterprise and self-dependent energy; a result exhibited, I might add, in the enfeebling and thoughtless social legislation of modern times—the artificial protection of a citizen against the troublesome effects of idleness and improvidence, which should, on the contrary, be permitted to operate as disciplines and incentives to a more responsible life, and a more conscientious industry.

The author deals with Social-Religious and Religious Gilds in Chapter X, and (in Chapter XI) with the forms of relief granted by the Craft Gilds; and, as he shows, the only condition, generally, of assistance in need (assuming the subscriptions were maintained) was the origination of the want from fortuitous circumstances and not as the effect of the applicant’s personal fault; in one Charter the expression is “without any self-guilt,” and in another, “through God’s sending.”

Part II of the book treats of Rents or Annuities, the System of the Montes, or the Modes of raising Funds for Loans to States or Towns, Gambling Insurance, the Scheme of Tonts and other Inventors, the Science of Life Contingencies, Friendly Societies, and the Advent of the Great Assurance Companies. Upon these chapters I have little to remark. With respect to Tontines, it has often been a subject of speculation to me that, in modern times, they have invariably, in my experience (and I have advised upon some extensive and promising schemes), failed entirely in attracting investors. Lands, houses, and properties generally, on a smaller or more extensive

scale, could be purchased on this plan, with productive results and the refund of capital, through the machinery of selected nominees and the combination of life assurance; but the public avoid the principle, possibly on the frequently valid ground that uniformity of return (and consequently the maintenance of the protecting policy) is not to be confidently expected where a considerable aggregate capital depends upon one class of property, and possibly, again, the apprehension that where the chosen nominees are comparatively young the investor will fail himself to see the termination of the scheme. In the chapter on Gambling Insurance, the vague term "gambling" demands a preciser definition than Mr. Jack appears to imply, for it might be shown that the example which he cites is in no way possessed of the gambling nature as commonly understood. All commercial businesses of every kind, and of the most legitimate character, are, in essence, speculations, though no careful observer would attach to them the title of gambling in the present significance of the word. Moreover, in a scientific investigation, it is imperative, before a valid judgment can be formed in any specific instance, to denude the term (in order to appreciate its sense and perceive its relevant application) of the immoral implications with which it is currently associated, and which have tended to deface and degrade its real meaning—to distinguish, in short, the meanings it may bear.

The chapter upon the Science of Life Contingencies will, I hope, be rewritten in a subsequent edition since the subject forms an integral part of the history, and should be sketched in a more adequate and connected form than this chapter presents.

I omitted, for the moment, to mention the interesting chapter upon Usury (Chapter XII) which demands a few words. The author, with praiseworthy research, traces the history of ecclesiastical and legal doctrines and proclamations upon usury, but does not appear to me to view the subject with sufficient clearness in relation to life assurance. It is evident that life assurance in any finished mode has been a function of, or has been historically conditioned by, (1) the advance in mathematical conceptions and processes, as embodied here in the doctrine of probabilities, and (2) the uncontrolled employment of capital on the basis of free demand and supply, as expressed here in the charge of interest—a condition possible only when the human mind, in every intellectual direction, and, here, in the commercial sphere, had emancipated itself from the oppressive dominion of an arbitrary ecclesiastical despotism which intruded into regions where (in the forms it adopted) its intrusion was the advent of a barbaric alien.

Bearing in mind the precise title of the book, and the author's definite statement that his object had been restricted to an examination of "certain main factors in the development of the Insurance idea," I have endeavoured to avoid any criticism which would assume that the volume formed a systematic treatise upon the fuller subject. My conclusions are, that the author has produced a very valuable and most instructive work, marked by careful and considerable

research, upon the various forms of social aid, and particularly upon this aspect of the system of guilds ; that the elements which he has so industriously exhibited, cannot, except in the very loosest and vaguest form, be regarded as anticipations or potential factors of life assurance ; and that, finally, any historical and continuous record of life assurance, consistently departing and expanding from primitive types in germ, is still indefinitely beyond our ken.

I may be permitted finally, in one's taste for exactness of terms, to advert to the author's proposition, in his General Remarks, that "Life Assurance is, above all, an economic arrangement." This, I suspect, is the casual use (to which we are all liable) of a word ("economic") which sounds excellently but has not been scrutinised as to its validity and relevancy of application to the question under discussion. The term "economic" has travelled from its primitive restricted significance, and now, according to the highest authorities, is employed as relating to the development and regulation of the *material* resources of a community or nation, and—I would suggest—to all elements, also, which affect that course or process. I fail to discern intelligibly any such meaning in "life assurance." The basis of the acknowledged significance of the term is not here involved, and life assurance is simply, in the long run—as at present organized—the equivalence of payments with receipts ; no production or regulation of material resources or wealth is even implicit. This irrelevancy of definition recalls the statement of a former able actuary that "the need of an equalization of the risks of death" originated the life assurance scheme. It is obvious, I think, that no such system equalizes the risks of death ; those risks remain precisely, after assurances are completed, as they existed before, and all that is effected is a uniformity of payment for a given contribution at a stated age at whatever interval the individually uncertain event may occur.

NOTICES OF NEW PUBLICATIONS.

Interest and Bond Values.

By Prof. M. A. MACKENZIE, M.A., F.I.A., A.A.S.

(University Press, Toronto ; C. & E. Layton, London.)

IN the opening chapters of the little text-book, "written mainly for the use of the Author's own classes in the elementary mathematics of finance," but deserving, and likely to obtain, a much wider circulation, Professor Mackenzie explains the nature and use of the ordinary interest functions, and demonstrates shortly their relations to one another, but the principal object of the book is to show how Interest Tables and Tables of Bond Values may be applied to the problems arising in connection with securities generally, but especially the various Bonds issued by commercial and municipal corporations in Canada and the United States.

The numerous examples are well chosen and the solutions easily followed. In many cases alternative solutions are given, based on ordinary Interest Tables alone, or on the special Tables used by the bond dealers. The Actuary—at all events in this country—would probably prefer to dispense with the assistance of these “Bond Tables,” which are extremely voluminous, of limited application, and sometimes inaccurate.

An interesting summary is given of the various approximations adopted, in practice, to fix the price, on a yield basis, of bonds sold between two interest dates, and an analysis is made of the errors involved. The statement on p. 53, however, that the error involved in the second method of approximation described “is greatest half-way between two coupon dates” is not strictly accurate, nor is the similar statement on p. 12, as to the maximum error introduced by the use of simple interest.

Chapter VI. contains a collection of nineteen complicated problems worked out in full, and Chapter VII., fifty “examples,” even more valuable to the examination student, inasmuch as he will have to evolve the solutions for himself.

The Student's Guide to Life Assurance.

By ARTHUR WYNDHAM TARN, F.J.A.

(Macdonald & Evans, London.)

MOST people, nowadays, are interested, as policyholders or otherwise, in the business of insurance, but comparatively few have any knowledge of how that business is carried on. Mr. Tarn aims at giving an insight into the principles of Insurance, and the practice of Insurance Companies, dealing at some length in Part I. with “Ordinary” Life Assurance, and very shortly with “Industrial” business.

After explaining mortality tables, premiums and valuations, the progress of an assurance is traced through the books of a typical life office, and its history recorded, from the completion of the proposal to the settlement of the claim.

The method adopted, and considerations of space, and lucidity made it impossible to indicate the considerable diversity of practice which exists in different offices, with the result that here and there the practice indicated will present an air of unfamiliarity even to those who have the advantage of a fairly wide experience; *e.g.*, it is surely very unusual for a Company, when settling a claim, to ask the claimants to pay its solicitors' charges for investigating the title (p. 88). Such a demand could hardly be enforced, except perhaps under a special condition in the policy.

Parts II. and III. are devoted to a very short summary of the principal features of Fire, Marine, Casualty, Burglary, Fidelity Guarantee, and Plate Glass Insurance.

S. G. D.

CORRESPONDENCE.

SICKNESS NOTATION.

To the Editors of the Journal of the Institute of Actuaries.

DEAR SIRS,—There are one or two points connected with Sickness Notation to which attention may, perhaps, usefully be drawn.

In some important sickness investigations there is an undesirable ambiguity as to the meaning of the words “rate of sickness.” Also, the symbol used is sometimes s_x and sometimes z_x . And in these cases, sometimes $s_x = z_x$ and sometimes s_x does not $= z_x$.

Here are some examples:

In the *Experience of Registered Friendly Societies* (Blue Book 303 of 1896, William Sutton) the “rate of sickness” is called s_x . On examining the mode by which the Exposed to Risk were computed one finds that the deaths between age x and age $x+1$ were regarded as at risk of sickness for the whole of the year of age x . Thus, this s_x denotes the rate of sickness as commonly understood.

In Mr. G. F. Hardy’s “Messenger Prize Essay on Friendly Societies,” the symbol z_x is used to denote the rate of sickness when, in computing the Exposed to Risk of Sickness, the deaths occurring between age x and age $x+1$ are regarded as at risk of sickness for the whole of the year of age x . Thus, Mr. Sutton’s s_x and Mr. Hardy’s z_x seem to mean the same thing.

In Mr. Watson’s admirable volume upon the “Manchester Unity Experience 1893–1897,” the “rate of sickness” is called s_x . On examining the mode by which the Exposed to Risk of Sickness were computed, one finds that “the number dying in each year were assumed to be at risk until the middle of the year.” That is to say, these deaths were regarded as at risk of sickness for one-half of the year, not for the whole of the year. Thus, the Manchester Unity “sickness rate,” s_x , is the central sickness rate, or the force of sickness in the middle of the year of age x to $x+1$. Thus, the Manchester Unity s_x is not the same thing as Mr. Sutton’s s_x , nor is it the same thing as Mr. Hardy’s z_x . But, as pointed out, Mr. Sutton’s $s_x =$ Mr. Hardy’s z_x .

Again, in the *Institute of Actuaries’ Text Book*, Part II., by Mr. George King, page 375, the “rate of sickness” is called z_x . And it is there defined in terms that agree with Mr. Hardy’s definition of z_x . Thus, Mr. King’s $z_x =$ Mr. Hardy’s z_x . The symbol s is used by Mr. King for the value of a sickness benefit.

Other examples could be quoted that show an existing confusion between the symbols s_x and z_x , and between the “rate of sickness” and the “central sickness rate.” I am not suggesting there is any confusion in the minds of the authors of the books here quoted. The confusion arises when a reader finds the same symbol used to denote different things. And unless the reader is well on his guard he may be making mistakes.

I venture to suggest that it is desirable for an authoritative pronouncement to be made by the Institute as to the meaning of

the two symbols s_x and z_x , including a statement as to the mode of computing the Exposed to Risk of Sickness. For the mode of computing these exposures has an appreciable effect upon the "rate of sickness" obtained.

Another point that emphasizes the desirability to attach an exact meaning to items of sickness notation, is that in comparing the actual and the expected time of sickness and cost of sickness in any Friendly Society that may be under observation, the expected sickness being based upon this or that standard Sickness Experience, it is necessary to compute the Exposed to Risk of Sickness in the Friendly Society under observation by the same mode as that by which the Exposed to Risk of Sickness in the Standard Experience were computed. For instance, in testing a Friendly Society's experience by the s_x of the Manchester Unity the Exposed to Risk of Sickness in the Friendly Society should give only half a year's exposure to the deaths. But in testing the same Friendly Society's sickness by the s_x of Mr. Sutton in Blue Book 303 of 1896, the Exposed to Risk of Sickness in the Friendly Society under observation must be recomputed so as to give the full year's exposure to sickness in respect of each death.

Then again, when constructing commutation columns, the present absence of standardization of sickness notation is a quite possible source of error. For, in addition to the confusion of notation herein mentioned there is the further point that commutation columns based upon a rate of sickness have to be prepared differently from commutation columns based upon a central sickness rate.

These matters seem to need an authoritative statement by the Institute.

Yours truly,

JOHN HOLT SCHOOLING.

*Fotheringhay House,
Twickenham,
1st May 1912.*

INTERNATIONAL CONGRESS OF ACTUARIES, 1912.

LIST OF PAPERS TO BE CONTRIBUTED BY MEMBERS OF THE INSTITUTE OF ACTUARIES.

Papers for Discussion—

"The Organisation in Public Administration of Old Age Pensions", by Mr. Vyvyan Marr.

"The Incontestability of Life Assurance Policies", by Mr. A. R. Barrand.

"The Course (since 1800) of the Mortality of Assured Persons, distinguishing, if practicable, the Sexes, Professions. Types of Assurance, etc.", by Messrs. J. Burn and W. C. Sharman.

“On the Mortality Experience of the Equitable Life Assurance Society during the Nineteenth Century”, by Mr. H. W. Manly.

“The Question of the Loading of Premiums: the Calculation of Office Premiums”, by Mr. H. W. Andras.

“The Question of the Loading of Premiums, and the Calculation of Office Premiums for Industrial Assurances”, by Mr. J. Bacon.

Papers not for Discussion—

“Tables of Mortality for the Assurance of Infantile Lives”, by Mr. S. J. H. W. Allin.

“The Course of Legislation, of the Teaching of Assurance, and of the Development of the Contract of Assurance since the Congress at Vienna”, by Mr. A. Wyndham Tarn.

ERRATUM.

British Offices Life Tables, 1893—Aggregate Tables.

$O^{M(5)} 3\frac{1}{2}$ per-cent Value of $a_{43:9}$ (Page 185). For 7·176
read 7·171.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Superannuation and Pension Funds of certain Metropolitan Borough Councils, their Establishment, Administration, and Actuarial Investigation. By HENRY WILLIAM MANLY, *Past President of the Institute of Actuaries*, and THOMAS GANS ACKLAND, *Fellow of the Institute of Actuaries and of the Faculty of Actuaries.* With Tables of Progress of typical Funds for Officers and Workmen, and Examples; by LOUIS ERNEST CLINTON, *Fellow of the Institute of Actuaries.*

[Read before the Institute, 29 April 1912.]

HISTORICAL AND INTRODUCTORY.

THE Metropolitan Boroughs in their existing form were established by the London Government Act, 1899 (62 and 63 Vict., Ch. 14), by the provisions of which the Vestries and District Boards and Metropolitan Board of Works, constituted under the Metropolis Management Act, 1855 (18 and 19 Vict., Ch. 120), were replaced by 28 Boroughs, of which 19 consisted of former parishes, or combined parishes, 2 of District Boards, and 7 of Parliamentary Boroughs, or combined boroughs or districts, included within the area dealt with by the Act of 1899. The following particulars as to the population and rateable value of each Borough are taken from Whitaker's Almanac for 1912. The yield from a rate of one penny in the £ of rateable value has also been added.

It will be seen that the population of individual Boroughs varies between the limits of 49,336 (Holborn) and 327,423 (Islington), the total rateable value between £353,131 (Stoke

Metropolitan Borough	Population	Rateable Value £	Yield from Rate of 1 <i>d.</i> in £
		£	£
Battersea . . .	167,793	1,019,959	4,250
Bermondsey . . .	125,960	919,998	3,833
Bethnal Green . . .	128,282	537,698	2,240
Camberwell . . .	261,337	1,386,960	5,779
Chelsea . . .	66,404	901,552	3,757
Deptford . . .	109,498	627,176	2,613
Finsbury . . .	87,976	1,010,893	4,212
Fulham . . .	153,325	896,675	3,736
Greenwich . . .	95,977	693,669	2,890
Hackney . . .	222,587	1,227,043	5,113
Hammersmith . . .	121,603	874,360	3,643
Hampstead . . .	85,510	1,099,823	4,583
Holborn . . .	49,336	1,103,990	4,600
Islington . . .	327,423	1,900,678	7,919
Kensington . . .	172,402	2,419,344	10,081
Lambeth . . .	298,126	1,908,590	7,952
Lewisham . . .	160,843	1,105,438	4,606
Paddington . . .	142,576	1,532,017	6,383
Poplar . . .	162,449	810,382	3,377
St. Marylebone . . .	117,844	2,121,787	8,841
St. Pancras . . .	218,453	1,759,432	7,331
Shoreditch . . .	111,463	810,559	3,377
Southwark . . .	191,951	1,248,994	5,204
Stepney . . .	280,024	1,512,164	6,301
Stoke Newington . . .	50,683	353,131	1,471
Wandsworth . . .	311,402	2,126,163	8,859
Westminster City . . .	160,277	6,422,553	26,761
Woolwich . . .	121,403	749,101	3,121
TOTALS . . .	4,502,927	39,080,129	162,833

Newington) and £6,422,553 (City of Westminster), and the yield of 1*d.* rate in the £ of rateable value from £1,471 (Stoke Newington) to £26,761 (City of Westminster), the average yield of 1*d.* rate over the 28 Boroughs (which exclude the City of London), being £5,816.

Prior to the passing of the Metropolitan Superannuation Allowances Act, 1866 (29 Vict., Ch. 31), there appear to have been no express powers vested in the Metropolitan Boroughs to grant superannuation allowances or pensions to employees retiring from infirmity or old age. The 1866 Act gave power to the Vestries and District Boards, and to the Metropolitan Board of Works, established under the 1855 Act, to grant to any officer "who shall become incapable of discharging the duties of his office with efficiency by reason of permanent infirmity of mind or body, or of old age, upon his resigning or otherwise ceasing to hold his office. an annual allowance not exceeding two-

“thirds of his then salary, regard being had to the scale of “allowances hereinafter contained.” No allowance was to be granted on retirement from old age at an age earlier than 60 years. The scale of allowances commences at ten-sixtieths of the salary and emoluments current at retirement, in respect of ten years’ completed service with the Vestry or District Board, or earlier authority superseded by such Vestry or Board; with the addition of one-sixtieth, in respect of each further completed year of service, up to a maximum of forty-sixtieths, in respect of forty years’ service and upwards. In cases where exceptional professional or other peculiar qualifications are required, an officer of upwards of 30 years of age may be granted by the Vestry or Board an addition of not more than ten years to the term of service counting for superannuation. Powers were also given to grant gratuities, not exceeding three months’ pay in respect of every two years of service, to those compelled to quit the service, “by reason of “any severe bodily injury occasioned, without his own default, “in the discharge of his public duty, or from infirmity of mind or “body, before the completion of the period which would entitle “him to a superannuation allowance.” Under the Metropolis Management Act, 1899, the powers and duties of the Vestries and District Boards were transferred to the respective Borough Councils established under that Act. It is to be noted that the Metropolitan Superannuation Act of 1866 dealt only with public authorities in the Metropolitan area; and Provincial Councils and other authorities outside the Metropolitan area have apparently no specific powers to grant superannuation or pension allowances or gratuities on retirement, other, of course, than any such powers included in special Acts of Parliament or Charters obtained by such provincial authorities.

Under the provisions of the Poor Law Officers Superannuation Act, 1896 (59 and 60 Vict., Ch. 50), a contributory scheme of Superannuation allowance was established, the conditions being generally similar, as to benefit, to those of the Metropolitan Superannuation Allowances Act, 1866, referred to above. The scale of contributions from the Officers and Servants was fixed at 2 per-cent on the salary or wages and emoluments, for those having less than 5 years’ service at the date of passing of the Act, or appointed subsequently; $2\frac{1}{2}$ per-cent for those having from 5 to 15 years’ service, and 3 per-cent for those having upwards of 15 years’ service. The Act applies only to Poor Law Officers and Servants, and to the managers, officers and servants of

district schools and sick asylums. No special superannuation funds were set up under this Act, the allowances being payable out of "the common fund of the Union", and the contributions of the employees being carried to the same fund.

In recent years there has been a considerable movement amongst Metropolitan Borough Councils towards the establishment of Superannuation and Pension Funds, upon a contributory basis; and in the case of several of these Councils, Acts of Parliament have been obtained for this specific purpose. The first that has come under our notice was the Act passed for the Borough of Stepney in 1905, and, in rapid succession, other Councils have secured similar powers. The following Acts are dealt with in the present Paper :

Stepney Borough Council (Superannuation) Act, 1905 (5 Edw. 7, Ch. 97).

Bethnal Green Borough Council (Superannuation) Act, 1906 (6 Edw. 7, Ch. 6).

Kensington Borough Council (Superannuation) Act, 1907 (7 Edw. 7, Ch. 94).

Camberwell and other Metropolitan Borough Councils (Superannuation) Act, 1908 (including Deptford and Hackney) (8 Edw. 7, Ch. 19).

St. Marylebone Borough Council (Superannuation) Act, 1908 (8 Edw. 7, Ch. 21).

Wandsworth Borough Council (Superannuation) Act, 1909 (9 Edw. 7, Ch. 9).

Westminster City Council (Superannuation and Pensions) Act, 1909 (9 Edw. 7, Ch. 59).

Paddington Borough Council (Superannuation and Pensions) Act, 1911 (2 Geo. V., Ch. 101).

Poplar Borough Council (Superannuation and Pensions) Act, 1911 (2 Geo. V., Ch. 102).

This is perhaps not quite a complete list of the existing Acts, and it is understood that Bills promoted by certain other Metropolitan Borough Councils will be submitted to Parliament during this and the ensuing Session.

It is to be observed that, in all the Acts up to that obtained by the Borough of Wandsworth inclusive, Officers and Servants were included under identical schemes as to benefit and contribution, an "Officer" being defined as "every Officer in the service of the Council in an established capacity" (Stepney, Bethnal Green, and Kensington); or "designated an Officer in an established capacity by a resolution of the Council passed or to be passed" (St. Marylebone); or "wholly in the service of

“ the Council in an established capacity ; provided that nothing
 “ in this definition shall be deemed to exclude an Officer who by
 “ consent of the Council holds other appointments ” (Camberwell,
 &c.) ; or “ in the service of the Council and placed upon the
 “ permanent establishment by a resolution of the Council passed
 “ or to be passed ” (Wandsworth). A “ Servant ” is defined as
 “ every servant or workman appointed by resolution and
 “ in the permanent employ of the Council ” (Stepney and
 Kensington) ; “ every servant or workman in the permanent
 employ of the Council ” (Bethnal Green) ; “ every servant or
 “ workman in the permanent employ of the Council, and
 “ appointed by the Council or by a committee or Officer of
 “ the Council duly authorized (either generally or specially) to
 “ make the appointment ” (St. Marylebone) ; “ every servant
 “ or workman placed upon the permanent establishment by
 “ resolution of the Council ” (Camberwell, &c.) ; “ every servant
 “ or workman in the service of the Council and placed upon
 “ the permanent establishment by a resolution of the Council
 “ passed or to be passed ” (Wandsworth).

In the later Acts granted to Westminster, Paddington, and Poplar, the benefits and contributions are on different scales for Officers and Servants ; an “ Officer ” being defined as “ every Officer
 “ wholly* in the service of the Council in an established capacity
 “ and every servant on weekly wages exceeding with emoluments
 “ other than overtime forty-five shillings a week in the permanent
 “ employ of the Council and duly placed on the establishment
 “ staff ; provided that nothing in this definition shall be deemed
 “ to exclude such an Officer who by consent of the Council holds
 “ other appointments ” ; the definition being uniform for the
 three Boroughs, but the proviso appearing in the Westminster
 Act only.

In the Westminster, Paddington, and Poplar Acts a
 “ Servant ” is defined as “ every servant or workman not
 “ less than 20 years of age on weekly wages not exceeding with
 “ emoluments other than overtime forty-five shillings a week
 “ in the permanent employ of the Council and duly placed on
 “ the establishment staff.” It will be seen that in these last
 named Acts, where benefits and contributions are discriminated
 for Officers and Workmen, the “ Officers ” comprise, generally
 speaking, the clerical staff with salaries payable monthly or
 quarterly, and the Workmen at weekly wages exceeding (with

* The word “ wholly ” is omitted in the Paddington Act.

emoluments) forty-five shillings weekly; and that the "Servants" comprise the Workmen of 20 years of age and upwards who receive a lower rate of weekly pay.

The Benefits provided and the contributions payable for Officers and Servants, under the Metropolitan Borough Council Acts which have come under our notice, are set out for comparison in the following tabular statements, marked (A) and (B) respectively. It will be seen that the conditions as to allowances and contributions are, generally speaking, similar to those set forth in the Poor Law Officers Superannuation Act, 1896.

ESTABLISHMENT AND ADMINISTRATION, AND ACTUARIAL INVESTIGATION, OF SUPERANNUATION AND PENSION FUNDS.

The provisions of the various Acts obtained by the several Metropolitan Borough Councils as to the establishment and administration of the Superannuation and Pension Funds are set out in detail in the appended tabular statement (C), from which it will be seen that the general lines laid down in this respect in the earliest Acts—those for Stepney and Bethnal Green—have been substantially followed in the Acts granted successively to Kensington, Camberwell, St. Marylebone, and Wandsworth, the only material difference being the introduction of a "primary annual contribution" (to be actuarially fixed and charged on the rates) in lieu of a fixed sum of one hundred pounds per annum. In the Westminster Act, the administration section (3), is stated somewhat more concisely, but with no material practical change. In the Paddington and Poplar Acts, section (3) is altogether omitted, and additional words are introduced in section (1), providing for the payment to and inclusion in the fund of lump-sum payments received by the employee from other public authorities, in respect of prior service with such authorities, and also (in the Poplar Act only) for the proportionate allocation of the primary annual contribution between the general rates and the funds of the Council's Electricity Undertaking.

The provisions for actuarial investigation of the Fund are set out in the appended statement (D). This section was first included in the Kensington Act, and was reproduced in the Camberwell, St. Marylebone, Wandsworth, and Westminster Acts, the only variation being as to the minimum amount of the primary annual contribution. In the Paddington and Poplar Acts, material modifications were introduced in the actuarial clause, the effects of which will be discussed later on.

The provisions for the administration of the Fund as set out in statement (C) are somewhat remarkable, and merit careful study. Their effect seems briefly to be that charges on the Fund are to be made (1) upon the income *arising solely during the year of charge* in respect of (a) the primary annual contribution (in the case of Stepney and Bethnal Green consisting of an annual sum of £100, and actuarially fixed in the other cases, with due regard to any specified minimum); (b) the percentage contributions of the Officers and Servants; (c) the dividends and interest accruing on any invested Fund; (2) out of capital, but limited in any year to an amount not exceeding 10 per-cent of the amount of the Fund for the time being; and (3) out of the general rates. These provisions involve the establishment and accumulation of a Fund, the capital of which can only be dealt with (and then only to a limited extent) in the event of all source of income *arising in the year of charge* proving inadequate for the allowances of the year; and the effect would seem to be that any excess income, arising in the early years of the Fund, when the income from contributions would be largely in excess of current allowances, would be capitalized, and practically locked up for all time, subject only to a possible draft to the extent of 10 per-cent per annum, in any quinquennial period during which the actuarial estimate of the primary annual contribution for that period (or the fixed sum in the case of Stepney and Bethnal Green) should prove inadequate for the actual current charges arising during the period. These are unworkable conditions, from an actuarial point of view; and, in the event of the Fund being at any time closed to new entrants, the income of the year and the limited draft on capital might not be sufficient to keep pace with the current allowances in respect of the existing members, and an undue charge upon the rates would appear in that case to be the only alternative course.

The payment of current charges out of current income may be regarded as an application of the "assessment" principle, and it is understood that some of the Boroughs have been advised, both actuarially and legally, that the provisions of the Acts (other than those for Paddington and Poplar) now being discussed, would be met by providing from the rates for the estimated charges actually arising during the current quinquennium, after allowing for actual receipts from contributions and interest during the period. This would, of course, involve (as is shown financially by Tables and examples given later on in the present Paper)

reduced charges on the rates during the early years of the Fund, which would, however, rapidly increase as the retiring allowances emerged in increasing numbers, so that the charge on the rates would ultimately increase to a maximum which would bear a very heavy proportion to the total amount of the salaries and wages payable, involving an unduly heavy charge on future generations, with an undue lightening of the charge on the present generation of ratepayers. The financial solvency of the scheme would be always well secured by the ultimate charge upon Parliamentary rates, which are largely in excess of all possible requirements; and it might be argued that in such circumstances, and under such conditions, the creation of a Fund would be unnecessary, especially if it is to be tied up by the remarkable conditions included in the Statutes now under consideration.

The attention of the Local Government Board was anxiously directed to the provisions of these Acts, and to their effect, as illustrated by the action of some of the Borough Councils in adopting, under actuarial and legal advice, a "primary annual contribution" which covered only the estimated charges during the current quinquennial period, and which would involve largely increased charges on future generations of ratepayers; and in February 1911 the Local Government Board invited the observations of the Institute of Actuaries on the provisions of the actuarial clause of the Paddington Borough Council Bill, then under consideration by the Parliamentary Committee on Local Legislation. The Institute of Actuaries, in a letter dated 8 March 1911, expressed their views on the questions submitted to them, and made important suggestions for the variation of the administration and actuarial clauses of the Bill, which were substantially given effect to in the Paddington Act as settled by Parliament, and in the Act obtained in the same Session for the Borough of Poplar. The Report of the Local Government Board on the Poplar Bill, which includes a copy of the letter from the Institute of Actuaries, referred to above, is printed as an appendix to the present Paper.

The important changes made in the Paddington and Poplar Acts can best be appreciated by giving the complete text of the actuarial clause, which runs as follows (omitting only references (in the Poplar Act only) to the separate electricity undertaking of the Council, with which our present discussion is not concerned):

"(17) Within six months after the commencement of this Act,

“ and at the expiration of every subsequent period of five years,
“ dating from the first day of January one thousand nine hundred
“ and twelve, the condition of the Superannuation Fund shall be
“ submitted by the Council to an Actuary being a Fellow either
“ of the Institute of Actuaries or of the Faculty of Actuaries in
“ Scotland, appointed by them and approved by the Local
“ Government Board, who shall consider the same, shall make an
“ actuarial valuation of the Fund, and on the basis of such valua-
“ tion shall certify what proportion in his opinion the primary
“ annual contribution shall bear to the total salaries of the
“ officers . . . so that without further recourse to the
“ general rates . . . the Superannuation Fund as con-
“ stituted under paragraphs (a) (b) and (c) of sub-section (1) of
“ section 16 [see Statement C] shall be solvent (having regard to
“ existing and prospective liabilities) and for the next quin-
“ quennial period the primary annual contribution shall be in
“ the proportion so certified, and shall be paid to the Super-
“ annuation Fund accordingly. Provided that in making a
“ certificate under this section the Actuary shall take into
“ account the sums which the Council are now paying under the
“ Superannuation (Metropolis) Act, 1866 [that is, current super-
“ annuation and pension allowances granted prior to the passing
“ of the present Act], or will hereafter pay under section 4 (4)
“ of this Act [in respect of superannuation and pension allow-
“ ances granted to employees who, at the date of passing of the
“ Act had either attained the age of 60 years after 40 years’
“ service or attained the age of 65 years], and make such certificate
“ as will cast upon the rates . . . as nearly as may be an
“ even annual charge so long as this Act is in operation in
“ respect of the expenditure both under this Act and under the
“ Superannuation (Metropolis) Act, 1866.”

It will be observed that, under this clause (a) the *proportion* of the primary annual contribution to the total salaries is to be ascertained on an actuarial valuation, instead of the *amount* of such contribution, and the valuation is to have regard to existing and prospective liabilities, and to secure the solvency of the Fund; (b) the charge upon the rates is to be “ as nearly as may be an even annual charge so long as this Act is in operation ”; (c) section (3) of the administrative clause, as included in all former Acts, limiting the charges in any year to the revenue of that year only, with a limited draft on the capital Fund, and ultimate recourse to the rates, is omitted altogether, the clause

being now limited to a statement of the several items which are to be carried and credited to, or charged upon, the Superannuation Funds respectively.

These modifications, as will be seen, sweep away the complicated and unworkable machinery of administration, established under all the previous Acts, and thus effect a great improvement. They also incorporate substantially the recommendations made by the Institute of Actuaries, and thus make it clear that in assessing the "primary annual contribution" the prospective, as well as the current, assets and liabilities, are to be brought into account by the Actuary, and apparently intend that the charges should be distributed equitably over the present and future generations of ratepayers. The words providing that the primary annual contribution is to be so ascertained as to cast upon the rates "as nearly as may be an even annual charge so long as this Act is in operation", which words are not included in the suggestions made by the Institute of Actuaries, nor in the printed Report of the Local Government Board on the Poplar Bill, are, however, somewhat puzzling, and their intention and effect are not altogether clear. The actual charges incident upon the Fund (as shown later on in the present Paper by examples) will be quite small at the outset, but will steadily increase, as the members existing at the establishment of the Fund retire from active service, and bring upon the Fund charges in respect of superannuation and pension allowances, based on terms of service which includes the periods running from entry up to the establishment of the Fund. These allowances will increase to a maximum, and will then steadily diminish, as the members so superannuated pass out by death. As the existing members on retirement will be replaced by new entrants, paying a minimum rate of contribution during the whole period of their service counting for superannuation, the charges on the Fund will ultimately fall, until, when all the members existing at the outset have passed out by death, the charges will be represented solely by the proportionate contribution actuarially required to secure the benefits to new members. It would not be equitable to meet these annual charges, first steadily rising and then as steadily falling to a minimum, by "an even charge so long as this Act is in operation", for this could only be provided by a uniform perpetual rent-charge, equal in present value to the amount of the actuarial liability as ascertained at the establishment of the Fund. This would spread the liability in respect of the past

service of the then existing members over the ratepayers of the present and all future generations, which would not be an equitable solution, and can hardly have been intended by the Parliamentary Committee who settled the provisions of the Paddington and Poplar Acts.

The intention of the added words providing for "as nearly as may be an even annual charge," was probably to make it still further clear (although it was already sufficiently plain) that the prospective liabilities in respect of members existing at the outset were to be brought into account, and not the current liabilities only ; and that such liabilities were to be provided by an equal annual charge, extending over the term of active service (or over the whole lifetime) of such existing members ; and it appears to have been overlooked that the rate-charge, if equitably assessed, must necessarily fall to a minimum rate, when all the members on the Fund are those paying contributions from the commencement of their service counting for pension. In this connection it is to be remarked that in considering the Bill promoted by the Urban District Council of Chiswick (outside the Metropolitan Area) in 1911, the Local Legislation Committee of Parliament expressed the opinion that such uniform annual charge should be so computed as to discharge the liabilities in respect of the members originally existing during a term not longer than the active service of such members, and declined to agree to a suggestion that the term might extend over the longer period of the whole future lifetime of such members. The opinion thus expressed appears to be quite inconsistent with the desire to make the amount of the annual charge, as far as practicable, uniform in amount, so long as the Act is in operation ; and it seems to be very desirable that in future Acts the reference to "as nearly as may be an even charge" should be omitted from the actuarial clause.

ON THE WORKING OF A FUND WHICH IS ESTABLISHED AND
MAINTAINED BY A UNIFORM NUMBER OF ENTRANTS,
JOINING THE FUND EACH YEAR AT A FIXED AGE, AND AT
A UNIFORM RATE OF CONTRIBUTION.

Confining our attention for the present to the consideration only of future entrants at a uniform rate of contribution, our object is to show in respect of such entrants :

- (1) The true actuarial contribution required to meet the various benefits in respect of officers and workmen, and the progress of the funds derived from (a) the true contributions ; and (b) the members' contributions only.
- (2) The charges on the rates that will be ultimately required, after
 - (a) applying in the first instance the accumulated contributions from the members towards the payments, until exhausted ;
 - (b) applying the accumulated contributions towards the payments until the Fund reaches its maximum, and thereafter applying the interest on the Fund and the members' contributions towards the outgoings.

The conditions assumed as to pensions in the illustrative Tables which follow are those outlined in Statement (A), appertaining to the Boroughs of Westminster, Paddington and Poplar.

The workmen's pensions, when compared with the average wages, work out almost exactly at one fifty-fifth for each year's service, with a maximum of forty fifty-fifths, and this rate has been adopted in the calculations.

On withdrawal and death before retirement it has been assumed that the members' contributions will be returned, without interest.

As regards the contributions, it has been assumed that the officers contribute at a uniform rate of $2\frac{1}{2}$ per-cent from entry, whilst the workmen contribute in accordance with the scale set out in Statement (B), appertaining to the Boroughs of Westminster, Paddington and Poplar, which works out on the average almost exactly at $1\frac{1}{2}$ per-cent of the average wages, and a contribution of $1\frac{1}{2}$ per-cent was therefore assumed in the subsequent calculations.

Table I, giving the "rates" per-cent per annum of withdrawal, mortality and retirement, which have been adopted for officers and workmen respectively, and the pensioners' rate of mortality, for both officers and workmen, has been compiled from a large amount of material collected from similar funds.

From the rates given in Table I, the Life and Service Tables shown in Tables II and III were constructed, and these Tables are the basis of all our subsequent calculations. The Average Salary Scales were deduced from the combined data of five of the Metropolitan Boroughs, and may be deemed to be the Average Salaries at exact ages. In order to obtain the average

salary during any year of age the scales were adjusted so that $\bar{s}_x = \frac{s_x + s_{x+1}}{2}$.

An investigation of the cases existing at the commencement of the Fund in each Borough brought out an average entry age approximating to 25 for the officers, and 30 for the workmen ; but in these data no withdrawals were included, and as no doubt a large number of withdrawals would have occurred in respect of junior officers and workmen, it was decided to take the average entry age, for the purpose of our calculations, as 20 for officers, and 25 for workmen ; thus making some arbitrary allowance for the effect of the withdrawals.

Proceeding on the lines indicated by Mr. Manly in his last paper (*J.I.A.*, vol. xlv, p. 149), Tables of Accumulations at $1\frac{3}{4}$ per-cent half-yearly related to 1 per-cent of Salaries or Wages were next formed, and the values at quinquennial intervals are shown in Tables IV and V.

It may be pointed out that by the method followed of constructing the Service Tables the convenient assumption has been made that all deaths, withdrawals and retirements take place at the end of the year of age, but throughout the following calculations it has been assumed that, in accordance with the conditions of practice, they are equally distributed over the year.

It will be observed that Table V presents a new feature, namely : " the returns on retirement during the first 10 years of membership." The pensions for both officers and workmen are subject to 10 years' service, and hitherto we believe it has been the practice to allow for this element in the construction of the pension factors by making $r_x=0$ for the 10 years following entry. This procedure is not, of course, strictly accurate, unless the other columns in the Life and Service Tables are adjusted. In order to overcome this difficulty therefore, it was decided to assume that a return similar to that on death or withdrawal was made in respect of retirements during the first 10 years of membership. A similar assumption was not necessary in Table IV, the entry age being 20, and the first retirements according to the Service Table taking place at age 30.

From Tables IV and V the true actuarial contributions required, as a percentage of the salaries or wages, were ascertained to be 8.10736 for officers entering at age 20, and 5.84264 for workmen entering at age 25.

TABLE I.

*Experience Table.**Rates per cent per annum of Withdrawal, Mortality and Retirement.*

Age	CONTRIBUTING MEMBERS - OFFICERS			CONTRIBUTING MEMBERS - WORKMEN			RATE OF MORTALITY AMONGST PENSIONERS - OFFICERS AND WORKMEN		
	Rate of Withdrawal	Rate of Mortality	Rate of Retirement	Rate of Withdrawal	Rate of Mortality	Rate of Retirement	Rate of Mortality	Age (Contd.)	Rate of Mortality (Contd.)
15	7.50	.36	65	5.000
6	8.92	.37	6	5.120
7	9.10	.38	7	5.240
8	6.5	.20	...	8.38	.39	8	5.360
9	6.0	.20	...	7.64	.40	9	*5.490
20	5.5	.21	...	6.98	.41	70	5.926
1	5.0	.21	...	6.30	.42	1	6.397
2	4.7	.22	...	5.71	.43	2	6.913
3	4.4	.23	...	5.18	.44	3	7.466
4	4.1	.24	...	4.72	.45	4	8.074
5	3.8	.25	...	4.31	.46	5	8.727
6	3.5	.26	...	3.96	.48	6	9.435
7	3.3	.27	...	3.64	.49	7	10.205
8	3.1	.28	...	3.35	.50	.02	10.0	8	11.035
9	2.9	.29	...	3.09	.52	.03	10.0	9	11.932
30	2.7	.30	.035	2.85	.54	.04	10.0	80	12.904
1	2.5	.32	.055	2.64	.56	.05	10.0	1	13.948
2	2.3	.34	.075	2.43	.58	.06	10.0	2	15.078
3	2.1	.36	.095	2.26	.60	.07	10.0	3	16.289
4	1.9	.38	.12	2.09	.62	.08	10.0	4	17.596
5	1.7	.40	.14	1.95	.64	.09	10.0	5	18.995
6	1.5	.42	.16	1.80	.67	.10	10.0	6	20.499
7	1.4	.44	.19	1.66	.70	.12	10.0	7	22.105
8	1.3	.46	.22	1.54	.73	.14	10.0	8	23.824
9	1.2	.49	.24	1.43	.76	.17	10.0	9	25.655
40	1.1	.52	.27	1.32	.80	.20	10.0	90	27.603
1	1.0	.55	.30	1.21	.84	.23	9.6	1	29.673
2	.9	.58	.34	1.12	.89	.27	9.2	2	31.862
3	.8	.62	.38	1.01	.94	.31	8.8	3	34.175
4	.7	.65	.42	.92	1.00	.35	8.4	4	36.610
5	.6	.69	.47	.82	1.06	.40	8.0	5	39.165
6	.5	.73	.53	.72	1.12	.45	7.6	6	41.835
7	.4	.77	.59	.62	1.18	.50	7.2	7	44.618
8	.3	.81	.68	.53	1.24	.55	6.8	8	47.502
9	.2	.85	.80	.43	1.31	.60	6.4	9	50.478
50	.1	.89	.92	.33	1.39	.66	6.0	100	53.536
194	1.08	.24	1.47	.73	5.8	1	56.657
298	1.27	.14	1.56	.81	5.6	2	59.825
3	...	1.03	1.50	.05	1.66	.90	5.4	3	63.019
4	...	1.09	1.74	...	1.76	1.00	5.2	104	100.000
5	...	1.15	2.00	...	1.87	1.23	5.0
6	...	1.22	2.29	...	1.99	1.59	4.8
7	...	1.33	2.60	...	2.13	2.08	4.6
8	...	1.49	2.95	...	2.27	2.70	4.4
9	...	1.66	3.33	...	2.42	3.50	4.2
60	...	1.86	3.90	...	2.58	4.40	4.0
1	...	2.12	4.60	...	2.76	5.40	4.2
2	...	2.42	5.50	...	2.98	6.60	4.4
3	...	2.74	6.60	...	3.26	8.00	4.6
4	...	3.07	7.90	...	3.64	9.60	4.8
65	100.00

* Oases Table, at ages 60 and higher ages.

TABLE II.

Life and Service Table.—Officers.

Based upon the Rates per-cent per annum of Withdrawal, Mortality, and Retirement given in Table I.

Age	Existing in Service	Withdrawals	Deaths	Retirements	Average Salary
					£
18	10,000	650	20	...	55
9	9,330	560	19	...	61
20	8,751	481	18	...	67
1	8,252	413	17	...	73
2	7,822	368	17	...	79
3	7,437	327	17	...	85
4	7,093	291	17	...	91
5	6,785	258	17	...	98
6	6,510	227	17	...	105
7	6,266	207	17	...	113
8	6,042	187	17	...	121
9	5,838	169	17	...	129
30	5,652	151	17	2	138
1	5,482	139	18	3	148
2	5,322	124	18	4	158
3	5,176	108	19	5	167
4	5,044	100	19	6	175
5	4,919	86	20	7	182
6	4,806	74	20	8	188
7	4,704	67	21	9	193
8	4,607	59	22	10	197
9	4,516	52	22	11	201
40	4,431	47	23	12	205
1	4,349	43	24	13	209
2	4,269	37	25	15	213
3	4,192	34	26	16	217
4	4,116	28	27	17	220
5	4,044	22	28	19	223
6	3,975	19	29	21	226
7	3,906	16	30	23	229
8	3,837	12	31	26	232
9	3,768	8	32	30	234
50	3,698	3	33	34	236
1	3,628	...	34	39	238
2	3,555	...	35	45	240
3	3,475	...	36	52	242
4	3,387	...	37	59	244
5	3,291	...	38	66	246
6	3,187	...	39	73	247
7	3,075	...	41	80	248
8	2,954	...	44	87	249
9	2,823	...	47	94	250
60	2,682	...	50	510	251
1	2,122	...	45	637	252
2	1,440	...	35	648	253
3	757	...	20	379	253
4	358	...	11	179	253
5	168	168	253

TABLE III.

*Life and Service Table.—Workmen.**Based upon the Rates per-cent per annum of Withdrawal, Mortality, and Retirement given in Table I.*

Age	Existing in Service	Withdrawals	Deaths	Retirements	Average Wages
					£
15	10,000	750	36	...	20
6	9,214	822	34	...	30
7	8,358	760	32	...	40
8	7,566	635	29	...	50
9	6,902	527	28	...	60
20	6,347	443	26	...	66
1	5,878	370	25	...	68
2	5,483	313	23	...	70
3	5,147	267	23	...	72
4	4,857	229	22	...	74
5	4,606	198	21	...	76
6	4,387	174	21	...	77
7	4,192	153	20	...	78
8	4,019	135	20	1	79
9	3,863	119	20	1	80
30	3,723	106	20	2	80
1	3,595	95	20	2	80
2	3,478	85	20	2	80
3	3,371	76	20	2	80
4	3,273	69	20	3	80
5	3,181	62	20	3	80
6	3,096	56	21	3	80
7	3,016	50	21	3	80
8	2,942	45	22	4	80
9	2,871	41	22	5	80
40	2,803	37	22	6	80
1	2,738	33	23	6	80
2	2,676	30	24	7	80
3	2,615	27	25	8	80
4	2,555	24	25	9	80
5	2,497	21	26	10	80
6	2,440	18	27	11	80
7	2,384	15	28	12	80
8	2,329	12	29	13	80
9	2,275	9	30	14	80
50	2,222	7	31	15	79
1	2,169	5	32	16	79
2	2,116	3	33	17	79
3	2,063	1	34	19	79
4	2,009	...	35	20	78
5	1,954	...	37	24	78
6	1,893	...	38	30	77
7	1,825	...	38	38	77
8	1,749	...	40	47	77
9	1,662	...	40	58	76
60	1,564	...	31	782	75
1	751	...	15	451	75
2	285	...	5	200	74
3	80	...	2	64	74
4	14	14	73

TABLE IV.—Officers.

Table of Accumulations, at 1³ per-cent half-yearly, of contributions of 1 per-cent of Salary, of the Return of 1 per-cent of Total Salary on Withdrawal, and on Death, and of the Payments for Pensions of 1 per-cent of Average Salary of last five years of membership for every year of membership (not exceeding 40) in respect of 875 contributors.*

Age of Fund in years at end of period	ENTERING ONCE ONLY AT AGE 20 (NO SUBSEQUENT ENTRIES)					ENTERING EACH YEAR AT AGE 20				Age of Fund in years at end of period
	Accumulations of Contributions of 1 per-cent of Salary	Accumulations of Return of 1 per-cent of total Salary on Withdrawal	Accumulations of Return of 1 per-cent of total Salary on Death	Accumulations of Pensions of 1 per-cent of average Salary of last 5 years of Membership	Accumulations of Contributions of 1 per-cent of Salary	Accumulations of Return of 1 per-cent of total Salary on Withdrawal	Accumulations of Return of 1 per-cent of total Salary on Death	Accumulations of Pensions of 1 per-cent of average Salary of last 5 years of Membership		
5	3,408.3	346.3	17.4	...	9,838.9	792.5	37.2	...	5	
10	7,976.8	1,167.2	83.8	...	40,038.6	4,832.5	298.9	...	10	
15	14,107.3	2,303.3	236.6	56.7	97,654.8	13,980.1	1,129.4	108.6	15	
20	21,724.5	3,566.3	542.2	457.8	190,495.4	29,254.4	3,153.5	1,367.4	20	
25	30,796.7	4,916.1	1,095.3	1,774.0	325,716.7	51,115.9	7,398.8	7,008.8	25	
30	41,469.4	6,214.7	2,024.9	5,194.0	511,034.9	79,621.6	15,486.4	24,960.3	30	
35	53,944.1	7,410.0	3,475.7	13,965.6	755,065.4	114,229.5	29,722.6	73,823.7	35	
40	68,277.6	8,813.8	5,688.0	35,330.0	1,067,088.1	155,393.7	53,323.5	200,491.9	40	
45	82,945.7	10,483.6	8,185.9	135,754.1	1,452,849.4	204,356.2	89,835.2	625,499.7	45	
50	98,659.9	12,469.6	9,736.7	292,596.6	1,913,628.9	262,594.4	135,309.4	1,774,907.9	50	
55	117,349.8	14,831.9	11,581.2	445,791.1	2,461,700.6	331,865.6	189,398.4	3,699,561.5	55	
60	139,581.1	17,611.7	13,775.2	593,502.9	3,113,601.4	414,259.7	253,734.3	6,373,675.4	60	
65	166,023.9	20,983.9	16,384.9	738,996.3	3,889,001.1	512,263.0	330,258.4	9,777,143.6	65	
70	197,476.2	24,959.1	19,488.9	891,315.5	4,811,296.0	628,832.3	421,279.6	13,923,947.8	70	
75	234,887.0	29,687.5	23,181.0	1,063,031.2	5,908,314.7	767,485.1	529,544.3	18,885,611.6	75	
80	279,385.4	35,311.6	27,572.5	1,264,709.7	7,213,157.7	932,404.9	658,319.1	24,792,075.9	80	
85	332,313.0	42,001.2	32,795.9	1,504,313.4	8,765,195.9	1,128,567.9	811,489.4	31,817,830.3	85	

* Strictly 875'l.

TABLE V.—*Workmen*

*Table of Accumulations, at 1½ per-cent half-yearly, of contributions of 1 per-cent of Salary, of the Return of 1 per-cent of Total Wages on Withdrawal, on Death, and on Retirement during the first ten years of membership, and of the Payments for Pensions of 1 per-cent of Average Wages of last five years of membership for every year of membership (not exceeding 40), in respect of 161 contributors.**

Age of Fund in years at End of Period	ENTERING ONCE ONLY, AT AGE 25 (NO SUBSEQUENT ENTRIES)					ENTERING EACH YEAR AT AGE 25					Age of Fund in years at End of Period
	Accumulations of Contributions of 1 per-cent of Wages	Accumulations of Return of 1 per-cent of total Wages on Withdrawal	Accumulations of Return of 1 per-cent of total Wages on Death	Accumulations of Return of 1 per-cent of total Wages during the first 10 years of membership	Accumulations of Pensions of average Wages of last 5 years of membership	Accumulations of Contributions of 1 per-cent of Wages	Accumulations of Return of 1 per-cent of total Wages on Withdrawal	Accumulations of Return of 1 per-cent of total Wages on Death	Accumulations of Return of 1 per-cent of total Wages during the first 10 years of membership	Accumulations of Pensions of average Wages of last 5 years of membership	
5	1,767.8	145.1	20.8	-6	...	5,281.0	339.7	45.5	-9	...	5
10	3,695.7	442.5	88.8	7.9	...	19,592.4	4,922.0	332.6	22.2	...	10
15	5,594.4	797.3	220.0	9.4	33.0	43,508.5	5,185.8	1,139.2	66.2	69.5	15
20	7,811.8	1,475.8	442.1	11.2	200.4	78,027.3	10,301.6	2,860.1	118.6	655.9	20
25	10,321.9	1,514.2	799.2	13.4	691.5	124,488.8	17,302.0	6,976.8	181.0	2,946.4	25
30	13,176.9	1,874.9	1,313.9	15.9	1,835.5	181,512.4	26,028.8	11,621.2	253.1	9,488.6	30
35	16,420.5	2,230.1	2,140.5	18.9	4,440.5	259,969.7	36,444.1	20,625.2	343.2	25,517.1	35
40	19,695.2	2,652.5	2,719.0	22.5	23,800.0	351,878.6	48,832.5	33,235.9	448.2	98,691.4	40
45	23,426.3	3,155.0	3,231.1	26.7	47,567.7	461,289.3	63,567.7	48,340.5	573.0	289,233.0	45
50	27,861.3	3,752.7	3,846.8	31.8	70,815.3	591,427.1	81,094.4	66,306.6	721.5	597,052.9	50
55	33,143.1	4,463.7	4,575.6	37.8	93,442.5	746,218.8	101,941.5	87,676.3	898.1	1,019,227.8	55
60	39,421.9	5,309.3	5,442.4	45.0	115,942.5	930,333.1	126,738.0	113,094.5	1,108.1	1,553,772.9	60
65	46,890.4	6,315.1	6,473.4	53.5	139,705.7	1,149,814.8	156,232.0	143,328.0	1,358.0	2,203,904.7	65
70	55,773.2	7,511.5	7,699.8	63.6	166,583.5	1,409,814.8	191,313.7	179,289.2	1,655.2	2,981,461.0	70
75	66,339.1	8,934.5	9,158.4	75.7	198,187.0	1,749,645.6	233,041.4	222,062.8	2,008.7	3,907,037.0	75
80	78,906.7	10,627.0	10,893.5	90.0	235,734.0	2,088,172.3	282,674.0	272,939.8	2,429.1	5,008,010.7	80

* Strictly 460.6.

Having ascertained the true actuarial contributions, Tables of Progress of Funds were then deduced, and the values for quinary groups are shown in Tables VI, VII, VIII, and IX.

Col. 2 in Tables VI, VII, VIII, and IX indicates what might be termed the *actuarial* primary annual contribution required, and if the Fund is to remain actuarially solvent throughout, the amounts set out in Col. 2 would have to be provided out of the general rates year by year.

Col. 8 in Tables VI and VII, and Col. 9 in Tables VIII and IX show the accumulated funds which would be derived from the members' contributions alone. These funds become exhausted in the 53rd year in the case of officers (entering every year), and in the 45th year in the case of workmen (entering every year). From these points onwards the balance of the amount required to meet the annual outgo, after applying the members' contributions, must be met by a charge on the general rates, and the amounts so required, together with the percentages of the total salaries or wages, in the case of the hypothetical funds under review, are shown in Tables X and XI.

It will be observed that under the conditions mentioned, the ultimate permanent charge on the rates, when the funds reach a stationary condition, is 19·067 per-cent of total salaries in the case of officers, and 14·349 per-cent of total wages in the case of workmen.

The funds derived from the members' contributions alone, after paying the outgoings, reach a maximum, in the case of officers in the 43rd year, when the charges of the year fully absorb the available income from the members' contributions and interest, and in the case of workmen in the 37th year. If, from those dates, the respective funds are maintained in a stationary condition, as seems to be implied in the conditions as to Administration (*see* Statement C (3) (d)). it will be necessary to provide out of the rates only the difference between the total assumed outgoings and the members' contributions together with the interest on the maximum Fund, the result of which is shown in Tables XII and XIII.

It must be clearly understood that the percentages shown in Tables X, XI, XII and XIII relate only to future entrants, and do not therefore, represent the full effect of the assessment principle. If the same principle be applied in respect of existing members, it is obvious that the percentages will be considerably greater.

TABLE VI.—Officers.

Table showing the Progress of a Fund formed in respect of 875 members entering at age 20 (no subsequent entries), the total contribution being 8-10736 per-cent of Salaries per member, the Pensions, subject to 10 years' membership, being 60th of average Salary of last five years of membership (not exceeding 40), and the Officers' contributions of 2½ per-cent being returned on Death in the service and on Withdrawal.

Age of Fund in Years at end of period	RECEIPTS DURING PERIOD			PAYMENTS DURING PERIOD			ACCUMULATED FUND AT END OF PERIOD		Age of Fund in Years at end of period	
	CONTRIBUTIONS			Interest	Returns on Withdrawal	Returns on Death	Pensions	From Officers' Contributions of 2½ per-cent only		From total Contributions of 8-10736 per-cent
	Officers' Contributions of 2½ per-cent	Balance of true actuarial Contributions to be met by a charge on the rates, being 5-00736 per-cent of Salaries	Total Contributions at 8-10736 per-cent							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)		
5	7,817	17,532	25,349	2,227	812	41	7,611	26,723	5	
10	9,001	20,189	29,190	7,514	1,738	146	16,814	61,543	10	
15	10,597	23,767	34,364	14,525	2,096	317	28,824	107,929	15	
20	11,323	25,396	36,719	23,543	1,888	602	43,277	165,094	20	
25	11,347	25,451	36,798	34,281	1,537	1,039	59,007	231,095	25	
30	11,073	24,836	35,909	46,607	831	1,663	74,418	306,952	30	
35	10,563	23,694	34,257	60,186	38	2,455	84,470	386,954	35	
40	9,393	21,069	30,462	73,584	...	3,580	75,556	458,413	40	
45	3,848	8,632	12,480	77,558	...	3,187	65,566	399,540	45	
50	56,656	—	296,528	50	
55	34,175	—	515,644	55	
60	17,387	—	718,761	60	
65	6,917	—	95,846	65	
70	1,924	—	49,904	70	
75	318	—	18,542	75	
80	24	—	4,209	80	
85	1	—	432	85	
Totals	84,962	190,566	275,528	457,427	8,940	13,030	Totals	

TABLE VII.—Officers.

Table showing the Progress of a Fund formed in respect of 875 members entering every year at age 20, the total contribution being 8-107-36 per cent of Salaries per member, the Pensions, subject to 10 years' membership, being 6-10 of average Salary of last five years of membership for every year of membership (and exceeding 10), and the Officers' contributions of 2-2 per cent being returned on Death in the service and on Withdrawal.

Age of Fund in Years at end of period	RECEIPTS DURING PERIOD			PAYMENTS DURING PERIOD				ACCUMULATED FUND AT END OF PERIOD		Age of Fund in Years at end of period
	Officers' Contributions of 2½ per cent	Balance of true actuarial Contributions to be met by a charge on the rates, being 5-607-36 per cent of Salaries	Total Contributions at 8-107-36 per cent	Interest	Returns on Withdrawal	Returns on Death	Pensions	From Officers' Contributions of 2½ per cent only	From total Contributions of 8-107-36 per cent	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)		
5	23,075	51,757	74,832	4,833	1,882	89	...	22,523	77,694	5
10	65,509	146,935	212,444	31,265	9,030	591	...	87,268	311,779	10
15	115,293	258,599	373,892	89,039	18,994	1,795	173	206,182	753,768	15
20	170,990	383,525	554,515	187,994	29,035	4,188	1,937	392,940	1,461,117	20
25	227,773	510,843	738,616	337,267	37,467	8,427	8,450	656,221	2,482,636	25
30	283,778	636,503	920,281	515,050	43,146	15,428	25,619	998,216	3,863,774	30
35	337,785	757,639	1,095,424	818,480	41,700	26,033	68,257	1,404,743	5,638,668	35
40	387,523	869,197	1,256,720	1,160,184	41,700	41,400	174,149	1,811,771	7,795,323	40
45	429,572	913,324	1,363,896	1,549,323	44,700	60,661	602,384	1,854,145	10,000,797	45
50	424,809	952,829	1,377,638	1,880,060	44,700	65,149	1,587,104	831,132	11,561,512	50
55	424,809	952,829	1,377,638	2,093,874	44,700	65,149	2,434,405	1,314,845	12,488,800	55
60	424,809	952,829	1,377,638	2,244,891	44,700	65,149	3,018,167	4,508,776	12,950,313	60
65	424,809	952,829	1,377,638	2,261,952	44,700	65,149	3,355,060	8,679,012	13,127,994	65
70	424,809	952,829	1,377,638	2,282,717	44,700	65,149	3,503,441	13,803,621	13,175,056	70
75	424,809	952,829	1,377,638	2,286,610	44,700	65,149	3,517,235	19,947,807	13,182,250	75
80	424,809	952,829	1,377,638	2,287,412	44,700	65,149	3,551,412	27,264,011	13,182,739	80
85	424,809	952,829	1,377,638	2,287,438	44,700	65,149	3,554,916	35,966,872	13,182,750	85
Totals	5,430,750	12,180,954	17,611,704	22,317,839	631,254	679,827	25,435,712	Totals

NOTE.—In col. (9) the Accumulated Fund is stationary after 85 years on the assumed conditions.

TABLE VIII.—*Withdrawal.*

Table showing the Progress of a Fund formed in respect of 461 members entering at age 25 (no subsequent entries), the total contribution being 5.84264 per-cent of Wages per member, the Pensions, subject to 10 years' membership, being $\frac{1}{5}$ th of average Wages of last five years of membership (not exceeding 40), and the Workmen's contributions of $\frac{1}{2}$ per-cent being returned on Death in the service, on Withdrawal, and on Retirement during the first 10 years of membership.

Age of Fund in years at end of period	RECEIPTS DURING PERIOD			PAYMENTS DURING PERIOD				ACCUMULATED FUND AT END OF PERIOD		Age of Fund in years at end of period	
	CONTRIBUTIONS			Interest	Returns on Withdrawal	Returns on Death	Returns on Retirement during first 10 years of Membership	Pensions	From Workmen's Contributions of 1½ per-cent only		From total Contributions of 5.84264 per-cent
	Workmen's Contributions of 1½ per-cent	Balance of true actuarial Contributions to be met by a charge on the rates, being 4.34264 per-cent of Wages	Total Contributions at 5.84264 per-cent								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)		
5	2,423	7,015	875	204	29	1	...	2,402	10,079	5	
10	2,060	5,965	2,625	372	372	10	...	4,600	20,258	10	
15	1,790	5,182	4,441	372	158	...	57	6,791	31,084	15	
20	1,588	4,598	6,397	311	249	...	273	8,910	42,834	20	
25	1,413	4,090	8,518	198	378	...	764	10,690	55,515	25	
30	1,233	3,571	10,776	50	513	...	1,701	11,576	68,798	30	
35	1,023	2,962	13,053	...	747	...	3,808	9,973	81,281	35	
40	215	620	13,018	...	227	...	31,199	21,821	63,708	40	
45	9,002	31,949	60,971	40,761	45	
50	5,426	23,588	98,405	22,599	50	
55	2,758	15,225	133,796	10,132	55	
60	1,097	7,901	167,867	3,328	60	
65	306	2,945	...	689	65	
70	51	669	...	71	70	
75	4	72	...	3	75	
80	3	80	
Totals	11,745	31,003	78,347	1,507	2,420	11	120,157	Totals	

TABLE IX.—*Workmen.*

Table showing the Progress of a Fund formed in respect of 461 members entering every year at age 25, the total contribution being 58,1264 per-cent of Wages per member, the Pensions, subject to 10 years' membership, being 53th of average Wages of last five years of membership for every year of membership (not exceeding 40), and the Workmen's contributions of 1½ per-cent being returned on Death in the service, on Withdrawal, and on Retirement during the first 10 years of membership.

Age of Fund in Years at end of period	RECEIPTS DURING PERIOD				PAYMENTS DURING PERIOD				ACCUMULATED FUND AT END OF PERIOD		Age of Fund in Years at end of period	
	CONTRIBUTIONS				Interest	Returns on Withdrawal	Returns on Death	Returns on Retirement during first 10 years of Membership	Pensions	From Workmen's Contributions of 1½ per-cent only		From total Contributions of 584264 per-cent
	Workmen's Contributions of 1½ per-cent	Balance of true-actuarial Contributions to be met by a charge on the rates, being 434264 per-cent of Wages	(2)	Total Contributions at 584264 per-cent								
5	7,420	21,481	28,901	1,924	484	65	1	...	7,342	30,275	5	
10	18,427	53,344	71,771	11,542	2,112	390	30	...	25,973	111,056	10	
15	27,878	80,708	108,586	30,066	4,009	1,034	54	120	55,549	244,491	15	
20	36,205	104,817	141,022	58,075	5,700	2,087	54	975	95,928	434,772	20	
25	43,615	126,272	169,887	96,355	6,948	3,702	54	3,664	146,036	686,646	25	
30	50,142	145,168	195,310	145,678	7,488	6,074	54	10,089	202,659	1,003,929	30	
35	55,708	161,282	216,990	206,441	7,534	9,368	54	23,979	257,441	1,386,395	35	
40	58,608	169,677	228,285	273,814	7,534	11,964	54	116,256	224,604	1,752,686	40	
45	58,725	170,015	228,740	327,105	7,534	12,100	54	288,297	— 2,667	2,000,546	45	
50	58,725	170,015	228,740	361,056	7,534	12,100	54	422,894	— 420,594	2,147,760	50	
55	58,725	170,015	228,740	379,787	7,534	12,100	54	515,628	— 1,019,588	2,220,971	55	
60	58,725	170,015	228,740	388,219	7,534	12,100	54	569,084	— 1,790,951	2,249,158	60	
65	58,725	170,015	228,740	391,024	7,534	12,100	54	592,585	— 2,734,481	2,256,649	65	
70	58,725	170,015	228,740	391,649	7,534	12,100	54	599,538	— 3,864,503	2,257,812	70	
75	58,725	170,015	228,740	391,727	7,534	12,100	54	600,697	— 5,209,905	2,257,894	75	
80	58,725	170,015	228,740	391,732	7,534	12,100	54	600,782	— 6,810,280	2,257,896	80	
Totals	767,803	2,222,869	2,990,672	3,846,194	102,681	131,514	787	4,314,588	Totals	

NOTE.—In col. (10) the Accumulated Fund is stationary after 80 years on the assumed conditions.

TABLE X.—*Officers.*

Table showing, in the case of a Fund established as in Table VII, but without provision of an Actuarial contribution, the Total Annual Charge on the Rates, and the percentage of such Charge on the Total Salaries, in the 53rd year, when the Accumulated Contributions of the Officers have become exhausted, and in following years.

Year of Fund	Total Annual outgo less the Officers' Contributions of 2½ per-cent	Ratio per-cent. of amounts in preceding column to total Salaries	Year of Fund }
53	326,147	9.597	53
4	453,602	13.347	4
5	479,043	14.096	5
6	502,351	14.782	6
7	523,546	15.405	7
8	542,664	15.968	8
9	559,756	16.471	9
60	574,889	16.916	60
1	588,148	17.306	1
2	599,632	17.644	2
3	609,620	17.938	3
4	617,906	18.182	4
5	624,794	18.384	5
6	630,424	18.550	6
7	634,948	18.683	7
8	638,513	18.788	8
9	641,263	18.869	9
70	643,336	18.930	70
1	644,860	18.975	1
2	645,950	19.007	2
3	646,706	19.029	3
4	647,215	19.044	4
5	647,545	19.054	5
6	647,750	19.060	6
7	647,848	19.063	7
8	647,919	19.065	8
9	647,957	19.066	9
80	647,977	19.067	80
1	647,986	19.067	1
2	647,991	19.067	2
3	647,992	19.067	3
4	647,993	19.067	4
5	647,993	19.067	5

TABLE XI.—*Workmen.*

Table showing, in the case of a Fund established as in Table IX, but without provision of an Actuarial Contribution, the Total Annual Charge on the Rates, and the percentage of such Charge on the Total Wages, in the 45th year, when the Accumulated Contributions of the Workmen have become exhausted, and in following years.

Year of Fund	Total Annual outgo less Workmens' Contributions of $1\frac{1}{2}$ per-cent	Ratio per-cent of amounts in preceding column to total Wages	Year of Fund
45	2,667	·341	45
6	67,333	8·599	6
7	72,390	9·245	7
8	77,110	9·848	8
9	81,492	10·408	9
50	85,533	10·924	50
1	89,236	11·397	1
2	92,603	11·827	2
3	95,640	12·214	3
4	98,355	12·561	4
5	100,759	12·868	5
6	102,865	13·137	6
7	104,689	13·370	7
8	106,258	13·571	8
9	107,575	13·739	9
60	108,660	13·877	60
1	109,554	13·992	1
2	110,273	14·083	2
3	110,839	14·156	3
4	111,276	14·211	4
5	111,605	14·253	5
6	111,848	14·284	6
7	112,020	14·306	7
8	112,140	14·322	8
9	112,221	14·332	9
70	112,274	14·339	70
1	112,307	14·343	1
2	112,326	14·346	2
3	112,338	14·347	3
4	112,344	14·348	4
5	112,347	14·348	5
6	112,348	14·348	6
7	112,349	14·348	7
8	112,349	14·348	8
9	112,349	14·349	9
80	112,349	14·349	80

TABLE XII.—*Officers.*

Table showing, in the case of a Fund established as in Table VII, but without provision of an Actuarial Contribution, the Total Annual Charge on the Rates, and the percentage of such Charge on the Total Salaries, in the 44th year, when the funds derived from the Officers' Contributions have reached a maximum, and in following years.

Year of Fund	Total Annual Outgo less Officers' Contributions of 2½ per-cent and Annual Interest on the Maximum Fund	Ratio per-cent of amounts in preceding column to total Salaries	Year of Fund
44	20,542	6.06	44
5	62,492	18.38	5
6	105,487	31.03	6
7	117,839	43.50	7
8	187,998	55.31	8
9	226,030	66.50	9
50	261,999	77.09	50
1	295,916	87.07	1
2	327,747	96.43	2
3	357,462	105.18	3
4	385,043	113.29	4
5	410,485	120.78	5
6	433,793	127.64	6
7	454,988	133.87	7
8	474,106	139.50	8
9	491,198	144.53	9
60	506,331	148.98	60
1	519,590	152.88	1
2	531,074	156.26	2
3	541,062	159.20	3
4	549,318	161.64	4
5	556,235	163.67	5
6	561,866	165.32	6
7	566,390	166.65	7
8	569,955	167.70	8
9	572,705	168.51	9
70	574,778	169.12	70
1	576,301	169.57	1
2	577,391	169.89	2
3	578,148	170.11	3
4	578,657	170.26	4
5	578,986	170.36	5
6	579,192	170.42	6
7	579,290	170.45	7
8	579,361	170.47	8
9	579,399	170.48	9
80	579,419	170.49	80
1	579,428	170.49	1
2	579,432	170.49	2
3	579,434	170.49	3
4	579,435	170.49	4
5	579,435	170.49	5

TABLE XIII.—*Workmen.*

Table showing, in the case of a Fund established under Table IX, but without provision of an Actuarial Contribution, the Total Annual Charge on the Rates, and the percentage of such Charge on the Total Wages, in the 35th year, when the funds derived from the Workmen's Contributions have reached a Maximum, and in following years.

Year of Fund	Total Annual Outgo less Workmen's Contributions of $1\frac{1}{2}$ per-cent and Annual Interest on the Maximum Fund	Ratio per-cent of amounts in preceding column to total Wages	Year of Fund
38	5,612	7.17	38
9	13,189	1.684	9
40	20,602	2.831	40
1	27,688	3.536	1
2	34,416	4.395	2
3	40,795	5.210	3
4	46,837	5.981	4
5	52,551	6.711	5
6	57,939	7.399	6
7	62,995	8.045	7
8	67,716	8.648	8
9	72,097	9.207	9
50	76,138	9.723	50
1	79,841	10.196	1
2	83,208	10.626	2
3	86,245	11.014	3
4	88,960	11.361	4
5	91,364	11.668	5
6	93,470	11.937	6
7	95,294	12.170	7
8	96,864	12.370	8
9	98,180	12.538	9
60	99,265	12.677	60
1	100,160	12.791	1
2	100,878	12.883	2
3	101,445	12.955	3
4	101,881	13.011	4
5	102,211	13.053	5
6	102,453	13.084	6
7	102,625	13.106	7
8	102,745	13.121	8
9	102,826	13.132	9
70	102,879	13.139	70
1	102,912	13.143	1
2	102,932	13.145	2
3	102,943	13.147	3
4	102,949	13.147	4
5	102,952	13.148	5
6	102,954	13.148	6
7	102,954	13.148	7
8	102,954	13.148	8
9	102,955	13.148	9
80	102,955	13.148	80

ON THE WORKING OF A FUND WHICH IS ESTABLISHED AT THE OUTSET WITH EXISTING EMPLOYEES OF VARYING AGES AND TERMS OF SERVICE, BRINGING PAST SERVICE, BUT NO PAST CONTRIBUTIONS, INTO ACCOUNT.

WE have, so far, investigated the working and administration of a fund, in respect of which all the members have contributed from their entry into service, and, therefore, in respect of each year which would be brought into account for superannuation allowance and for returnable benefit. This does not, of course, represent the conditions in practice, as regards the Metropolitan Borough Council Funds. These Councils have been established for many years, either as vestries, district boards, or borough councils, and have steadily developed, as regards the municipal work involved, and the staff required for the conduct of such work; and the officers and servants employed have rendered varying terms of service already qualifying them, in some cases, for superannuation or pension allowances. No funds for such purposes have, however, been established in connection with current or contingent allowances, prior to the earliest Act obtained by the Stepney Borough in 1905; but superannuation and pension allowances have been granted from time to time, payable under the 1866 Act out of the rates, on conditions practically identical as to retirement with those laid down in the recent Acts—that is, on permanent breakdown after 10 years' service, or on attainment of 60 years of age, the allowance being at the rate of one-sixtieth of the average salary current at retirement, for each year of service, with a minimum of ten-sixtieths, and a maximum of forty-sixtieths.

On the establishment of a fund under the conditions of practice, the members at the outset would thus comprise (1) officers and servants at ages ranging, say, within the limits of 20 and 65, who have rendered varying terms of past service, which will be reckoned for superannuation or pension allowance, but in respect of which no contributions have been paid, and no fund has been set aside in the past; (2) retired officers and servants, at varying ages, mostly over 60, who have been superannuated or pensioned under the 1866 Act; and (3) members still on the active list, over 60 years of age, with upwards of 40 years' service, or over 65 years of age, at

the date of establishment of the fund, who are qualified for immediate retirement. The contingent allowances and benefits for class (1) would be payable, under the conditions of the Borough Superannuation Acts, partly from the superannuation fund to be established and administered as therein provided, and partly from the Borough rates; the current allowances under (2) would continue to be payable (under the 1866 Act) out of the Borough rates; and, as regards superannuation allowances and pensions to be granted in the future in class (3), it is provided, in nearly all the Borough Superannuation Acts, that future allowances in this class are to be payable out of the Borough rates, and not from the superannuation fund. It is also to be noted that, as regards classes (2) and (3) above, the Paddington and Poplar Acts provide that the actuary, in computing the proportionate primary annual contribution, is to bring into account current allowances under (2) and contingent allowances under (3), as well as contingent allowances under (1), and to make the primary annual contribution "as nearly as may be an even annual charge," having regard to the liabilities in respect of these several classes of members.

The actuary entrusted with the task of investigating the financial position of a fund newly established would, in practice, have to make a complete valuation of the liability, in respect of (1) each officer and servant who joined the fund, on account of allowances earned by past service, and to be earned by future service, and of returnable benefits in respect of future contributions, and of the assets estimated as accruing from the future contributions of the member, payable at a specified rate on his salary, determined according to his term of service at the outset, and varying in most cases between the limits of 2 and $3\frac{1}{2}$ per-cent for officers, and approximating to $1\frac{1}{2}$ per-cent for workmen. The liability in respect of (2) incumbent superannuation and pension allowances, granted prior to the establishment of the fund, and of (3) allowances and benefits in respect of employees who at the date of establishment had attained the age of 60 years, after 40 years' service, or had attained the age of 65 years, would also be calculated; the last-named allowances and benefits being deemed (on a conservative estimate) to be immediately incumbent. Combining the amounts of the liability thus computed in these several classes, the total initial liabilities on the fund would be ascertained, representing

the amount at which the fund should stand at the outset, in order that full provision should be made for the incumbent and contingent allowances and benefits. As no fund is available at the outset for such purpose, the amount of the liability must be provided from the Borough rates, and, as it cannot conveniently be raised in a lump sum, it must, in practice, be provided by an annual charge upon the rates, extending over a term of years. This term may be an arbitrarily fixed period, or one depending on the duration of active service, or future lifetime, of the members existing at the outset, or may take the form of a perpetual rate-charge. In selecting the most suitable period, the actuary must, we think, give reasonable consideration to the equity of the charge, as regards the generations of ratepayers upon whom it is imposed. Equitably, some proportion of the charge should have been provided out of the rates paid in the past; but, as this has not been done, the present and future generations of ratepayers must share the burden between them in more or less equitable proportions. If the whole charge is liquidated during the remaining term of active service of the members existing at the outset (usually about 30 years) the whole weight of the liability arising from their prior service will fall upon the ratepayers in the present and the immediate future, involving a very heavy, and, it may be thought, an unduly heavy burden of charge. If, on the other hand, the rate-charge is extended over a period of, say, 100 years, or in perpetuity, future generations of ratepayers would be involved, who were in no way responsible for the genesis of the scheme, and upon whom such charge could hardly equitably be imposed. An intermediate course which has been suggested is that the period of rate-charge should be determined, not by the future active service, but by the future lifetime, of the members existing at the outset. This period would usually extend over about 60 years; and it may be considered that the ratepayers may fairly and equitably bear their proportion of the liability for pensions to the initial members, so long as such pensions are actually payable. It has, however, been already pointed out in this Paper, that the Parliamentary Committee on Local Legislation have expressed the view (on the Chiswick Bill) that the term of charge in respect of members existing at the outset should not exceed the term of their future active service. This opinion has not, however, been included as a provision in any of the Acts with which we are acquainted; and we consider that the actuary may fairly and equitably

adopt, either the future term of active service, or of future lifetime, of the initial members, as a basis for the calculation of the primary annual contribution.

Assuming that one or other of these courses is followed, the question arises whether such term of active service, or future lifetime, is to be based upon the *individual* expectations of members existing at each attained age, or upon the *average* expectations of members of all ages, taken as a whole. The former course involves the grave practical inconvenience of proportionate rates of charge, differing for each attained age: and this may also be deemed to be inconsistent with the provisions of the Paddington and Poplar Acts, which prescribe that the primary annual contribution shall bear a certain definite proportion to the total salaries. There seems to be no practical or theoretical objection to the adoption of the average term of future service, or of future lifetime: and this will have the effect of relieving the ratepayer in the immediate future from the very heavy charges arising in respect of members who at the outset had only a few years to run before retirement. The primary annual contribution would then represent the uniform proportion of the total salaries (separately computed for officers and servants) which should be provided out of the rates during the ensuing quinquennium, and which, if continued during the average term of future service (or future lifetime) of the existing members, considered as a whole, would provide for the initial liability. This average proportion of salary can readily be calculated by dividing the aggregate amount of the liability in respect of officers (or servants) existing at the outset, including allowances then incumbent, by the value of an annuity, computed at the average age of the members, of 1 per-cent upon their estimated salaries, payable throughout active service (or continued as a uniform charge for the period from retirement to the end of life).

The full actuarial contribution required to provide the allowances and benefits to newly appointed officers and workmen must necessarily be a matter of separate assessment, and can readily be computed at an average entry age, either by special commutation columns, allowing for mortality, withdrawal and retirement, or by accumulation methods; and this contribution, expressed as a proportion of the salary, after deducting the proportion payable by the employee, will represent the uniform charge on the rates, in respect of

future members joining the fund, during the term of their active service.

The primary annual contribution, for the ensuing quinquennium, deduced as above, will thus consist of two parts : (1) a uniform proportion of the salaries of officers and servants existing at the outset, payable out of the Borough rates during their average term of active service (or future lifetime ; and (2) a (smaller) uniform proportion of the salaries of new officers and servants, taken on after the establishment of the fund, also payable out of the Borough rates, during their individual terms of active service. At the outset, the total proportionate charge will be entirely at the higher rate required in respect of existing officers and servants under (1), and as time goes on this total charge will be steadily diminished, first, by the passing out by death of the retired members whose allowances were incumbently payable at the outset, and, more gradually, by the retirement and ultimate death of the existing officers and servants under (1), and their replacement by new employees under (2), in respect of whom a smaller charge on the rates is required. The total proportion of the salaries charged on the rates will thus approximate gradually to the minimum rate for new members, and on the passing out by death of all the members existing at the outset, will ultimately reach a uniform charge, equivalent to such minimum rate, which uniform charge will continue unaltered so long as the fund is in existence.

The matter can be more readily appreciated by an illustration in figures : but the data relating to any particular Borough, or combination of Boroughs, could hardly be regarded as typical, even if available for our present purpose, as the age-distribution, the salaries, and the terms of past service of the officers and servants existing at the date of establishment of the fund, would vary materially in the case of individual Boroughs. We have, therefore, thought it preferable, and sufficient for the purposes of illustration, to take the case of the normal funds whose progress are shown in Tables VII and IX, constituted and maintained by a uniform number of members entering annually, at age 20 for officers, and at age 25 for workmen. Under these conditions, the numbers existing at each age, after the expiration of any period of n years, would precisely represent a portion of the Service Table for officers extending from age 20 to age $(20 + n)$, and at the expiration of $(\omega - 20)$ years, and uniformly afterwards, the numbers

at all ages in the Service Table would be exactly reproduced. Thus, for example, after a period of 45 years from the entry of the earliest members, the numbers in the Service Table from age 20 to age 65 would be precisely reproduced, and at this point the entrants in the first year would, on the assumptions made in the retirement column, all have passed into the retired list. We should thus have lives on the active list at all ages from 20 to 64 inclusive; and this may be taken sufficiently to illustrate the condition of affairs in a Borough staff at the date when it is decided to establish a formal superannuation fund, bringing into account the past service of the members (uniformly in our illustration from age 20 for officers, or 25 for workmen) but to be credited with their future contributions only.

To trace the progress of a fund, established under such conditions, through successive years, is a laborious task, as it involves the tracing out of the financial effect in respect of each separate age, the accumulation starting, not, as in Tables VII and IX, uniformly at age 20 (or 25), but at each separate age from entry to 64. The work can, probably, be somewhat reduced by groupings of the ages; but, in the brief time at our disposal for the preparation of the present Paper, we have not been able to attempt the task. It is, however, comparatively easy to ascertain, by suitable modifications in the normal Tables, calculated by Mr. Manly's methods, showing the accumulation of a unit of contribution and benefit to the end of life, the rate of actuarial contribution which should be charged, at each age from 20 to 64, in order to provide for the benefits with inclusion of past service, such contributions being payable during the future terms of active service only. The only difficulty arises in connection with the pension accumulations, as it is difficult to separate or eliminate, in the accumulation Tables calculated as above, the continuing allowances to be made in the future, in respect of pensions which vested prior to the establishment of the fund, from the payments which arise in respect of pensions vesting in the future. In view of this difficulty, we have preferred to leave these continuing allowances as a charge on the fund, after of course eliminating the *past* payments in respect of such allowances. The future payments in this class will then be analagous to those arising on and after the establishment of a Borough Council Fund, in respect of allowances previously

granted under the 1866 Act, and, as these will continue to be a charge on the Borough rates, and are to be brought into account in estimating the primary annual contribution, there seems to be no practical objection to this course.

Table XIV shows the full rates of actuarial contribution required, at each age from 20 to 64, to provide for the future allowances to existing officers, under these conditions. It will be seen that the rates steadily increase from 8·107 per-cent at age 20 (which is the actuarial contribution for new entrants at that age) to 11·655 per-cent at age 30, 20·410 per-cent at age 40, 46·756 per-cent at age 50, and 320·805 per-cent at age 60, rising to a very high percentage at age 64, when there is practically no future salary or contribution payable, and provision has to be made for the continuing allowances in respect of pensions which vested at all previous ages.

If we now assume that the contributions payable by the members are on the scale fixed by the Poplar Act, the officers in our illustrative Fund at ages 20 to 24 inclusive (whose duration does not exceed 5 years) will contribute to the fund $2\frac{1}{2}$ per-cent on their salaries: those at age 25 to 34 inclusive (whose duration lies between 5 and 15 years) will contribute 3 per-cent, and those at ages 35 to 64 (whose duration exceeds 15 years) will contribute $3\frac{1}{2}$ per-cent; the balance required at each age to make up the actuarial contribution being found from the rates. By taking the aggregate accumulations of benefit and contributions to the end of life, under the conditions now obtaining, the average rate in respect of officers existing at all ages from 20 to 64 inclusive, at the outset (and including also provision for continuing allowances granted in the past) is found to be 19·154 per-cent upon their total salaries, or 16·118 per-cent, after allowing for the contributions payable by the officers to the fund, and this would represent the "primary annual contribution" for the ensuing quinquennium, calculated as payable over the average term of future service.

The effect of taking a single average rate on the total salaries is, of course, to spread the heavy liability, arising in respect of members existing at the ages where the retiring rates are heavy, over a term corresponding to the average term of future service of the whole of the members. An intermediate course would be to deduce separate average rates in respect of officers paying $2\frac{1}{2}$ per-cent, 3 per-cent, and $3\frac{1}{2}$ per-cent. This would give average charges on the rates of 6·118 per-cent, 8·413

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TABLE XIV.—*Officers.*

Table showing, at each age from 20 to 64, the accumulations to the end of life of (a) contributions of 1 per-cent of future salaries; returns of 1 per-cent of future total salaries, (b) on withdrawal, (c) on death; (d) pensions of 1 per-cent of average salary for the last 5 years of membership, reckoning full service (not exceeding 40 years) from the uniform entry age of 20, and including also the continuing allowances, in respect of such pensions vesting at earlier ages; also the actuarial contribution required at each age to provide the benefit on the above bases.

Age at Establishment Fund	Accumulations to end of life of Contributions of 1 per-cent of Salary	Accumulations to end of life of Returns of 1 per-cent of total Salary on Withdrawal	Accumulations to end of life of Returns of 1 per-cent of total Salary on Death	Accumulations to end of life of Pensions of 1 per-cent of average Salary of last five years of Membership	Members' rate of Contribution per-cent of Salary (K)	Actuarial Contribution K [Col(3) + Col(4)] + $\frac{1}{6}$ Col(5)] ÷ Col(2)	Age at Estab- lishment of Fund
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
20	332,313·0	42,001·2	32,795·9	1,504,313·4	2½	8·10736	20
1	321,147·0	37,952·0	32,104·6	1,504,313·4		8·35234	1
2	310,076·5	34,182·3	31,377·6	1,504,313·4		8·61429	2
3	299,125·3	30,685·3	30,616·9	1,504,313·4		8·89408	3
4	287,313·8	27,461·5	29,827·9	1,504,313·4		9·22480	4
5	277,605·8	24,483·9	29,008·2	1,501,313·4		9·60954	5
6	266,961·4	21,732·2	28,154·2	1,504,313·4		9·95218	6
7	256,351·9	19,190·1	27,266·3	1,504,313·4		10·32393	7
8	245,755·1	16,810·4	26,341·9	1,504,313·4		10·72912	8
9	235,199·7	14,686·9	25,385·9	1,504,313·4		11·17096	9
30	224,668·6	12,719·7	24,396·4	1,504,313·4	3	11·65511	30
1	214,111·1	10,921·3	23,370·0	1,501,297·8		12·19011	1
2	203,522·6	9,285·7	22,305·9	1,504,245·4		12·78409	2
3	192,969·2	7,818·3	21,212·7	1,504,137·1		13·44250	3
4	182,525·4	6,516·5	20,100·5	1,503,951·6		14·17029	4
5	172,260·5	5,375·8	18,979·8	1,503,670·1		15·04327	5
6	162,230·0	4,388·8	17,859·8	1,503,265·8		15·92377	6
7	152,474·8	3,538·8	16,746·9	1,502,715·1		16·89148	7
8	143,030·7	2,815·6	15,651·0	1,501,994·6		17·95389	8
9	133,909·6	2,205·4	14,576·0	1,501,078·3		19·12135	9
40	125,096·6	1,694·9	13,524·0	1,499,946·7	3½	20·40965	40
1	116,577·4	1,271·4	12,494·8	1,498,576·4		21·83793	1
2	108,345·3	927·5	11,492·3	1,496,946·0		23·42861	2
3	100,390·1	653·8	10,516·9	1,495,012·3		25·20950	3
4	92,722·4	442·5	9,574·0	1,492,734·2		27·20969	4
5	85,348·6	284·8	8,666·5	1,490,087·4		29·46514	5
6	78,254·4	171·1	7,796·8	1,487,028·1		32·02718	6
7	71,430·1	91·4	6,965·4	1,483,493·0		34·95987	7
8	65,038·5	40·7	6,173·6	1,479,421·2		38·24584	8
9	58,745·6	13·4	5,424·8	1,474,729·4		42·16343	9
50	52,728·1	2·6	4,721·3	1,469,295·7	4	46·75609	50
1	46,975·5	...	4,063·4	1,462,981·5		52·20859	1
2	41,482·0	...	3,452·4	1,455,636·6		58·77596	2
3	36,245·3	...	2,888·4	1,447,083·1		66·82010	3
4	31,267·4	...	2,372·5	1,437,109·3		76·86875	4
5	26,549·4	...	1,904·7	1,425,494·3		89·73801	5
6	22,101·5	...	1,485·8	1,412,030·8		106·71606	6
7	17,799·0	...	1,118·0	1,396,523·9		130·98785	7
8	13,905·5	...	801·4	1,378,788·3		165·45866	8
9	10,287·7	...	537·2	1,358,652·0		220·29221	9
60	6,944·2	...	328·4	1,335,952·5	See above	320·80541	60
1	4,115·0	...	176·6	1,301,114·6		527·13060	1
2	2,080·7	...	79·8	1,243,144·6		995·90859	2
3	866·6	...	27·7	1,161,047·4		2,233·06716	3
4	271·1	...	5·8	1,062,865·1		6,534·34932	4
20-24	1,549,975·6	172,282·3	156,722·9	7,521,567·0	2½	8·61850	20-24
25-34	2,299,670·8	144,195·0	247,512·0	15,042,512·3	3	11·41297	25-34
35-64	1,979,473·6	23,918·5	200,406·0	42,758,418·3	3½	36·39814	35-64
20-64	5,829,120·0	340,395·8	604,670·9	65,322,497·6	See above	19·15447	20-64

TABLE XV.—*Workmen.*

Table showing, at each age from 25 to 63, the accumulations to the end of life of (a) contributions of 1 per-cent of future wages; returns of 1 per-cent of future total wages, (b) on withdrawal, (c) on death, (d) on retirement during the first 10 years' membership; (e) pensions of 1 per-cent of average wages for the last 5 years of membership, reckoning full service from the uniform entry age of 25, and including also continuing allowances, in respect of such pensions resting at earlier ages; also the actuarial contribution required at each age to provide the benefits on the above bases.

Age at Establishment of Fund	Accumulations to end of life of Contributions of 1 per-cent of Wages	Accumulations to end of life of Returns of 1 per-cent of total Wages on Withdrawal	Accumulations to end of life of Returns of 1 per-cent of total Wages on Death	Accumulations to end of life of Returns of 1 per-cent. of total Wages on Retirement during first ten years of Membership.	Accumulations to end of life of Pensions of 1 per-cent of average Wages of last five years of Membership.	Actuarial Contribution $\left[\frac{1}{2} \left\{ \text{Col}(3) + \text{Col}(4) \right. \right.$ $\left. \left. + \text{Col}(5) \right\} + \frac{1}{5} \text{Col}(6) \right]$ $\div \text{Col}(2)$	Age at Establishment of Fund
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
25	78,906·7	10,627·0	10,893·5	90·0	235,731·0	5·843	25
6	73,480·4	9,092·7	10,280·8	90·0	235,731·0	6·230	6
7	68,415·2	7,761·8	9,684·9	90·0	235,731·0	6·649	7
8	63,671·3	6,606·3	9,104·9	52·0	235,731·0	7·103	8
9	59,217·2	5,602·0	8,541·2	39·3	235,734·0	7·597	9
30	55,050·8	4,733·7	7,996·6	28·8	235,734·0	8·133	30
1	51,169·3	3,987·1	7,473·7	19·0	235,734·0	8·713	1
2	47,515·1	3,345·6	6,971·0	11·3	235,734·0	9·341	2
3	44,155·3	2,793·8	6,489·0	5·6	235,734·0	10·022	3
4	40,979·1	2,321·3	6,026·5	1·9	235,734·0	10·765	4
5	37,998·7	1,916·5	5,581·4	...	235,734·0	11·576	5
6	35,198·9	1,571·5	5,156·3	...	235,720·8	12·463	6
7	32,565·6	1,278·2	4,747·6	...	235,684·3	13·436	7
8	30,087·1	1,029·5	4,358·1	...	235,623·6	14·507	8
9	27,750·5	819·9	3,985·3	...	235,538·5	15·692	9
40	25,548·1	611·5	3,630·3	...	235,419·2	17·005	40
1	23,469·7	499·0	3,292·3	...	235,257·7	18·468	1
2	21,508·8	379·5	2,969·7	...	235,052·3	20·103	2
3	19,658·4	282·2	2,664·8	...	234,891·4	21·941	3
4	17,911·3	203·8	2,376·8	...	234,495·3	24·020	4
5	16,262·4	143·0	2,105·2	...	234,127·0	26·383	5
6	14,705·6	96·5	1,849·9	...	233,687·6	29·091	6
7	13,237·3	62·7	1,611·1	...	233,170·1	32·216	7
8	11,850·5	37·8	1,389·0	...	232,565·9	35·862	8
9	10,543·3	21·9	1,182·4	...	231,869·2	40·157	9
50	9,316·9	11·2	994·1	...	231,071·9	45·255	50
1	8,197·7	4·8	823·8	...	230,166·2	51·389	1
2	7,084·1	1·5	660·0	...	229,147·1	58·954	2
3	6,063·7	...	530·6	...	228,008·4	68·499	3
4	5,109·6	...	408·9	...	226,738·5	80·802	4
5	4,218·0	...	304·0	...	225,330·1	97·237	5
6	3,387·6	...	215·1	...	223,759·9	120·191	6
7	2,617·7	...	142·9	...	221,976·9	154·261	7
8	1,902·3	...	86·1	...	219,909·5	210·253	8
9	1,247·4	...	45·0	...	217,473·2	317·038	9
60	656·8	...	19·5	...	214,573·3	594·035	60
1	250·1	...	6·6	...	206,675·2	1,502·531	1
2	75·5	...	1·6	...	191,986·8	4,623·434	2
3	16·8	174,582·1	18,894·185	3
25-63	971,000·8	65,875·9	134,610·0	427·9	8,917,486·0	17·008	25-63

per-cent, and 32·898 per-cent respectively, after allowing for the officers' contributions at the above rates, and would seem to be more equitable, as regards the incidence of the charge on the ratepayers, than the single average rate prescribed by the Paddington and Poplar Acts. As, however, this rate would be subject to revision every five years, on an actuarial investigation of the cases actually surviving and subsisting, the inequities attaching to a single average rate would, in practice, to some extent be mitigated.

As regards new members entering after the establishment of the fund, and paying contributions of $2\frac{1}{2}$ per-cent from the commencement, the only proper course appears to be that the charge on the rates on their account should be the true actuarial contribution for such members, which is in our illustrative Fund equal for officers to 8·107 per-cent at age 20, or 5·607 per-cent, after allowing for the $2\frac{1}{2}$ per-cent paid to the fund. The average net rate in respect of members existing at the outset being 16·118 per-cent on their salaries, the total charge on the rates would at the outset be in this proportion, but would gradually approximate, as new members were introduced in place of old ones retiring, withdrawing or dying, to the rate of 5·607 per-cent applicable to such new members, and would ultimately be fixed at the latter rate for all time.

In Table XV, the results are shown of calculations made, on precisely the same principles and methods, in respect of workmen. Column (7) shows the rate of actuarial contribution required, at each age from 25 to 63, in order to provide for the benefits, with inclusion of past service, such contributions being payable during the future term of active service only. As in the case of officers, the continuing allowances, in respect of pensions vesting at earlier ages, are included in the calculations. The rates of contribution, thus calculated for workmen, increase from 5·843 per-cent on the wages at age 25 (which is the actuarial contribution for new entrants at that age) to 8·133 per-cent at age 30, 17·005 per-cent at age 40, 45·255 per-cent at age 50, and 594·035 per-cent at age 60, rising to a maximum rate at age 63. The average rate of contribution over the ages 25 to 63 inclusive, deduced from the aggregate accumulations of benefits and contributions to the end of life, is found to be 17·008 per-cent upon the total wages, or 15·508 per-cent, after allowing for the contributions (at the approximate rate of $1\frac{1}{2}$ per-cent) payable by the workmen to the fund, and this latter percentage would

represent the "primary annual contribution" for the ensuing quinquennium. The charge on the rates, commencing at this proportion of the total wages, will gradually diminish, by the introduction of new members to replace those passing out of active service, and will ultimately be reduced to the minimum rate of 5·843 per-cent, applicable to new members, when all the members existing at the outset have passed into the retired list.

The actual charges in each year, gradually diminishing in rate as above, can only be shown financially by Tables for officers and workmen, showing the Progress of a Fund, constituted and worked under the special conditions set forth above; and we must defer the calculation and publication of such Tables, and of Tables showing the effect of making provision only for current allowances as they emerge for payment, until a later period.

APPENDIX.

IN PARLIAMENT.

1 and 2 Geo. V, Session 1911.

REPORT OF THE LOCAL GOVERNMENT BOARD ON THE POPLAR BOROUGH COUNCIL (SUPERANNUATION AND PENSIONS) BILL, 1911.

Clause 8.—"RETURN OF CONTRIBUTION AND POWER TO GRANT GRATUITIES AND SUPERANNUATION ALLOWANCES IN CERTAIN CASES."

The words "loses his office or employment" in paragraph (a) of sub-clause (1) should apparently be transferred to the beginning of paragraph (b).

Clause 16.—"SUPERANNUATION FUND."

The Board may refer to the latter part of their observations on Clause 17 of the Bill with respect to certain suggestions made on behalf of the Institute of Actuaries for the amendment of Clause 16 and 17. The Board suggest that the two clauses should be considered together.

Clause 17.—"ACTUARIAL INVESTIGATION."

The Board think it right to draw the attention of the Committee to a question which has arisen in the interpretation of this Clause as contained in the previous local Acts from which it is taken.

The Clause was first inserted in the Kensington Bill of 1907 when that Bill was in Committee of the House of Commons, and has been included in all the Acts since passed empowering Metropolitan Borough Councils to establish superannuation funds, viz.: the Camberwell and other Metropolitan Borough Councils (Superannuation) Act, 1908, the Saint Marylebone Borough Council (Superannuation) Act, 1908, the Wandsworth Borough Council (Superannuation) Act, 1909, and the Westminster City Council (Superannuation and Pensions) Act, 1909.

The Clause provides that the condition of the superannuation fund shall every five years be submitted to an actuary. His duty is to report as to the solvency of the fund, and to certify what sum should be contributed annually out of the rates under paragraph (a) of Clause 16 (1), so as to avoid any deficiency arising which would have to be met by a further contribution out of the rates under paragraph (d) of that Clause. The sum which he certifies is to be paid to the fund accordingly for the next five years.

The question which has arisen is whether in determining the amount of this annual sum the actuary is to have regard only to the solvency of the fund during the next five years or to its permanent solvency.

Each of these views has, in fact, been taken by actuaries in the exercise of their duties under clauses similar to the present one. In the case of the Metropolitan Borough Councils of Camberwell and Hackney, the actuaries have adopted the former view, and have fixed the annual contribution at £500, although they appear to have considered that no contribution was really needed during the current quinquennium to comply with the requirements of the statute; and the same view has also been acted upon in regard to the superannuation fund of certain other Metropolitan Borough Councils. In the case of the Deptford Borough Council, on the other hand, the actuary has had regard to the permanent solvency of the fund, and has fixed the amount of the annual contribution at £1,900. It may be mentioned by way of comparison that the percentage amounts of salary or wages and emoluments deducted under section 15 (1) (b) of the Camberwell and other Metropolitan Borough Councils (Superannuation) Act, 1908, in the year 1909-10, were £561. 2s. 1d. in the case of Deptford and £1,867. 6s. 5d. in the case of Camberwell.

It is clear both from these figures and from general considerations that the question at issue is an important one. The charges upon the superannuation fund are, in the nature of the case, at their minimum in the earlier years of the existence of the fund, and do not reach their maximum for a considerable period. The payments to be made by the officers under paragraph (b) of Clause 16 (1), although they are on a higher scale than in the precedents, would not of themselves be sufficient to provide the benefits which the Bill proposes to secure to the officers, and in the absence of satisfactory provision for the equalization of the charge there would ultimately be a large deficiency to be met out of the rates. If in determining the annual contribution for the next five years, regard is only had to the solvency of the fund during that period, the result will be that the ultimate deficiency will in the main be left to be made good at the time when it accrues, whilst if regard is had to the permanent solvency of the fund the annual burden on the rates would as far as possible be equalized from the commencement of the fund.

Mr. A. Carson Roberts, the District Auditor for the Metropolitan Boroughs, in a report made to the Kensington Borough Council with reference to the superannuation fund of that Council which has been established under the Kensington Borough Council

(Superannuation) Act, 1907, sets out some figures which show the importance of the question. He estimates that the charge on the fund will ultimately be some £9,100 per annum, and that the income from the contributions of the officers and servants, which starts at £1,576 per annum, will eventually settle down to some £1,210 per annum. Hence there will ultimately be an annual deficiency of £7,000 and upwards to be met out of the general rate. The auditor contends that this ultimate annual deficiency should be taken into account in calculating the amount of the primary annual contribution out of the general rate from the commencement of the fund, and he estimates that if the calculation was made on this basis the amount required would be some £3,500 per annum as against £300 (the minimum prescribed by the Act) certified by the actuary.

The question at issue is, shortly, whether the intention of the provision is that the ratepayers in the first 30–40 years of the existence of the fund should bear a due proportion of the burden thrown upon the rates by the establishment of the fund, or whether practically the whole of the burden should be left to be borne by the ratepayers of the time when the deficiency actually accrues.

It appears to the Board that the intention of the Clause as contained in the previous local Acts was to secure the former of these objects. They find it difficult to think that the obligations of the Clause would have been imposed on the promoters if the object were merely to equalize the contribution during each quinquennium.

In any case it appears to the Board that the annual charge on the rates should be equalized from the commencement of the fund so far as is practicable; and for their own part they would have been disposed to regard the language of the Clause as sufficient to provide for this. But as doubt has in fact been cast upon the meaning of the provision, they suggest for the consideration of the Committee whether it would not be advisable so to amend the Clause as to prevent any doubt arising in the present case.

The Board suggest also that the proviso to the Clause, which prescribes a minimum primary annual contribution, and the effect as it appears to them may be misleading, should be omitted.

The Board have communicated with the Institute of Actuaries with respect to the corresponding clause in the Paddington Borough Council Bill of this Session, and they subjoin in the Appendix a copy of a letter which they have received from the Institute. It will be seen that amendments are suggested both in this and the preceding Clause, with consequential amendments in other clauses of the Bill.

The Board do not express any opinion on these amendments, but they think that the Committee should carefully consider them as well as any others that may be suggested with the view of settling definitely the principle to be adopted in the present case.

Clause 22.—“ARBITRATION.”

The words “or at the option of such officer to the President of the Local Government Board for the time being whose decision shall be final and conclusive”, are not contained in the precedents for this

Clause—*see, e.g.*, section 21 of the Westminster City Council (Superannuation and Pensions) Act, 1909—and the Board submit that they should be omitted. Section 18 of the Poor Law Officers Superannuation Act, 1896, empowers the Board (not the President for the time being) if they think fit, to determine questions arising between guardians or other authorities to whom that Act applies and their officers or servants, but it is to be borne in mind that the Board stand in a closer relation to poor law officers than to officers of the Borough Councils.

(Signed) F. J. WILLIS,
Assistant Secretary.

LOCAL GOVERNMENT BOARD, WHITEHALL.

21 April 1911.

Copy of Letter from the Institute of Actuaries to the Local Government Board.

THE INSTITUTE OF ACTUARIES,
STAPLE INN HALL,
HOLBORN, W.C.
8 March 1911.

SIR.—With further reference to your letter of the 13th ultimo, I beg to inform you that a Committee has been appointed by the Council of the Institute of Actuaries which have now carefully considered Sections 16 and 17 of the Paddington Borough Council (Superannuation and Pensions) Bill. The Committee note your remark that the provisions in the Bill are similar to the provisions in Local Acts recently obtained by certain Borough Councils, and that these provisions have been construed in different senses by the actuaries appointed under them. The Committee are agreed as to the ambiguity of the Sections, under which it would seem open to individual actuaries to take different views of the proper construction to be placed on them, and in order to remove such ambiguity suggest that Sections 16 and 17 should be re-drafted. I beg to enclose the modifications which the Committee suggest would meet the case in the hope that they may be of assistance to your Board.

The Committee have, on the present occasion, confined their attention to Sections 16 and 17, as they understand that these are the only Sections on which their observations have been invited. They desire to point out, however, that if the modifications suggested by them are adopted, some modification in other Sections would be rendered necessary.

I am, Sir,
Your obedient Servant,
(Signed) G. H. RYAN,
President.

F. J. WILLIS, Esq.,
Assistant Secretary,
LOCAL GOVERNMENT BOARD,
WHITEHALL, S.W.

(ENCLOSURE.)

Modifications suggested by the Council of the Institute of Actuaries :—

Section 16.

- (1) Substitute in paragraph (a) of Sub-section (1) the words :
“ and to bear such a proportion to the total salaries of the officers as hereinafter provided ”

for the words :

“ and to be of such amount as hereinafter provided.”

- (2) In paragraph (d) omit words “ as in this Act provided.”

- (3) Delete Sub-section (3) as unnecessary and misleading.

Section 17.

Delete the words following the word “ consider ” in line 17 to the end of the Section, and substitute therefor the following :

“ the same and shall make an actuarial valuation of the Fund, and on the basis of such valuation shall certify what proportion in his opinion the primary annual contribution shall bear to the total salaries of the officers so that without further recourse to the general rate the Superannuation Fund as constituted under paragraphs (a), (b) and (c) of Sub-section (1) of Section 16 shall be solvent (having regard to existing and prospective liabilities) and for the next quinquennial period the primary annual contribution shall be in the proportion so certified and shall be paid to the Superannuation Fund accordingly.”

Metropolitan Borough	ADDED YEARS"	Remarks
STEPNEY	<p>et of (a) Prior service Public Authorities; special professional or other qualifications</p> <p>To be aggregated reckoned for purpose of Superannuation Fund. No allowance.</p> <p>cap or, (af or, yes me or, yes</p>	<p>The following additional provisions are included in most of the Borough Council Acts:—</p> <p>(a) Allowances to the following classes of employees, existing at the date of passing of the Act, are to be paid out of the general rates, and not out of the Superannuation Fund: (i) Officers and servants who had then attained 65 years of age, or 60 years of age after 40 years' service (St. Marylebone, Wandsworth); (ii) Officers and servants who had then attained 65 years of age (Kensington); (iii) Officers who had then attained 65 years of age, or 60 years of age after 40 years' service, and workmen who had attained 60 years of age (Westminster, Paddington, Poplar):</p>
BETHNAL GREEN	Do.	
KENSINGTON	<p>As above.</p> <p>per term not exceed- to years may be added wh service, on the direc- be the Council, provi- We the officer had at- Act 0 years of age at the the his appointment.</p>	
CAMBERWELL, HACKNEY and DEPTFORD	<p>As above.</p> <p>No allowance.</p>	
ST. MARYLEBONE	provision for added	
WANDSWORTH	Do.	
WESTMINSTER	<p>As in Stepney. Officers only)</p> <p>SEs in Kensington. Officers only)</p>	
PADDINGTON	<p>As in Stepney. cap Officers only) (oth No allowance.</p> <p>from of has Wo Act ther men But paid Com that this to b</p>	
POPULAR	Do.	

his scale is approximately

to forty fifty-fifths (or eight-elevenths) after 40 years' service.

The following additional provisions are included in most of the Borough Council Acts:—

(a) Allowances to the following classes of employees, existing at the date of passing of the Act, are to be paid out of the general rates, and not out of the Superannuation Fund: (i) Officers and servants who had then attained 65 years of age, or 60 years of age after 40 years' service (St. Marylebone, Wandsworth); (ii) Officers and servants who had then attained 65 years of age (Kensington); (iii) Officers who had then attained 65 years of age, or 60 years of age after 40 years' service, and workmen who had attained 60 years of age (Westminster, Paddington, Poplar):

(b) Allowances to holders of joint appointment, where loss of office arises from death, resignation, or insanity of one of the holders:

(c) As to contracting out of scheme, and retaining the benefits of the 1866 and 1896 Acts:

(d) As to non-assignment of superannuation allowance:

(e) Investment clause: (i) In trustee securities (Stepney, Bethnal Green, Kensington, Camberwell); (ii) in trustee securities, or statutory Municipal securities in Great Britain, or other Local Authority as defined in § 34 of Local Loans Act, 1875 (St. Marylebone, Wandsworth); (iii) in trustee securities, or any securities, authorized by § 44 of Friendly Societies Act, 1896 (Westminster); (iv) in "any statutory securities" with power of statutory borrowing from Superannuation Fund (Paddington, Poplar):

(f) As to registration of Scheme under Friendly Societies Act.

(g) As to establishment or continuance of Provident or Sick Funds.

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STATEMENT (B).

Metropolitan Borough Superannuation and Pension Funds.

Rates of Contribution payable by Officers and Servants, according to duration of Service and Wages.

Metropolitan Borough	OFFICERS					SERVANTS OR WORKMEN					
	Years' Service			In respect of Salaries below £120	"Added Years" in respect of (a) Service with Public Authorities ; (b) Special Qualifications	Amount of Weekly Wages					
	Under 5 Years	5 to 15 years	Upwards of 15 years			Under 25s.	25s. to 30s.	30s. to 35s.	35s. to 40s.	40s. and upwards	In respect of wages below 20s. weekly
Percentage of Salary deducted					Weekly contribution deducted†						
STEPNEY AND BETHNAL GREEN }	2	2½	3	...	(a) 1 per-cent for every 10 years or part of 10 years added	Contributions of Servants or Workmen computed on same percentage basis as that provided for Officers, according to the duration of service at the date of establishment of the Fund.					
					(a) as above ; (b) 1 per-cent for any term added up to 10 years						
KENSINGTON . . .	2	2½	3	...	(a) as above						
CAMBERWELL . . .	2	2½	3	...	(a) as above						
ST. MARYLEBONE } AND WANDSWORTH }	2	2½	3						
WESTMINSTER . . .	2	2½	3	1½	(a) ½ per-cent for every 5 years or part of 5 years added	4d.	5d.	6d.	7d.	8d.	Weekly Contribution 1d. ; but only on written request of Workman, and future service only to count for pension allowance.
					(b) as Kensington						
PADDINGTON AND POPLAR }	2½	3	3½	2	(a) 1 per-cent for every 5 years or part of 5 years added*	4d.	5d.	6d.	7d.	8d.	

* (Paddington and Poplar) Or, alternatively, the aggregate amount of any contributions and gratuities which may have been returned or paid to the employee, on the termination of any period or periods of service which are aggregated and reckoned, by the local authority or authorities under whom such service had been spent.

† The scale of contributions for Servants or Workmen is approximately equal to 1½ per-cent of the average wages in each group.

Metropolitan Borough Councils Acts.

Conditions as to Establishment and Administration of Superannuation and Pension Funds.

DURNEY AND BETHNAL GREEN	KENSINGTON, CAMBERWELL, ST. MARYLEBONE AND WANDSWORTH	WESTMINSTER	PADDINGTON AND POPLAR
(1) The Council shall establish and administer a Superannuation Fund, to which shall be carried and credited—	(1) As in Stepney and Bethnal Green, modified as under:	(1) As in Kensington, Camberwell, St. Marylebone, and Wandsworth.	(1) As in Kensington, Ac., modified as under—
(a) A sum of one hundred pounds to be raised annually in and by the general rate;	(a) A sum hereinafter called "the primary annual contribution . . .	(a) A sum hereinafter called "the primary annual contribution" to bear such proportion to the	(a) A sum hereinafter called "the primary annual contribution" to bear such proportion to the
amounts together with the amount of such dividends and interest (if any) not being sufficient to fully satisfy such payments then to the extent to which such sums are insufficient for that purpose out of a part of the Superannuation Fund not exceeding in the aggregate a sum equivalent to ten per centum of the amount standing to the credit of the Superannuation Fund on the 31st day of March immediately preceding the dates at which such payments shall accrue due and become payable; and	(c) In the event of the sources above mentioned not being sufficient . . .	(b) In the event of the income of the fund not being sufficient to fully satisfy such payments, then out of the money or investments standing to the credit of the fund on the 31st day of March immediately preceding to the extent (if any) which the Council in their discretion may direct, not exceeding in any case ten per cent of the total amount thereof:	(b) In the event of the income of the fund not being sufficient to fully satisfy such payments, then out of the money or investments standing to the credit of the fund on the 31st day of March immediately preceding to the extent (if any) which the Council in their discretion may direct, not exceeding in any case ten per cent of the total amount thereof:
(c) In the event of the aggregate of such amounts as aforesaid not being sufficient to fully satisfy such payments then to the extent to which such aggregated sums are insufficient out of the portion of the Superannuation Fund taken from the general rate to make up the deficiency.	(c) As in sub-section (c) in Kensington Act.	(c) As in sub-section (c) in Kensington Act.	(c) As in sub-section (c) in Kensington Act.

ST. MARYLEBONE, 31 March 1909.
Wandsworth, 31 March 1910.
Westminster, 1 January 1910.
Paddington and Poplar,
1 January 1912.

who, at the passing of the Act, had attained the age of sixty years, and completed a service of aggregated service of forty years, or had attained the age of sixty-five years; which allowances are charged on the general rates, and not on the Superannuation Fund.

STATEMENT (D).—Metropolitan Borough Council Acts.

Clauses providing for Actuarial Investigation.

STEPNEY AND BETHNAL GREEN	KENSINGTON	CAMBERWELL AND WESTMINSTER	ST. MARYLEBONE AND WANDSWORTH	PADDINGTON AND POPLAR
(No clause providing for actuarial investigation).	Within three months after the passing of this Act and at the expiration of every subsequent period of five years dating from the First day of January, One Thousand Nine Hundred and Eight,* the condition of the Superannuation Fund shall be submitted by the Council to an Actuary being a fellow either of the Institute of Actuaries or of the Faculty of Actuaries in Scotland, appointed by them and approved by the Local Government Board, who shall consider the same and shall report as to the solvency thereof, and certify what sum in his opinion is necessary to be the amount of the primary annual contribution so that the Superannuation Fund shall be solvent as constituted under paragraphs (a), (b) and (c) of sub-section (1) of the immediately preceding section without further recourse to the general rate, and the sum so certified shall be the primary annual contribution for the next quinquennial period, and shall be paid into the Superannuation Fund accordingly, but the primary annual contribution shall not be less than <i>Three Hundred Pounds</i> .	As in Kensington, but no provision as to minimum amount of primary annual contribution.	As in Kensington, but minimum amount of primary annual contribution fixed at <i>Four hundred pounds</i> .	As in Kensington, but wording modified as under : — who shall consider the same, shall make an actuarial valuation of the Fund and on the basis of such valuation shall certify what proportion in his opinion the primary annual contribution shall bear to the total salaries of the officers [of the General Staff of the Council, and of the Electricity Undertaking respectively], so that without further recourse to the general rate [for the funds of the Electricity Undertaking] the Superannuation Fund as constituted under paragraphs (a), (b) and (c) of sub-section (1) of the preceding section shall be solvent (having regard to existing and prospective liabilities), and for the next quinquennial period the primary annual contribution shall be in the proportion so certified and shall be paid to the Superannuation Fund accordingly. Provided that in making a certificate under this section the Actuary shall take into account the sum which the Council are now paying under the Superannuation (Metropolis) Act, 1866, or will hereafter pay under Section 4 (4)† of this Act and make such certificate as will rest upon the rate [and funds of the Council's Electricity Undertaking respectively], as nearly as may be an even annual charge so long as this Act is in operation in respect of the expenditure both under this Act and under the Superannuation (Metropolis) Act, 1866.

† In respect of superannuation allowances to officers who, at the passing of the Act, had attained the age of sixty years, and completed a service of aggregated service of forty years, or had attained the age of sixty-five years; which allowances are charged on the general rates, and not on the Superannuation Fund.
(The clauses in square brackets are in the Poplar Act only.)

Camberwell, 1 January 1901.
St. Marylebone, 31 March 1900.
Wandsworth, 31 March 1910.
Westminster, 1 January 1910.
Paddington and Poplar,
1 January 1912.

ADDENDUM.

As to methods employed in computing the figures in Tables XIV and XV.

DEALING first with Table XIV, in respect to Officers, in order to obtain the column of accumulations of contributions from the corresponding column in Table IV, the accumulation after any period n (n being less than 45) in the latter Table was accumulated to the end of life (*i.e.*, to the end of Table IV) by multiplying by $1.0175^{2(s-5-n)}$. This accumulated figure was then deducted from the final accumulation of Table IV, and the resulting net figure represents the accumulation, from age $20+n$ inclusive to the end of life, of the contributions payable from age $20+n$ inclusive.

A similar operation to the above in regard to the "returns" on withdrawal and death would only eliminate the accumulations of *past* RETURNS up to age $20+n$, and the resulting figures would represent the accumulations, from age $20+n$ to the end of life, of the "returns" from age $20+n$, on the basis of contributions having been paid from the commencing age 20. A further adjustment is therefore necessary, in order to eliminate the accumulation of the future returns from age $20+n$, in respect of *past* CONTRIBUTIONS paid prior to age $20+n$. This last adjustment is rather troublesome, and may be represented mathematically as follows:

At age $20+n$ the further deduction would be

$$\sum_{t=0}^{n-1} (s_{20+t} \times .01) ; \sum_{t=n}^{44} (w_{20+t} \times 1.0175^{2(s-5-t)+1}) ;$$

n being the year of accumulation in Table IV.

With regard to the pensions a deduction was made in precisely the same manner as for the contributions, and the resulting figures obviously represent the accumulations, to the end of life, of the pension payments from age $20+n$ to the end of life, arising from past and future pensioners, and counting service from age 20. This is the condition assumed in regard to pensions in Table XIV.

The remaining columns in Table XIV speak for themselves.

Bearing in mind that the commencing age for workmen is 25, and that the final year of accumulation in Table V is 80, instead of 20 and 85 respectively for officers, the process of deducing Table XV from Table V was otherwise entirely the same as that above described for Table XIV.

ABSTRACT OF THE DISCUSSION.

Mr. E. C. THOMAS thought that in considering whether the entire liability for future payments under superannuation schemes should be borne by the present generation, or what proportion, if any, could be legitimately thrown on future generations, distinction must be drawn between the schemes of the various classes of industrial undertakings and those of the State. In private business houses and small companies with small staffs concentrated under the immediate supervision of the directors of the company or principals of the business, where the staff had been practically stationary for some time, there might be no necessity for making any provision for future liabilities. If, however, the staff were large, the only satisfactory guarantee for the future was a properly constituted fund on a contributory basis. In the case of a national scheme it might fairly be argued that in a country like England each generation should look after its own old people, and the only reason for a fund would be to protect those taxpayers who indirectly were paying for their pensions from being deprived of them when they reached the pension age by some future Chancellor of the Exchequer who might repudiate on behalf of his own generation the burden of the pensions then falling due. Such a contingency was probably remote, and could be ignored.

Borough Council schemes might be regarded as intermediate between industrial schemes and those of the State, but, for the protection of the employees, it would probably be desirable in most cases that a proper fund should be established to ensure that the benefits when due should be paid. It was possible for a parish or town to dwindle and decay to such an extent that the charge for pensions, although it might be approximately constant from year to year, could no longer be borne by the depleted number of ratepayers, and, in such a case, however well a man might have served the Council in the past, he would have no security for his pension and presumably no remedy for its loss. Moreover, the contributions were a distinct help and were by no means to be ignored.

Referring to the illustrative tables in the paper, he noted that the rates of mortality, withdrawal, and retirement, were the same as those used in Mr. Manly's previous paper, although the salary scale differed rather widely. In spite of that, the rates of contribution brought out were almost identical in the two cases, affording a striking illustration of the fact that two widely different salary scales

might produce very similar results, owing to the ratios of increase being similar. It would be well, he thought, to emphasize the fact that elaborate statements of accumulation were not necessary in order to determine the amount of liability and the rates of contribution, for these could be just as effectively obtained by the ordinary methods of present values. The authors, however, had a double purpose in view; first, to fix the actuarial rate of contribution, and, second, to trace the progress of a fund through successive years during which those rates of contribution were being paid. The preliminary work necessary for the second purpose was used for the first, and the authors were thus saved the necessity of constructing commutation columns.

With regard to the question of the primary charge upon the rates, it was satisfactory that the ambiguities of the earlier Acts had now been cleared up, and the Institute was to be heartily congratulated on the share it had had in obtaining that result. It would have been very unfortunate that a system under which actuaries could have given widely different estimates should be perpetuated. The non-expert would never have appreciated the true reason for those differences and would have attributed them to the inexactness of actuarial science. When it had been decided that the charges were to provide for future as well as present liabilities, all the actuary needed to do was to make a valuation by the ordinary present values, and from the commutation columns used in the valuation he would be able to assess the actuarial rates of contribution for future entrants. The principal point at issue was as to the term over which the liabilities for the older entrants should be spread. The Authorities seemed to favour the future term of service while the authors apparently recommended the total after-lifetime of the existing members. There was no valid objection that he could see either theoretically or practically to the latter course. Theoretically the Corporation might well be expected to have to make some payments during the actual years that the pensions would be payable, while from a practical point of view the cash liability might be considered as a loan, bearing interest subject to repayment by means of a sinking fund, and a period of 60 years for a sinking-fund in such a case would be by no means an unduly long one.

The tables followed very closely the lines of Mr. Manly's previous paper and emphasized the lessons to be deduced from that paper. It was interesting to know that if only the contributions of the members themselves were drawn upon, the ultimate charge on the rates after the fund accumulated from those contributions had been exhausted would be 19 per cent. of the total salaries, and that without making any provision for the liability in respect of the past services of the older employees. The authors had made a very ingenious attempt to deal with the question of the liability for past services by means of an adaptation of the Accumulation Tables, and in Table XIV. they had a novel and somewhat curious statement. In view of its special features he wished the authors had

given a fuller explanation of the method by which it had been obtained.* On first reading the paper he was considerably mystified by the table. The rate of contribution at age 64 was set down as 6,534 per cent. and he found it very difficult to understand how a rate of 65 times the total salary could be required at any age. On turning to the text he found the explanation was that "Provision had to be made for the continuing allowances in respect of pensions which vested at all previous ages." That increased his difficulty. He then tried to conceive how the table had been formed, and he came to the conclusion that the method was somewhat as follows—the authors would correct him if he was wrong: In Mr. Manly's previous paper, and presumably in the present paper, the fundamental table was a statement of the accumulations to the end of life of the contributions and benefits for a certain number of entrants at age 20. If from such a statement one deducted the amount to which they had accumulated at the end of one year, together with all the future interest to be earned on that amount, one had a similar statement of accumulations to the end of life for the survivors to age 21 of those original entrants at 20. A similar proceeding would produce a statement for the survivors at age 22 and succeeding ages. With regard to the contributions, the matter was comparatively simple, as only future payments were being dealt with; but with regard to the benefits, the matter was not quite so simple, because they included the future accumulations of benefits arising in respect of past service, and in regard to the pension payments those past services might refer either to existing employees or current pensioners, and the pension payments would go on being payable and accumulated long after the last age in the Service Table had been reached. The consequence was that at the higher ages was included the accumulation not only in respect of the members at that age, but, in varying degree, of some part of the liability for old members who had left the service and been pensioned. At age 64, for instance, accumulations were included in respect of those who would retire in the following year, plus the accumulation in respect of those who had retired one year previously at age 63, those who had retired two years previously at age 62, three years previously at age 61, and so on, down to those who retired thirty-four years ago at age 30; while at age 31 was included, in addition to the future liabilities, only the liability arising in respect of those who were pensioned in the previous year, and for ages under 30 nothing whatever was included on that account. The authors had assumed a fund started forty-five years ago and having now just reached a stationary condition with regard to those in active service, so that there were no employees or pensioners over the age of 65. The probability was that the actual conditions at entry would have been somewhat different, and that when the service was first started it would have been commenced with employees of various ages and not all at the assumed entry age (20). He thought a better result might perhaps have been obtained by assuming that the fund had been in existence

* See Addendum to Paper, on pages 369 and 370.

for a longer term, say eighty-five years instead of forty-five, so that it would be stationary to the end of life, the l_x column thus representing the numbers in active service up to 65 and pensioners at all ages over 65. In order to present the whole case on that assumption it would be necessary to continue the statements of accumulation for all ages up to the end of life, from 64 onwards, by the methods previously followed. It was true that for ages over 64 the benefits could not be shown as a percentage of the salary payable at each age, but they could be included in the total and an average percentage of the total salary for the whole service could be worked out, representing the contribution requisite to provide the current liabilities and all future liabilities in respect of existing employees and pensioners of all ages. He did not quite understand why the authors objected to the principle of the average rate for the whole service. It seemed to him rather less lenient than the course they had apparently recommended in their early remarks of spreading the liability obtained by a valuation over the whole after-lifetime. He thought that the existing generation of ratepayers might well consider themselves entitled to a method of dealing with the case not less favourable to their interests than that based on the average rate.

Mr. CARSON ROBERTS (of the Local Government Board) said that pension legislation presented a series of four phases. In the first phase the Local Authorities were without statutory power to grant pensions at all and those who voted pensions, even in most deserving cases, were apt to get into trouble.

In the second phase they had voluntary powers—the system being still non-contributory and of course upon an “assessment” basis. A good example was to be found in the Metropolitan Act of 1866. These conditions were still largely extant, but they had not given satisfaction; for they left the officer entirely in the hands of those who happened to be his employers at the date when he had to give up his work—*i.e.*, of men who had probably not known him when he was giving his most useful service.

The third phase was contributory and obligatory, but still on the assessment basis. Strange to say it had been entirely on the initiative of the officers that this contributory system had grown up. One reason for this was that men were willing to pay in order to have a claim to that which they would probably obtain in like measure as an act of grace, but subject to the vote of a body corporate, the *personnel* of which was continually changing. Another reason was that there was no such thing as a non-contributory system. This thesis had received strong confirmation from the report of the Royal Commission on Civil Service Pensions, in which it was granted that the statutory pension was deferred pay.

As samples of this third phase they might take the Acts applicable to Poor Law Officers and Asylum employees, and some of the earlier local Acts of the municipalities. The combination of the contributory with the assessment principle, which characterized this phase of legislation, had a striking result which could hardly commend itself to the members of this Institute. Where a pension system

was established on this basis, no capitalization of any sort took place. In the early years, when contributions exceeded claims, these surplus contributions were not carried to a separate fund, but were spent upon other purposes, such as relief of the poor and mending of roads, so that the assumption of a new and heavy liability had in some cases actually resulted in a temporary reduction of rates—a proof, if proof were needed, that the “assessment” system was not the correct principle to follow in the case of any pension fund.

It was in the last decade that they had seen the fourth phase reached and sounder financial principles gaining ground. The history of the introduction of the principle of capitalization was an interesting one. It was with it and with its results that the valuable paper which they had heard that day was concerned.

The idea of an accumulating fund to meet and equalize the incidence of pension charge in the case of a rating authority was to be found in its earliest embryonic stage in the Stepney Act of 1905. But a poor little embryo it was, for this and a number of later Acts appeared to have been obtained by the Councils under the impression that a 2 per-cent deduction from the earnings of the employed, together with a nominal contribution of £100 to £300 a year from the rates, would actually meet the nominal cost of pension and invalidity allowances similar in scale to those of the Civil Service combined with return of contributions in certain cases. It must appear incredible to the members of that Institute that any Local Authority should go to Parliament under an impression that this charge which had now been actuarially assessed at 15 to 18 per-cent of the pay-roll could be covered by a normal contribution of 2 per-cent plus a few hundreds a year: or that any Council could be advised that £100 would do where £5,000 was necessary. But nevertheless it was on record on the printed minutes in several cases.

They must, however, remember that a voluntary system on a similar scale had been in operation for about 40 years, and in some cases had been applied as universally as if it had been compulsory. The resultant charge in no case came to as much as 2 per-cent of the pay-roll. He believed this fact had given rise to a widespread idea that the Actuaries or the Auditor who talked about 15 per-cent were simply demented persons. He had tried, though not with complete success, to dispel this notion by pointing out these three facts:—(1) that the mean pay-roll of the period to which the pensions related was but a fraction of the present pay-roll; (2) that it was only in the last few years that the term “officer” in the 1866 Act had been taken to include a staff workman with a definite “office” to perform—a decision which in a case that came before him had the effect of extending the pay-roll to which pension might apply by about 200 per-cent; and (3) that there was a vast difference in cost when pensions were claimable at fixed ages and when there was only a hope of pension grant, however good that hope might be, and that that difference was still more marked where there was a compulsory retirement clause.

If they knew how completely this conception of the developed cost as something comparable with the present charge under the 1866 Act filled the minds of those concerned, they would understand him when he claimed that the Stepney and Bethnal Green Acts marked the first effort at instituting accumulating funds, and that they honestly aimed at capitalization and not at assessment. To his mind, Section 15 of the Kensington Act of 1905 was the real landmark in this legislation. This section required an actuarial investigation at stated periods and a certificate to fix the annual rate contribution in such manner that the fund "shall be solvent." He had no intention of reviving the arguments over the meaning of these words "shall be solvent." It was with the resultant position that they were concerned. In two of the seven cases to which this clause applied it had been taken to require continuous solvency, in the other five it was at present taken to require that the income should meet the outgo for the succeeding five years.

He thought that the mind responsible for introducing this first actuarial clause intended it to apply to permanent solvency and equal charge, whether the words definitely conveyed that meaning or not. They had the opinion of an eminent counsel that they did not; they had the opinion of the Local Government Board, as quoted in the Appendix to the paper, that they did. The Local Government Board said, in effect, that Parliament would not have imposed all this cost and trouble in order that the contribution should be accurately equalized throughout each quinquennium while it was mounting in about fifty years by a series of sudden steps from nothing to ten or twenty thousand pounds per annum. The difference of reading practically involved the difference between capitalization on the one hand and assessment with a temporary modification on the other. The difference was one of extreme importance: if they included the pension funds of all the local bodies of London, it would mean in fifty years' time something like a threepenny rate where a penny rate or a little more would now suffice. Fortunately, they now knew that Parliament would take care in future to make its meaning clear by adopting the Kensington clause in the form to which it had been amplified by the Institute of Actuaries—a form under which the great principle of capitalization was established beyond any doubt. He thought the real credit for this good work was due to the gentleman who insisted upon the original introduction of the Actuarial Clause.

Two minor matters of considerable actuarial interest arose out of the additional words introduced in the Paddington and Poplar Acts. He alluded to the last eight lines of clause 17 and to the re-casting of clause 19. These two alterations were made at his suggestion. Clause 19 allowed the Councils to invest the pension funds in their own securities and set up adequate safeguards. This had an important effect on the rate of interest which could be taken into calculation: it placed the Municipal fund in a position practically as strong as that of the Railway funds which invest in 4 per-cent debentures at par. Presumably the loans by the Pension

Fund to the Council would be free of income tax and in that event their yield would be about equal to that of 4 per-cent investments, but subject of course to fluctuation.

As to the last eight lines of clause 17 which required the rate contribution in respect of all pension costs to be made "As nearly as may be an even annual charge", he was afraid that only one interpretation was admissible, viz., that the initial deficiency, together with the remaining pensions under the old voluntary system, were to be spread out as a perpetual rent charge. On pages 336 and 337 and again on pages 356-358 of the paper strong reasons were given for holding this to be inequitable. It was urged that the initial deficiency should be discharged by a sinking fund of limited duration, and 60 years was shown to be the most reasonable period. He agreed absolutely that these arguments were financially sound and that 60 years was a very right and proper selection.

It gave him great pleasure to find Mr. Ackland and himself in reversed positions in regard to this question of what is the sound financial principle for a municipal pension fund. Mr. Ackland was now out-Heroding Herod in his condemnation of the "Assessment" principle and in pointing out the inequity of allowing any part of the initial deficiency to remain as a charge on future generations (page 356). He entirely agreed with Mr. Ackland and Mr. Manly: let them by all means have a 60-year discharge definitely prescribed in the next Act if they could. Then they would have reached perfection in the financial basis of this pension legislation.

There remained one matter of comparatively small consequence to which he would like to refer, simply in order to remove a misunderstanding. Several allusions were made in the paper to the clause relating to the administration of the Pension Fund which immediately preceded the actuarial clause. It was spoken of as "complicated and unworkable" and as importing the "assessment" principle. He agreed that it was cumbrous and unnecessary, but he thought that it was not "unworkable" and that it in no way imported the "assessment" principle. The clause about the order in which the income is applicable appeared in a score or more of Acts relating to the funds of Local Authorities, and he was familiar with the interpretations which had been placed upon it, and he could vouch for it as a most innocent provision—somewhat old-fashioned, perhaps, but so inoffensive that its removal in many cases would make no possible difference.

The question whether the "assessment" principle was imported depended entirely upon the regulation of the annual contribution. If it and the officers' contributions and the interest on capital were together sufficient continuously to meet the annual outgo, there was a complete application of the "capitalization" principle as opposed to the "assessment" or "re-partition" principle. He had never seen any good reason for calling it unworkable; it certainly would not prove to be so in cases such as Westminster or Deptford, where it applied to an accumulating fund, nor could he see that it would cause the least inconvenience. Nevertheless, he admitted that it

was not wanted and that its limitation of the capital draft to 10 per cent was, in view of the actuarial safeguards, entirely unnecessary.

The PRESIDENT asked whether Mr. Roberts could state the name of the gentleman to whom credit was due for the introduction of the actuarial idea.

Mr. CARSON ROBERTS said there was no objection to stating it. He took it to have been introduced by the late Chairman of the Local Legislation Committee, Mr. Cornwall.

Mr. THOMAS TINNER, referring to the distinction between a "workman" or "servant" and an "officer", drew attention to the undesirable complication in administration which arose from the fact that it was necessary to deduct contributions at two or three different rates from the pay of the employees who were paid weekly. Where a large number of pay lists had to be prepared at short notice, it seemed to be advisable, if possible, to arrange that all men so paid should contribute at a uniform rate, and it had been found very convenient, in one case, at all events, to provide that everybody who was paid weekly should be classed as a workman and everybody paid a yearly salary as an officer. There was no real difficulty about that, because if there were any employees whom it was desirable to treat as officers they could easily be paid yearly salaries. It made an appreciable difference in the cost of administration if the number of rates at which deductions had to be made could be reduced. The matter was not so important in the case of officers, because they were not paid so often and there was more time available for the necessary calculations.

The authors remarked (p. 339) that "The pensions for both officers and workmen are subject to ten years' service, and hitherto we believe it has been the practice to allow for this element in the construction of the pension factors by making $r_x = 0$ for the ten years following entry." In investigating the experience of any particular fund it was necessary to base the rate of superannuation not only upon the numbers who had actually retired on pension, but also upon those who would have received pensions if at the time of retirement they had been in service long enough to have become entitled thereto, because in some cases the inclusion of the latter class materially affected the rates of contribution required. In Table I he noticed that the authors gave only one table of mortality rates for pensioners, and therefore did not distinguish between the mortality of those who retired in the ordinary course through old age or on completion of a certain number of years' service and those who retired owing to ill-health before 60, or whatever the inferior limit of age for normal retirement might be. It would be rather interesting to see whether any difference had been observed in the case of these local authorities' funds between the mortality of the two classes. He thought there could hardly be any doubt there was such a difference, but presumably the funds had not been in existence long enough to furnish the numbers requisite for such an analysis.

With reference to the suggestion for meeting the initial deficiency

due to the establishment of a fund, it might be interesting to call attention to another method which had been adopted. A certain body to which the 1866 Act applied had a number of pensioners and existing officers subject to that Act. The body itself decided to discontinue the payment of pensions but as the rights of the existing officers at the time of this decision were protected by statute, it was not possible to deprive them of their pension rights. Some years later an accumulating fund was formed for the benefit of the non-pensionable employees, and for some years there were two pension systems in operation. There was the 1866 Act, or what Mr. Carson Roberts called the voluntary system, and there was the Accumulating Fund. While the two systems were in existence side by side the rates had to bear a double charge, namely the cost of pensions to the older generation and the contributions to the pension fund established for their successors. There was also an initial deficiency to be made good in respect of the employees who were to be provided for by the fund. On the one hand there was the burden of the old pension system, and on the other hand it was undesirable to allow the initial deficiency to grow, partly because it was a liability which should have been met in the past and partly because if nothing were done it would accumulate at compound interest. It was ultimately decided to ascertain the initial deficiency as at the date when the fund was started, and in order to prevent the deficiency from growing there was paid into the fund just enough to meet the interest on it. The annual charge for pensions to the older generation under the Act of 1866 was growing year by year, and it was arranged that after the maximum had been attained and the charge began to decline the amount by which the maximum exceeded the diminishing charge should be applied each year towards the extinction of the initial deficiency in the accumulating fund.

Mr. A. W. J. RUSSELL (Town Clerk of Paddington) said he had not intended to take part in the discussion that evening, but there were one or two points that occurred to him which he should like to mention. Reference had been made in the discussion to a double charge upon the ratepayers, but instead of that being so the fact was that the ratepayers were going on with a single charge and the single charge was being reduced by the aid of the employees' contributions. Looking at it in that way put a very different aspect upon it. He had never been satisfied yet that the financial clauses which had been introduced into the Acts would be absolutely necessary at any rate to the extent to which they were being interpreted by some actuaries. There was a large Railway Fund (Great Eastern) that was quoted at the time the Paddington Bill was going through, and it was proved that the Railway Employees' contributions at $2\frac{1}{2}$ per-cent of their salaries and wages, the Company paying an equal percentage, gave such large accumulations that the interest on the accumulation in the course of a large number of years paid more than half the outgoings, so that the Company had now nothing to provide towards the Superannuation Fund.

The PRESIDENT asked whether the liabilities were not running up during that time.

Mr. RUSSELL said they were not running beyond the interest on the accumulations and employees' contributions. One point often lost sight of was that although the workmen might retire at age 60, it would be found in the working of the Act that they would not retire, but that many of them would continue working until the age of 70. Many of the workmen could go on working until that age, and would rather do so and receive their full week's wages. A man could not live on the pension with so much comfort as he could on his full wage, and therefore would be more content to work as long as he was able to do so. The officers would be practically compelled in the Paddington case to retire at the age of 65, and he thought it was a wise thing that they should do so. Indeed, many of them could not go on beyond that age: the life was such a strenuous one and took so much out of a man that it was really necessary he should retire at that age. But even in that case they were allowed to go on from year to year by permission of the Borough Council who had to pass a resolution each year for that purpose.

He thought the improvement that was started in the financial aspect of the Superannuation Acts was due to Mr. Carson Roberts. He had been through the Acts to such an extent that he saw the weakness of the first Act and pounced upon Paddington as the one that should bear the brunt of the improvement. The Paddington Authorities were rather fearful for their Bill at the time, but by securing the passing of a special clause by the Borough Council, and ultimately by the Committee of the House of Commons, matters had ended satisfactorily. By the clause referred to the amount of superannuation paid by Paddington at the present time will be spread over a series of years and the equivalent annual charge represented thereby will be added to the amount of the new primary annual contribution under the Act.

Mr. H. E. W. LUTT in closing the discussion, said that, with regard to the evolution of municipal schemes generally, the Act of 1866, which provided for voluntary pensions only, was followed by the Poor Law Officers' scheme of 1896, under which both contributions were paid into, and pensions paid out of, the union fund. He thought it was the Corporation of Birmingham that next adopted the principle to which Mr. Carson Roberts had referred, namely, that the rates were to be relieved by the contributions which were received, and to be burdened with the pensions when they came to be paid. The Stepney Council started a small present contribution from the rates as a subsidy to the Fund. The Kensington Act was the first that provided for the actuarial certificate. In referring to the certificate he noticed the Authors mentioned that both actuarial and legal advice was obtained to the effect that the assessment principle was permissible. He had been unable to come to that conclusion. It seemed to him that the question of five years involved in the clause had reference merely to the time over which the contribution was payable, not to the actuarial certificate required. It might be

supposed that the Corporation were going to be relieved of their liabilities by payment, theoretically, to an outside body of such a sum as would enable that body, with the contributions of the officers, eventually to pay out the pensions, and of course the periodical revision of that sum was a necessity similar to that of a life office to value its assets and liabilities every five years, so as to declare its bonus. The Borough Accountants of the various Councils were by no means inexpert in figures, and it seemed to him that it did not require the calling in of actuarial advice to say that the contributions would exceed the payments for a short period of years ; they would probably have been able to find that out for themselves. The advice that nothing was necessary, but that £300 or £400 a year might be paid by the Council did not seem to him to be really meeting the case from a scientific point of view. As Mr. Carson Roberts had pointed out, the question had now been settled by the incorporation in the new Acts of a reference to existing and prospective liabilities. The Paddington Act was the first to include that clause. He would suggest that the references to the Paddington Act in the paper should be altered so as to refer to the Act as it now stood instead of as outlined in the original Bill.

Then there was the question that had been raised as to the "even annual charge." The new clause referred to existing and prospective liabilities, and it seemed to him there was a possibility of getting over the difficulty mentioned by Mr. Carson Roberts. Instead of taking the period of sixty years, or the future lifetime of the recipients of present pensions, if regard were paid to the *prospective* liabilities, *i.e.*, to the liabilities for replacements, and they were valued as at the present time, one could not go very far wrong if one spread the deficiency over the entire future. The revision every five years would have the effect of leaving out of account those pensions which had ceased through the death of the recipients, and thus automatically, on the death of the last pensioner, and in the meantime as the pensioners died off, the liabilities for pensions existing when the Act came into force would be satisfied. Similar remarks applied to the valuation of pensions accruing to existing officials. An assumption could be made as to the average age at which replacements would take effect, either on the withdrawal or the superannuation or the death of present holders of office, and the loss incurred on each replacement and the deficiency which the Council incurred at the commencement of the scheme could be valued to include those sums in perpetuity as successive endowment assurances payable on death or withdrawal or pension. It was a case of the valuation of successive endowment assurances just as an advowson involved a succession of lives—or rather deaths.

He noticed that the working of Tables X and XI involved the assumption of members' contributions being exhausted first. He took it that any reference to the employment of 10 per-cent of the accumulated fund was intentionally excluded. In the ordinary course under the Acts 10 per-cent of the fund would be available each year. He had some difficulty in understanding the exact meaning

of "continuing allowances." He noticed it was mentioned that the payment of pensions which had emerged from the past had been eliminated, and he should like some information upon that point. If the Service Table given in the paper was used, would it not have the effect of including amongst the allowances on death or withdrawal actual returns of assumed contributions, which had really not been paid? He did not quite follow the construction of the table in that respect.

In Table XV the authors referred to the allowance for past service of servants of the Borough Council. In Paddington he thought no past service in respect of workmen was aggregated for the purpose of pension. In Statement B in the Appendix a rather interesting column was inserted with regard to "Added years in respect of service with public authorities", and he noticed in the various Acts there were different ways of dealing with the service of existing employees with public authorities other than the predecessors of the particular Borough Councils, and perhaps it would be well for the completeness of the table if it were amplified a little. For instance, in Stepney the aggregate scale was applied to all existing members of the staff, and both old and new members had to pay extra contribution for past service; but in Camberwell it was only new appointments that involved the payment of further extra contribution for past service with other authorities. In Westminster the past service with other authorities was not aggregated for the scale contribution but involved an extra $\frac{1}{2}$ per-cent for each five years only. In the Paddington Act service with preceding authorities was aggregated for the assessment of the scale contribution; and service with other authorities required an additional 1 per-cent for each five years, or the refund of any allowance received from such other authority, at the Council's option.

The **PRESIDENT** thought the members would agree that the discussion had been a very interesting one. He had been struck with Mr. Thomas's criticism of the tabular matter, and no doubt Mr. Ackland, in his reply, would pay some attention to what he had said. With regard to the broader questions involved in the paper, he thought everyone must have listened with very great interest to the remarks of Mr. Carson Roberts and Mr. Russell. The successful working of the Superannuation Acts was in great measure due to the fact that there happened to be at the Local Government Board a public official who was not to be frightened by having to grapple with subjects which lay a little outside his ordinary work, but who attacked them in a courageous fashion and had that evening proved himself well able to cross swords on actuarial matters, even with their doughtiest champions. He thought it was a matter of great congratulation that at such a time so capable a public official as Mr. Carson Roberts had had under his charge this great public question.

In the course of the discussion reference had been made to what might be regarded as the ultimate or ideal solution of the pension difficulty, and there was little doubt, he thought, that in the interests

both of the officer and of the employer, whether a personal employer, a municipal employer, or the State itself, the best solution would be found in establishing an accumulating fund which should be kept solvent from time to time in accordance with actuarial principles. Then every officer on that fund would feel sure that his pension would be properly paid in due time, and the employer would be able to know exactly the extent of the charge which would fall upon him to keep the fund in proper order. There were, of course, grave public difficulties in the matter. The accumulation of a very large fund in the hands of certain bodies might in times of stress arouse the cupidity of the authorities—whether ratepayers or Chancellors of the Exchequer—and it might be thought undesirable, on this or other grounds, to parade large accumulating funds. There was, however, one step that could and should be taken, which fell short of the establishment of an adequate accumulating fund, and that was that each Authority should be advised by competent actuaries as to the probable future charge which would be incumbent upon them in respect of their liabilities for pensions. He had observed an awakening sense of responsibility on the part of Departments of the Government, Municipal Authorities and large employers of labour, who, for reasons which he had stated or for other reasons best known to themselves, had decided not to set up an accumulating pension fund, and had found a desire on their part to know more exactly the effect of their policy with regard to such pensions and to be advised as to the extent to which their commitments and contracts with their employees would carry them in the future. That was a great thing achieved. It was highly desirable that public bodies should know, not only the amount of the obligations which would emerge in the first few years after the step had been taken, but the eventual charge to which by such act they were committing future ratepayers. He welcomed the discussion that evening as throwing much-needed light on a very important public question. In conclusion, he proposed a cordial vote of thanks to the authors for their paper.

The motion was carried unanimously.

Mr. T. G. ACKLAND, in replying, said the discussion had covered a good deal of ground, and he trusted that those who had taken part in it would not feel that their remarks had been ignored, if he was unable specifically to refer to them in the short time at his disposal. In opening the debate Mr. Thomas spoke of the authors advocating the spreading of the initial charge over the whole after-lifetime of the existing officers and servants, but suggested that apparently in the latter part of the paper they rather advocated spreading it over the shorter term of active service. That was not quite the case. It was left open, in the earlier part of the paper, for the actuary to adopt either course, but, in the latter part, when the authors were dealing with Tables XIV and XV, it was more convenient to follow Mr. Manly's methods, which proceeded on the lines of a contribution payable during active service only. The difficulty of bringing in ages over 64, as Mr. Thomas had pointed out, was that there was no

salary payable on which to assess the contribution, and that would involve a re-casting of the methods and tables to some extent. Personally, he did not see the logical necessity for carrying the initial data for the fund very much further than the 45 years which was proposed in the paper. It appeared to the authors that it was at the point when pensions began to become numerous that the Borough Councils would be most likely to desire to start a fund, and the period had been chosen by the authors to accord with that principle, which had they thought been borne out in practice.

Mr. Carson Roberts had made a most valuable contribution to the discussion, and his remarks would require careful consideration before being dealt with by way of criticism or reply. His analysis of the different phases of the growth of the superannuation funds was extremely interesting, and had some points new to the authors, including his reference to the assessment principle probably dating back as far as twenty years ago. It was the case, as Mr. Roberts remarked, that in some Corporations—Birmingham and Liverpool he thought were illustrations—the assessment principle was applied in the most bare form, and that in some instances an actual reduction of rates had been brought about by the temporary excess of contributions over and above the charges of the year. The authors had thought it was probable that, in the Stepney and Bethnal Green Acts, the compilers had been somewhat misled by not tracing out the term of the liabilities long enough. He believed it was the case in Stepney that the term was traced out for about fifty years by methods which were perhaps not entirely actuarial, and, of course, it would be recognized by those who were familiar with Mr. Manly's figures that fifty years was not nearly long enough, and that the liabilities only began to emerge then, so that such an amount as £100 a year was quite inadequate. Mr. Roberts had said that he was responsible for the phrase introduced in later Acts, "involving as near as may be an even annual charge." Mr. Roberts was quite right in saying that these words, literally interpreted, must necessarily involve a perpetual rent charge, and therefore the throwing on the ratepayers in perpetuity of the initial obligations of the fund. He understood, however, that Mr. Roberts would not demur to the sixty years' term that had been suggested in the paper, and possibly in future Acts, with Mr. Roberts's good help, the objectionable words referred to might be omitted.

Mr. Tinner, as well as other speakers, had given an alternative method of dealing with the initial liability by spreading it over future years. The authors would like to have an opportunity of studying more fully the method Mr. Tinner had expounded, which was rather difficult to follow verbally.

With regard to the working of Tables XIV and XV, it would have materially extended the paper if the methods adopted had been fully dealt with. Mr. Thomas's explanations of the methods which he thought had been adopted in the paper were, however, almost identical with those adopted by the authors.

In reply to Mr. Lutt, the authors made due allowance in Tables

XIV and XV for the fact that contributions which had not been paid in the past would not involve any future return. There was a modification in respect of the accumulation of the returns, as well as in respect of the accumulation of pensions, modifications which were rather difficult and complicated in their nature, but which perhaps should have been explained in the paper.

Mr. Russell had referred in an interesting way to certain funds which had been started and worked successfully apparently for many years, and be believed one of the funds referred to was that of the Great Eastern Railway. There was no greater authority on that fund than Mr. Manly, who had already shown by tables the present position of affairs and the probable future of that fund.

Mr. Lutt advocated the spreading of the initial liability over a perpetual term, but pointed out very cogently that that was really only done in name. That was one of the difficulties to be contended with in such calculations, that while the actuary at the moment decided that the liability was to be spread over a perpetual term, the next actuary, who took up the work five years afterwards, found different conditions existing, and, although he might apply the same methods, he did not really spread the original liability over a perpetual term, as several of the members had passed away and were necessarily excluded from his calculations. Mr. Lutt had also referred to the fact that Tables X and XI excluded the 10 per-cent draft on capital, but those tables were really based on the assumption that the actuarial estimate of liabilities would strictly accord with the facts, and that therefore there would be no question of any draft on capital, so long as that were the case.

The Medico-Actuarial Investigation of the Mortality of American and Canadian Life Assurance Companies.

[Communicated by Mr. ARTHUR HUNTER, A.I.A., F.F.A., Chairman of the Central Bureau.]

THE Joint Committee which is undertaking this Investigation on behalf of the Actuarial Society of America and the Association of Life Insurance Medical Directors, has just published its first Report. It deals with two subjects :

- (a) Standard Tables of Height and Weight ; and
- (b) Standard Mortality Ratios to be used for obtaining the expected deaths in the various classes to be studied.

There were 43 companies which contributed their data on Policies issued in the United States and Canada, the insurance in force in these companies being about 93 per-cent of the total in

all old-line companies during the years covered by the investigation (1885-1909).

In the first part of the report the Committee states that the investigation was only intended to include types of risks which were likely to be of practical insurance value, and that classes of purely academic interest would be excluded; that the mode of procedure planned by the Committee was such that each Company might apply it to its own business for individual investigations, and that the investigations should extend back only so far as to include cases accepted under modern conditions of medical selection and life environment. The Committee then states the means adopted to obtain the opinions of the actuaries and medical directors of all the companies as to the classes to be investigated. Incidentally the reasons are given why the preparation of a mortality table among standard lives was not undertaken. The implication, however, is that the Actuarial Society will undertake such a work after the present investigation is completed. The present standard Table—the American—was prepared in 1869, and was based on a small amount at risk. At a meeting in 1911 of the Insurance Commissioners of the various States the Actuarial Society of America was urged to undertake the preparation of a new standard Table. This suggestion has been placed before the Society, which has favourably acted upon it. On account of the marked difference in the climate and sanitary conditions in various parts of the United States and Canada and also the difference in mortality under the various plans of insurance, careful consideration must be given to the plans for obtaining the data for a new standard table.

The Hollerith system of perforated cards and of electric sorting and tabulating machines was used.

TABLE OF HEIGHTS AND WEIGHTS.

In order to obtain a new table of average weight at each age according to inches in height, the companies supplied their data on the issues of January of the odd years, and July of the even years 1885 to 1900 inclusive, these months in the year being selected so as to counteract the differences between summer and winter weights. The data for men and women were treated separately, and all cases on which an extra premium has been charged or which were treated as under-average were excluded. The necessary records were received from the companies on 221,819 policies on men. The number of policies on women was

less than 10,000, as the companies did not insure women freely prior to 1890. Four companies accordingly contributed additional material of more recent date, so that there were in all 136,504 policies on women. In the report it is brought out that no reliable tables have been published giving the relation of weight to height by age in English-speaking countries, with the exception of the National Fraternal Congress Tables, based upon the experience of American Fraternal Orders, and the Medical Directors' Table (1897) based on lives insured in the regular companies in the United States. The latter Table is frequently referred to as the "Shepherd" or the "Wells" Table. The German companies have recently published a table of height and weight based on their experience, and an Austrian company in 1908 published a similar table. In Great Britain there does not seem to be any reliable table in existence based on insured lives, giving the height and weight according to age. Through the courtesy of Mr. J. J. M'Lauchlan, the Committee was given a synopsis of the practice of eighteen prominent companies, from which it appeared that a number of the companies used the statistics of the Medical Directors' Association of America, as the tables for Great Britain did not make allowance for age.

The following is the graded weight table in pounds for men by five-age groups based on the data of the American and Canadian Companies :

Age Group	5 ft.	5 ft. 1 in.	5 ft. 2 in.
15-19	113	115	118
20-24	119	121	124
25-29	124	126	128
30-34	127	129	131
35-39	129	131	133
40-44	132	134	136
45-49	134	136	138
50 and over	135	137	139
Age Group	5 ft. 3 in.	5 ft. 4 in.	5 ft. 5 in.
15-19	121	124	128
20-24	127	131	135
25-29	131	134	138
30-34	134	137	141
35-39	136	140	144
40-44	139	142	146
45-49	141	144	148
50 and over	142	145	149
Age Group	5 ft. 6 in.	5 ft. 7 in.	5 ft. 8 in.
15-19	132	136	140
20-24	139	142	146
25-29	142	146	150
30-34	145	149	154
35-39	148	152	157
40-44	150	154	159
45-49	152	156	161
50 and over	153	158	163
Age Group	5 ft. 9 in.	5 ft. 10 in.	5 ft. 11 in.
15-19	144	148	153
20-24	150	154	158
25-29	154	158	163
30-34	158	163	168
35-39	162	167	172
40-44	164	169	175
45-49	166	171	177
50 and over	168	173	178
Age Group	6 ft.	6 ft. 1 in.	6 ft. 2 in.
15-19	158	163	168
20-24	163	168	173
25-29	169	175	181
30-34	174	180	186
35-39	178	184	191
40-44	181	187	194
45-49	183	190	197
50 and over	184	191	198
Age Group	6 ft. 3 in.	6 ft. 4 in.	6 ft. 5 in.
15-19	173	178	183
20-24	178	183	188
25-29	187	192	197
30-34	192	198	203
35-39	197	203	209
40-44	201	208	214
45-49	204	211	217
50 and over	205	212	219

The foregoing table was found to agree very closely with that of the Fraternal Congress and that of the Medical Directors' Association, there not being a variation of more than one pound except at six feet and over.

From the foregoing were prepared tables of average weight for all ages at entry from 15 to 55 inclusive for each inch in height from five feet to six feet five inches.

Tables are given in the report showing the increase in weight for each consecutive five-age group, which is naturally found to be a decreasing amount. The average weight of 7,406 men aged 50 to 54 is 165.5, and of 5,018 men from age 55 to 84, 165.3. It was accordingly decided to assume that the weight for ages at entry 55 and above was the same as for age 55.

A table is given showing the percentage of entrants at each unit figure in weight for a large group chosen at random, in order to determine approximately the proportion of recorded weights which were estimated. It was found that 33 per-cent of the applicants gave the last integral figure of their weight as 0, and 28 per-cent gave 5 as the last integral figure, from which it may be assumed that in about two-fifths of the cases the weights were estimated.

Tables are also given of the average weight by 5-pound groups from which it appears that the range of the weights is greater than was anticipated. A brief example of this is now given.

AGES 20-39		HEIGHT 5 FT. 8 IN.	
Weight Group		Number	Ratio to Total
Below 123		99	.7
123-127		259	1.9
128-132		602	4.3
133-137		1,238	8.9
138-142		1,574	11.3
143-147		1,558	11.2
148-152		1,770	12.7
153-157		1,270	9.1
158-162		1,350	9.7
163-167		1,022	7.4
168-172		842	6.1
173-177		681	4.9
178-182		507	3.6
183-187		342	2.5
188-192		270	1.9
Above 192		524	3.8
Total	13,908	100.0
Average Weight 155 pounds.			

Several tables similar to the foregoing are given, and the Committee makes the following interesting statement: "It is probable that the normal weight for any age and height may vary considerably from the average weight for that age and height. We have used the word 'normal' in the sense of typical of the group." Readers are warned not to come to the conclusion that those at the average weight have the lowest mortality. The next report of the Committee will deal with the influence of build on longevity.

Comparisons are made of the average weight under the new standard with the experience of the German Companies, and with the Austrian Company, Assicurazioni Generali, from which it appears that excluding the extremes of both height and weight, the average weight of the German insured is about 10 per-cent higher than among the insured in the United States and Canada, and that the smaller the stature of the German the more does he exceed in average weight the American. The statement is made that judging from these and also from unpublished statistics the average weight in one country may not be applicable to another country, especially where there is a difference in race.

WOMEN.

Tables similar to those for men were also prepared for women, and we give here the graded weights for quinquennial age groups.

Age Group	4 ft. 8 in.	4 ft. 9 in.	4 ft. 10 in.
15-19	104	106	108
20-24	107	109	111
25-29	110	112	114
30-34	113	115	117
35-39	116	118	120
40-44	120	122	124
45-49	123	125	127
50-54	125	127	129
Age Group	4 ft. 11 in.	5 ft.	5 ft. 1 in.
15-19	110	112	114
20-24	113	115	117
25-29	116	118	120
30-34	119	121	123
35-39	122	124	126
40-44	126	128	130
45-49	129	131	133
50-54	131	133	135
Age Group	5 ft. 2 in.	5 ft. 3 in.	5 ft. 4 in.
15-19	117	120	123
20-24	120	123	126
25-29	122	125	129
30-34	125	128	132
35-39	129	132	136
40-44	133	136	139
45-49	136	139	142
50-54	138	141	144
Age Group	5 ft. 5 in.	5 ft. 6 in.	5 ft. 7 in.
15-19	126	130	134
20-24	129	133	137
25-29	132	136	140
30-34	136	140	144
35-39	140	144	148
40-44	143	147	151
45-49	146	151	155
50-54	148	152	157
Age Group	5 ft. 8 in.	5 ft. 9 in.	5 ft. 10 in.
15-19	138	141	145
20-24	141	145	149
25-29	144	148	152
30-34	148	152	155
35-39	152	156	159
40-44	155	159	162
45-49	159	163	166
50-54	162	166	170
Age Group	5 ft. 11 in.	6 ft.	
15-19	150	155	
20-24	153	157	
25-29	155	159	
30-34	158	162	
35-39	162	165	
40-44	166	169	
45-49	170	173	
50-54	174	177	

While the weight for age 55 and above is taken as of age 55, there were indications that above age 60 the average weight was less; the average weight in the group 50 to 59 was about four pounds more than for ages 60 to 74, but there were only 688 cases in the latter group.

COMPARISON BETWEEN WEIGHTS OF MEN AND OF WOMEN.

A comparative table is given in the Report of the average weight of men and women 5 ft., 5 ft. 4 in., 5 ft. 8 in., and 6 ft. in height, for quinquennial age groups, from which it appeared that the difference in weight at the same height is slight below age 20, and above that age young men are distinctly heavier than young women, the difference becoming less marked as they grow older. Tall women are markedly lighter than men of the same height.

The ungraded statistics were also combined for each inch in height irrespective of age, from which it appeared that the increase in pounds for each increase of an inch in height was practically the same among men as among women.

It should be borne in mind that the average weights are taken in the street costume, the overcoat being removed, and in some instances the coat also being removed.

HEIGHT.

The average height of the men was found to be 5 ft. 8½ in., and of the women 5 ft. 4¼ in. It should be remembered in making comparisons that the Insured are measured in their shoes.

MORTALITY RATES TO BE USED IN DETERMINING THE EXPECTED DEATHS IN THE VARIOUS CLASSES.

In order to obtain select rates of mortality which represent the average mortality experience of the combined companies in the United States and Canada, with sufficient accuracy to be used as a measure of standard mortality in the various classes, the Committee analysed the statistics used in obtaining the average height and weight. There were 229,971 entrants, with deaths of 15,180. It was found that the actual deaths were only 81 per-cent of the expected according to the select table used in the Specialized Investigation. A test was then made to determine the applicability of the O^M Table. The expected

deaths by that Table were found to be 18,930, with actual of 15,180, a ratio of actual to expected of 80 per-cent. It should be borne in mind that the basis of the new investigation consists of policies, and that there is a goodly proportion of Endowments and Limited Payment Life insurance in the Medico-Actuarial statistics. The experience was also tested by the Select Table of the New York Life Insurance Company, known as the Compound Progressive, which is based on amounts insured, the ratio of actual to expected deaths being 87 per-cent. Many tests were made to determine whether there was any error, as the mortality was lower than the Committee expected, such tests including the experience of three companies taken separately, in respect of which it was known that all chances of error had been practically eliminated. In order to obtain confirmation of the low rates of mortality and in order that the rates of mortality to be used in calculating the expected deaths in the several classes of risks to be investigated should cover the same range of years of issue as these classes, the companies were asked to supply statistics on the issues of January of the odd years, and July of the even years 1901 to 1908 inclusive, carried to the anniversaries in 1909. While this delayed the investigation several months it put the Committee in possession of the complete experience of the companies for January of the odd years and July of the even years 1885 to 1908 inclusive, carried to the anniversaries in 1909, and from this material the standard mortality ratios were derived. The extent of the statistics can be seen from the following table :

Years of Issue	Number of Entrants	Total Exposures	Deaths
1885-1892	80,976	781,852	7,180
1893-1900	148,995	1,106,216	8,000
1901-1908	270,404	926,180	5,042
	500,375	2,814,248	20,222

Graded rates of mortality obtained either by inspection or by a graphic process were then constructed, and it was found that the rates for the 11th and succeeding policy years for the same attained age differed so very slightly from those for the 6th to the 10th years that ultimate rates were prepared from the data

for the 6th and succeeding years. The ultimate values from attained age 70 and upward are equal to those of $O^{(M)}$ after 10 years. Finally, the data for the 5th policy year were included in the ultimate. The Committee states that it should not be inferred that the effect of medical selection passes off in 4 years, but rather that it endures for a much longer period: but owing to the improvement in the health of the community the effect of selection cannot be clearly traced in the table. It is pointed out that there are two independent forces at work, both tending to improve the mortality, namely:

- (a) More intelligent methods of medical selection.
- (b) Better sanitary conditions and advancing knowledge, resulting in continued improvement in the general health of the community.

Other forces undoubtedly affect the mortality, but the foregoing are probably the principal ones. As the effects are intermingled it is difficult to determine the degree in which each affects the mortality.

Examples of the graded rates of mortality per 1,000 based on the experience by policies are now given:

Ages at Entry	POLICY YEARS					Attained Ages
	1st	2nd	3rd	4th	5th and later	
22	3.3	4.5	4.6	4.7	4.8	26
32	3.7	4.8	4.9	5.0	5.2	36
42	4.7	6.4	6.9	7.5	8.0	46
51	8.9	12.2	13.4	14.6	15.8	55
61	20.5	27.5	32.1	35.4	39.0	65
70	36.0	60.0	72.3	78.3	84.8	74

Rates of mortality for all ages of issue are not given because it is intended to obtain the expected deaths in the various classes according to five age groups at issue up to ages 45 to 49, the groupings thereafter covering few ages at issue. Interesting statistics are given of the average age at issue in various companies and of the average ages of the insured after the policies have been 10, 15 and 25 years in force.

Tables appear in the Report showing the results of applying the new graded rates to the data when split up into three groups of eight years of issue each. For the period 1885 to 1892 the ratio of actual to expected deaths is 105 per-cent, for 1893-1900

101 per-cent, and for 1901-1908 93 per-cent, showing a distinct improvement in the mortality of recent years.

The Committee closes this part of the report with a warning against using the rates of mortality for any other purpose than that for which they were prepared. They should not be used in the preparation of premiums, reserves, or other monetary values, because the basis is policies and not amounts insured, the mortality by the former being generally lower in the United States than by the latter.

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., *Barrister-at-Law*.

Income tax in respect of reserve for unexpired risks. THE case of *Ernest Clark (Surveyor of Taxes) v. The Sun Insurance Office* is concerned with the question of the liability of an insurance company to income tax in respect of the provision made for unexpired risks, when that provision is, admittedly, the proper one to make in the circumstances. The case was reported in these Notes when it came before the Court of Appeal (*J.I.A.*, vol. xlv, p. 389). That Court, reversing the decision of Mr. Justice Bray, came, reluctantly, to the conclusion that the case was covered by the decision of the House of Lords in the case of *General Accident Fire and Life Assurance Corporation v. McGowan* ([1908] A.C. 207, *J.I.A.* vol. xlii, p. 407), and held that the insurance society was liable to tax in respect of the reserve in question, although the Master of the Rolls, in delivering judgment, said that: "Apart from authority he should have had no hesitation in agreeing with the argument of the respondents (the insurance office)."

The insurance office appealed against this decision, and the House of Lords upheld the appeal, reversing the decision of the Court of Appeal. The Lord Chancellor, in delivering judgment, expressed his surprise at the view taken by the Court of Appeal of the decision in the case of the *General Accident Corporation v. McGowan*, and anyone who has compared the two cases is likely to share in that surprise. All the Law Lords agreed in allowing the appeal, and it will be sufficient for the purposes

of these Notes if an extract from the judgment of Lord Haldane is given. He said: "The appellants carry on the business of fire insurance. It has been their practice to carry forward annually 40 per-cent of their yearly premium receipts, in order to bring about the correct proportion, as between year and year, of the premium income which has to answer to the annual incidence of fire losses, and on the balance of which income, over or under the losses or charges for the year, the profit or loss attaching to their business depends. If the premium income were stationary this would make but little difference as regards income tax, for a uniform amount deducted would be carried forward to profit in the ensuing year. But the business of the appellants and their premium income are increasing, with the result that the amount escaping the tax in each year varies progressively. The Commissioners for Taxes in the City of London, after examining the case, were of opinion that the percentage carried forward in each year was a reasonable and proper allowance, and did not form part of profits or gains for the year. The surveyor admitted that he was not in a position to contest that the percentage carried forward was accurately estimated, but contended that the appellants were liable in respect of the entire premium income, on the ground that the Courts had laid down that no such deduction was permissible. A case was stated for the opinion of the High Court of Justice, and Bray, J., decided in favour of the appellants. The Court of Appeal reversed his judgment and decided in favour of the Crown, on the ground that they were bound by the decision of this House in the case of *General Accident Fire and Life Assurance Corporation v. McGowan*.

"It is plain that the question as to what is or is not profit or gain must primarily be one of fact, and of fact to be ascertained by the tests applied in ordinary business. Questions of law can only arise when (as was not the case here) some express statutory direction applies and excludes ordinary commercial practice, or where, by reason of its being impracticable to ascertain the facts sufficiently, some presumption has to be invoked to fill the gap. Such a presumption was made in the appeal referred to, where, as pointed out in the judgment delivered by the Lord Chancellor, it was not shown that the percentage deducted represented the real value of risks yet to run, and was so unearned, and where it appeared further that the Company had actually divided the entire amount without

“deduction among its shareholders as part of the profit of the year. In *General Accident Fire and Life Assurance Corporation v. McGowan*, it was, therefore, in the absence of evidence to the contrary, presumed that the whole of the premium income received in the year represented income earned by bearing the risks of the year. It is difficult to see how, on the materials before the House, the decision could have been otherwise, but it seems to me equally clear that the reason of the decision has no application where, as here, it has been accurately ascertained that part of the premiums are not in the nature of profits earned. As Bray, J., puts it, the case is analogous to one in which if goods are bought their value cannot be treated as profit without deducting the value of the liability to pay for them which the buyer has incurred. . . . The judges in the Court of Appeal appear to have thought themselves bound to hold that this House had laid down a rule of law so rigid that, although introduced for the purpose of supplying deficient evidence of fact, it must still apply even where there was no deficiency of proof. I have come to a different conclusion. I think that Bray, J., has correctly interpreted the decision in question, when he says that if in the earlier case such facts had been before this House as have been established in the present case, it would, in his opinion, have decided against the Crown. The present appeal ought, in my judgment, to succeed.”

Right of
assurance com-
pany to delivery
of policy before
paying claim.

I am indebted to Dr. A. E. Sprague, F.I.A., F.F.A., of the Edinburgh Life Assurance Company, for particulars of a case dealing with the question of the right of an assurance company to demand the production of the policy before paying the claim, when such policy is known to be in existence, and in the hands of a third party. The case referred to is that of *Scott v. Edinburgh Life Assurance Company*, and the material facts are as follows :—

A policy for £500 was taken out by one Walter Paton Scott on his own life, and on his death the policy moneys were claimed by his widow and executrix, Rachel Scott, who produced confirmation in her favour. She was unable to produce the policy, and it then transpired that the assured had, during his lifetime, deposited the policy with the Clydesdale Bank, together with a letter agreeing to allow it to remain there as security for a

cash credit drawn by assured upon the bank, and an undertaking to grant an assignation in favour of the bank, if required at any time. The executrix pressed for payment without the production of the policy, and was willing to give a letter of indemnity to the company. The latter, however, declined, in the circumstances, to pay without the policy, and action was accordingly taken against them in the Sheriff's Court for the recovery of the amount.

On the case coming before the sheriff, he found in favour of the assurance company, and dismissed the action with costs, the note to the judgment being to the following effect: "The insurance policy, during Mr. Scott's life, was a contingent document of debt. When he died it became an absolute document of debt, which his executor became entitled to present for payment. The pursuer, when confirmed as executrix, became defender's creditor. Defenders are willing to pay, upon the policy being handed up to them. This is a reasonable condition. The policy may have been assigned, or pledged, or it may lie subject to a lien, but if it exists it must be produced by the party claiming payment under it, for I think a creditor is bound to deliver up to his debtor a document of debt, if the document of debt exists, as this policy admittedly does. When a document of debt has been lost, it may in practice be customary and convenient for a creditor to give the debtor an indemnity letter in place of delivering the document of debt. But I do not think a debtor under an existent document of debt is legally bound to pay upon an indemnity letter, and I never heard of an insurance company being bound to pay without receiving delivery of their admittedly existent policy."

Right of company to costs in foregoing circumstances when estate was sequestered.

This case had a somewhat interesting sequel in regard to the costs of the action. The sheriff disposed of the case by intimating his opinion on 21 February 1907, but delayed issuing judgment until 20 March.

Between those dates sequestration of the estate of the late Mr. Scott had been applied for, and as it was ultimately granted, the sequestration dated back to 13 March, the date of the application. On the trustee in the sequestration satisfying the assurance company as to his title, and he being in a position to deliver up the policy, the company expressed their willingness to pay over the policy moneys to him, but intimated their intention of retaining thereout the amount of the costs of the action. The

trustee declined to admit the company's right to do this, and the dispute became the subject of a small debt action. This was decided on 3 July 1907, by the sheriff in favour of the trustee, on the ground that the company's claim for expenses in the earlier action was against the executrix personally, and that, therefore, there was no *concursus* of debit and credit between them and the trustee. It is understood that the company's legal advisers considered that this decision was wrong in law, but as there is no right of appeal against an error in law under the Small Debt Act, nothing further could be done in the matter.

Rights of parties
under an
endowment
settlement policy
issued under
Married Women's
Policies of
Assurance
(Scotland Act,
1880.

Mr. H. C. Thistleton, F.I.A., F.F.A., has been kind enough to furnish me with particulars of a case which came recently before the Scottish Courts, and which is concerned with the rights of the various parties to the proceeds of certain endowment assurance policies coming under the Married Women's Policies of Assurance (Scotland) Act, 1880. The case came before the First Division of the Court of Session as a special case for James Bogle Gibson, Trustee of the sequestrated estate of the deceased David Chrystal, and the facts, so far as they are material for the purpose of these Notes, are as follows: David Chrystal, by an ante-nuptial settlement dated 27 June 1894, bound himself, *inter alia*, to pay to the trustees of the settlement certain sums amounting in all to £3,000. In September 1894, the said David Chrystal effected two "with profits" policies for £1,000 each with the Equitable Life Assurance Society of the United States, such policies being declared payable at the company's London office. Certain options were given by these policies, all exercisable by the assured and including a right on his part to the payment of the policy moneys on the expiration of twenty years, and by each of them it was provided that in the event of the death of the said David Chrystal before the expiration of twenty years from the date thereof, and while the policy was in force, the society would, immediately on receipt of satisfactory proof of death, pay the sum assured (£1,000) to his wife (who was a party to the policy), if living, and if not, then to the said David Chrystal's executors, administrators or assigns. The policies provided, further, that should the annual premiums paid thereon,

compounded annually at four per-cent, exceed the sum assured, such excess should be added to such sum and paid therewith.

The wife of the assured gave no price or consideration to him in respect of his effecting and keeping up the policies, nor did she accept them in lieu of any of her rights under her marriage settlement. The premiums were paid out of the husband's own funds, the wife having no separate estate. The assured died before the expiration of the period of twenty years named in the policy.

The said David Chrystal had, during his lifetime, and for the purposes of his business borrowed certain sums from the assurance society on the security of the policies, the wife being in each case a party to the mortgage in favour of the society. These loans, at the time of assured's death, amounted, with interest, to £1,077. The policy moneys, less this amount, were paid by the society to the trustee of the sequestrated estate of David Chrystal and to assured's wife, on their joint receipt, and were then placed on deposit in their joint names to await the decision of the Court as to the rights of the parties. The actuarial value, as at the time of David Chrystal's death, of the provision under the marriage settlement in favour of the wife, which the settlor had failed to carry out, was £1,640.

The question presented for the decision of the Court was as to whether the trustee of David Chrystal's estate, or the wife of the assured, was entitled to the balance of the proceeds of the policies, after deducting the amount of the loans. For the purposes of the case it was agreed that David Chrystal was a domiciled Scotsman at the date when the policies were effected, and remained so continuously until the time of his death, and that the question was to be determined on the same footing as if the contracts contained in the policies had been entered into in Scotland, and were to be construed in accordance with the law of Scotland.

The case came before the First Division of the Court of Session on 30 May 1912, when judgment was given in favour of the wife of assured. Lord Johnston, in delivering judgment to this effect, said: "This is a policy of a peculiar description. "It is a combination of the ordinary endowment insurance, and "of the 'Married Women's Policy of Assurance', and the only "difficulty in the case arises from that combination. The "methods of the Equitable of New York are not those to which "we are accustomed in the practice of our insurance companies.

‘But they are only more complicated. They are perfectly legitimate developments of the principles of life insurance.

“The policy in the present case is, as regards the primary interest of the assured, David Chrystal, a twenty-years endowment policy—that is to say, it secures him payment of the sum assured with accrued share of profits, on his maintaining the policy for twenty years and surviving the term. That it allows him to take payment in four alternative ways is immaterial. What is material is that at and after the expiry of the endowment period he, and he alone, is entitled to the benefits under the policy. But within the endowment period, by which I mean the period of twenty years, which is the primary currency of the insurance, the policy is a provision for the wife of the assured. In the event of the death of the assured within the twenty years, the policy being meanwhile duly maintained, the sum assured is made payable to the wife of the assured *nominatim*, if surviving, and failing her, to the assured’s representatives. The difference between the policy in question and the usual policy taken out by a married man for the benefit of his wife, is that the latter confers a right on the wife contingent merely on her husband predeceasing, while this policy confers a right on the wife contingent on a double event, viz.: (1) the husband predeceasing her, and (2) dying during the endowment period, that is, before the expiry of the twenty years during which the policy is primarily current.

“I think, notwithstanding this double contingency, that the policy in question comes under the protection of the Married Women’s Policies of Assurance Act, 1880. It is a policy effected by a married man on his own life, and it is expressed on the face of it to be for the benefit of his wife. The statute does not restrict the benefit of the wife to any specified interest in the policy. The statute does not say that the benefit of the wife must be absolute and void of contingency, so as to leave nothing in the husband. . . . I read the enactment as providing that the policy and all the benefit thereof shall be deemed a trust for the benefit of the wife for her interest, as that benefit is defined or expressed in the policy. That is, I think, the meaning of ‘in trust for the purpose or purposes so expressed,’ and I hold that that benefit may be a contingent benefit.”

Bankruptcy.
Protection of
transactions
taking place
before date of
receiving order.
Dating back of
receiving order.

Under section 49 of the Bankruptcy Act, 1883, certain transactions with a bankrupt are protected, and are not invalidated by the bankruptcy, provided they are bona fide transactions without notice of the bankruptcy, and provided, also, that they take place before the date of the receiving order. The date of the receiving order thus becomes of great importance, and the recent case of *In re Teale, ex parte Blackburn* [1912] 2 K.B. 367, which deals with this point is of some interest from that point of view. It was decided in that case that "where, on appeal from the dismissal of a bankruptcy petition, a receiving order is made, and the order, in accordance with the usual practice of the Court, is antedated as of the date when the petition was wrongly dismissed, for the purposes of section 49 of the Bankruptcy Act, 1883, . . . the date of the receiving order is the date on which it was made, and not the date which it bears."

ACTUARIAL NOTES.

I.

A practical formula for the value of a loan repayable by a cumulative sinking fund operating at every p th interest date.

A Loan of 1 bearing interest at the nominal rate of j per interval is repayable by a cumulative sinking fund of z operating at the end of each p th interval and repaying the entire loan in n intervals. What is the present value of the loan to yield the rate of i per interval?

Previous discussions of this problem (*T.B.*, Part I, pp. 94-6; *J.I.A.*, vol. xliii, p. 107) have not left it in a completely satisfactory condition, because the solutions involve an annuity-value of a special character which is a little troublesome to calculate accurately. This special annuity-value is eliminated in the following solution, which takes a remarkably neat form.

First let us find the value of z ; and to abbreviate, call a cycle of p intervals a "period" (usually, though not necessarily, a year). During the first period, the interest will be j each interval, and the capital repayment at the end of the period will be z , leaving $1-z$ outstanding. During the next period the interest will be $j(1-z)$ each interval, and as the total payment during the period for principal and interest remains unchanged, namely, $pj+z$, the capital repayment at the end of the second period will be $pj+z-pj(1-z)=z(1+pj)$; and the capital outstanding will be $1-z(1+1+pj)=1-zs_2^{pj}$. Continuing in the same way it will easily be seen that the capital repayment at the end of the t th period is $z(1+pj)^{t-1}$, and the capital outstanding is $1-zs_t^{pj}$. Thus as the loan is to be totally repaid at the end of n intervals, or n/p periods, we must have $1-zs_{n/p}^{pj}=0$ or $z=1\div s_{n/p}^{pj}$. Thus the sinking fund z is the same as if the interest were payable once per period instead of p times.

At the end of the $(t-1)$ th period the amount outstanding will be $1-zs_{t-1}^{pj}$; therefore the interest for each interval during the following period, *i.e.*, the i th, will be

$$j(1-zs_{t-1}^{pj})=j\left(1-z\frac{(1+pj)^{t-1}-1}{pj}\right)=\left(j+\frac{z}{p}\right)-\frac{z}{p}(1+pj)^{t-1}.$$

These payments will be equivalent at rate i to $\left(j+\frac{z}{p}\right)$ each interval less a payment of

$$\frac{z}{p}(1+pj)^{t-1}s_{p}^{i'}=z(1+pj)^{t-1}\frac{s_{p}^{i'}}{p}$$

at the end of the period. But the capital payment at the end of the period is $z(1+pj)^{t-1}$. Therefore the total payments (interest and capital) of the t th period will be equivalent to $\left(j+\frac{z}{p}\right)$ each interval less (full capital repayment at end of period) $\times \left(\frac{s_{p}^{i'}}{p}-1\right)$ at the end of the period. Let us write ϕ for $\left(\frac{s_{p}^{i'}}{p}-1\right)$ and F for $\left(j+\frac{z}{p}\right)a_{\overline{n}|i}$; and let K =the present value of the capital repayments and I =the present value of the interest payments.

Then the present value of the whole loan

$$= K + I = F - \phi K . \quad . \quad . \quad . \quad (1)$$

But by the ordinary formula

$$K + I \frac{i}{j} = 1 \quad . \quad . \quad . \quad . \quad (2)$$

Solving these equations we find

$$K = \frac{j - iF}{j - i(1 + \phi)}$$

$$\text{whence} \quad K + I \text{ or } F - \phi K = F + \phi \frac{j - iF}{i(1 + \phi) - j} \quad . \quad . \quad . \quad (3)$$

which is the total value of the loan. Now

$$\phi = \frac{s_p^i}{p} - 1 = \frac{(1+i)^p - 1}{pi} - 1 = \frac{(1+i)^p - (1+pi)}{pi}$$

$$\text{and} \quad 1 + \phi = \frac{(1+i)^p - 1}{pi}$$

Hence (3) becomes

$$F + \frac{(1+i)^p - (1+pi)}{(1+i)^p - (1+pi)} (j - F) \quad . \quad . \quad . \quad (4)$$

which is the solution of the problem. F is equal to

$$\left(j + \frac{z}{p}\right) a_n^i = \frac{a_n^i}{p a_{n/p}^{ij}}$$

In the most common case, the interest is payable half-yearly and the sinking fund applicable yearly, so that $p=2$. In that case the formula becomes

$$\frac{a_n^i}{2a_{\frac{n}{2}}^{\frac{2i}{2}}} + \frac{i^2}{(1+i)^2 - 1+2j} \left(j - \frac{a_n^i}{2a_{\frac{n}{2}}^{\frac{2j}{2}}} \right) \quad . \quad . \quad . \quad (5)$$

The following is perhaps a slightly simpler form of the demonstration, and it leads directly to an alternative but equivalent result, which might be obtained by transformation of (4). It has been shown that the capital repayments are $z, z(1+pi), z(1+pi)^2 \dots$ at the end of the 1st, 2nd, 3rd . . . period. If interest were payable at rate i instead of rate j on

the same outstanding balances, we should have unity for the present value of the loan, *i.e.*,

$$K + \frac{i}{j}I = 1 \left(\text{whence } K + I = 1 + \frac{j-i}{j}I \right) \quad . \quad . \quad (6)$$

If, however, interest were payable at the end of each *period* at rate pj , we should have the ordinary case of interest and sinking-fund payable concurrently, and the value of the loan would be the present value of an annuity of $z + pj$ payable at

the end of each period, *i.e.* $(z + pj) \frac{s_{\overline{n}|i}}{s_{\overline{n}|p}} = F \frac{p}{s_{\overline{n}|p}}$. But in this case the present value of the interest, I , would be reduced in the proportion of $s_{\overline{n}|p}$ to p : thus we should have, writing s for $s_{\overline{n}|p}$,

$$K + \frac{p}{s}I = F \frac{p'}{s} \quad . \quad . \quad . \quad . \quad (7)$$

Solving (6) and (7) we shall find

$$I = \frac{\frac{s}{j} - F}{\frac{i}{j} \cdot \frac{s}{p} - 1}$$

Hence, the value of the loan $= 1 + \frac{j-i}{j}I$

$$= 1 + (pj - pi) \frac{\frac{s_{\overline{n}|i}}{s_{\overline{n}|p}} - F}{(1+i)^n - (1+pj)} \quad . \quad . \quad (8)$$

The above are the complete theoretical solutions, which are very easily evaluated. It may, however, be pointed out that, except in extreme cases, a close approximation is $\frac{a_{\overline{n}|i}}{a_{\overline{n}|p}}$, which is the value of the loan when the equivalent sinking fund is payable each *interval* over the same number of periods.

As an example, take $n=72$, $i=.0275$, $j=.025$, $p=2$. Formula (5) gives

$$\begin{aligned} & \frac{31.2069}{33.0937} + \frac{.0275^2}{1.05576 - 1.05} \left(\frac{.025}{.0275} - \frac{31.2069}{33.0937} \right) \\ &= .94299 + \frac{.0275^2}{.00576} (.90909 - .94299) = .93854. \end{aligned}$$

Formula (8) gives

$$1 - \cdot 005 \frac{1 \cdot 01375 - \cdot 94299}{\cdot 0057562}$$

$$= 1 - \cdot 005 \times \cdot 07076 \div \cdot 0057562 = 1 - \cdot 06146 = \cdot 93854.$$

The approximate value is $\frac{31 \cdot 2069}{33 \cdot 2401} = \cdot 93883$

showing an error of $\cdot 00029$, or $7d.$ per £100 bond.

The above and some further examples are given in the following Table. It will be seen that the error involved in the approximate formula $\frac{a_{\frac{n}{j}}^i}{a_{\frac{n}{j}}}$ increases as n decreases: also that for any given values of n and j the error is nearly proportional to $(j-i)$.

n	$j = \cdot 025, i = \cdot 0275, p = 2$			$j = \cdot 0225, i = \cdot 0275, p = 2$		
	True Value	Approx. Value	Error	True Value	Approx. Value	Error
96	92·848	92·863	·015	85·881	85·916	·035
72	93·854	93·883	·029	87·867	87·932	·065
48	95·273	95·325	·052	90·653	90·762	·109
24	97·210	97·293	·083	94·458	94·626	·168
12	98·402	98·503	·101	96·816	97·018	·202

If the loan is valued after the redemption has begun, say after t cycles or tp intervals, the value of z will not be the original value, but we must take

$z' = (\text{next repayment of principal}) \div (\text{total loan outstanding}).$

This being done, the same demonstration and formula will apply: and using $\frac{a_{n-tp}^i}{pa_{n/p-t}^{ij}}$ for F , the value of z' does not appear and need not be considered, except so far as it is involved in $n-tp$, the remaining term of the loan. It may easily be shown that $z' = 1 \div s_{n/p-t}^{ij}$.

II.

Solution of a Problem in Reversions.

A life (x) is entitled to the life interest in a fund X.

A life (y) is entitled to the life interest in a fund Y.

Subject to these respective life interests, A is absolutely entitled to a sum N payable out of the funds; and B is entitled to the balance of the two funds after satisfying A's claims. Find the present value of A's and B's reversionary interest.

I. Suppose both X and Y not less than N.

Present value of A's interest $= N A_{xy}$

Present value of B's interest

$$= (X - N) A_x + (Y - N) A_y + N A_{x\bar{y}}.$$

II. Suppose one of the two funds, say X, to be not less than N, and the other Y, to be not greater than N.

Present value of A's interest

$$= (N - Y) A_x + Y A_{xy}$$

Present value of B's interest

$$= (X - N) A_x + Y A_{x\bar{y}}.$$

III. Suppose both X and Y to be not greater than N.

Present value of A's interest

$$= (N - Y) A_x + (N - X) A_y + (X + Y - N) A_{xy}.$$

Present value of B's interest

$$= (X + Y - N) A_{x\bar{y}}.$$

The terms are all positive, and each represents an absolute reversion payable on the failure of a definite status, so that the present value of the reversionary interest is in each case

either a single reversionary interest or the sum of a number of partial reversionary interests. Moreover—and this is the important point—these partial reversionary interests in each case accurately represent the actual facts as to the times at which the amounts will fall in, whether (x) or (y) die first.

G. J. L.

REVIEW.

The Insurance Guide and Handbook. Fifth Edition. Edited by
HENRY WALSHINGHAM ANDRAS, F.I.A.

(C. & E. Layton, London.)

THE first and second editions of this work, edited by the late Mr. Cornelius Walford, appeared in 1857 and 1867 respectively, and treated of Life Assurance only. The third and fourth editions (1900 and 1901), were edited by Mr. A. W. Tarn, who introduced several improvements, notably chapters dealing with other branches of Insurance. The fifth edition, which has now appeared, is on a much more ambitious scale. The Editor, Mr. H. W. Andras, has invited "Actuaries and Insurance Officials to contribute chapters upon subjects with which they are pre-eminently conversant", and the work now extends to two main volumes and a supplement.

Vol. 1 is devoted to Life Assurance, Vol. 2 to Fire, Accident, Employers' Liability, Motor Car, Burglary, and Fidelity Insurance, and a still wider range of subjects is foreshadowed in the next edition. The smaller supplementary volume contains a historical review of Life Assurance, reprinted from the fourth edition, with additional chapters by the Editor, bringing the account up to date.

The previous editions were designed more particularly to meet the needs of insurance agents, and this object has been kept in view, but the work should prove at least equally useful to insurance officials, and as a textbook for the examinations of the Chartered Insurance Institute. A very valuable feature from the point of view of actuaries, actuarial students, and all who require a more detailed and technical knowledge, will be found in the references, at the end of each chapter, to the best sources of further information on the subject. One would like to suggest that, in any future edition, the lists of authorities should be made more complete by the inclusion, in all cases, of the references given in the text.

The reader who overcomes a not unnatural hesitation at the prospect of some 600 pages of closely printed matter will find his interest stimulated by the attractive presentation of the subject,

and his industry rewarded by a clear and accurate insight into the practical working of insurance business which he will hardly need to supplement for purposes of general information. He must indeed be very expert already, or very dull, who fails to acquire a great deal of useful knowledge, in the course [of a careful perusal.

The Editor opens Vol. 1 (Life Assurance), with a historical review of the principal events in the insurance world since the beginning of the present century, and contributes articles on the Calculation of Office Premiums, and on Reserves and Surrender-Values. He shows that fair average rates of with profit premiums can be obtained by the use of Dr. Sprague's well known formula, substituting $O^{(M)} 3\frac{1}{2}$ per-cent for $H^{(M)} 4$ per-cent and doubling the allowance for initial expenditure, but that a similar substitution in the formula for without profit premiums produces rates considerably higher than those now in common use.

Mr. James Bacon deals with industrial premiums, and Mr. C. W. Kenchington gives a concise, but clear, account of the historical mortality tables. The chapter on under-average lives is written by Mr. F. L. Collins, who has made good use of the exceptional opportunities at his disposal for studying the subject. Mr. Collins emphasizes the importance of considering the incidence of the extra mortality, and incidentally refers to an experience of rated up lives, apparently comparatively recent and unpublished. The subject is of such interest and importance, that it is hoped that opportunity will be found of submitting some particulars of this experience in the *Journal*. Mr. H. E. W. Lutt contributes a valuable paper on the practice of offices in connection with extra premiums for occupation, and much information on extra risks on account of climate in various parts of the world. We read, with conflicting emotions, that "In North-East and North-West Rhodesia are many very unhealthy places, *peopled chiefly by government officials and missionaries.*"

The non-technical reader of Mr. C. H. Maltby's contribution on the Distribution of Surplus, will be much assisted by the clear and logical arrangement, but he will need to bear in mind that the word "profit" in connection with Life Assurance is capable of many shades of meaning. Under the heading "Sources of Profit", the author is considering the apparent sources of *surplus* on a given valuation basis. The actual *profit* arises from a working experience more favourable than that assumed in the calculation of the premiums. The relative importance of the various "apparent" sources of surplus depends very much upon the valuation basis adopted. Mr. Maltby also discusses the question of Income Tax, and shows the difficulties under which Insurance Companies labour owing to the uncertainty of the law.

Mr. P. C. Crump writes on the Investments of Insurance Companies, and Mr. J. R. Hart on Mortgages. Mr. Arthur Taylor and Mr. H. E. Melville discuss the legal questions which arise in daily practice. Mr. A. G. Cudlipp describes and explains the Accounts which have to be returned to the Board of Trade, and contributes a chapter on Life Assurance Bookkeeping, which will be read with interest by those who have had practical experience, but for the student the old saying will always hold good, that he who would learn bookkeeping must keep books, rather than read them.

The subject matter of Vol. 2 does not come so directly within the province of most readers of the *Journal*, and it will be sufficient to state that Mr. H. S. Bell writes on Fire Insurance, Mr. Frederick Thoresby on Accident, Employers' Liability and Motor Car Insurance, Mr. F. D. Macmillan on Burglary Insurance, and Mr. A. J. Small on Fidelity Guarantees.

S. G. D.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS, 1912.

BOARD OF EXAMINERS.

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EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART I).

First Paper.

$$\left. \begin{array}{l} 1. \text{ If } y^2 + yz + z^2 = a^2 \\ \quad z^2 + zx + x^2 = b^2 \\ \quad x^2 + xy + y^2 = c^2 \end{array} \right\} \text{ then prove that } a^2x^2 = b^2y^2 = c^2z^2.$$

and $yz + zx + xy = 0$

2. Solve the equations

$$(i) \quad \left(\frac{x+a}{x-a}\right)^{\frac{1}{3}} + \left(\frac{x-a}{x+a}\right)^{\frac{1}{3}} = 3$$

$$(ii) \quad x^2 + (y-z)^2 = a^2$$

$$y^2 + (z-x)^2 = b^2$$

$$z^2 + (x-y)^2 = c^2.$$

3. In the British Mercantile Marine the number of seamen employed in 1909 was greater than the average number for the ten years 1897-1906 by one-nineteenth. British seamen alone had increased by one-thirteenth, and the number of Lascars and other foreigners taken together was unchanged. The number of Lascars alone had increased by 6,000, and was equal in 1909 to one-fourth of the number of British seamen. In the period 1897-1906 the average number of Lascars had been equal to the average number of other foreigners. Required the number of British seamen, of Lascars, and of other foreigners employed in 1909.

4. Show that if x and m are positive and less than unity the error involved in neglecting terms after the r th in the expansion of $(1+x)^m$ is less than x times the r th term.

Find by the binomial theorem the square root of 110 correct to 4 places of decimals.

$$5. \text{ Prove that if } x < 1, \quad \log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots$$

Calculate the value of $\log_e 4$ to four places of decimals.

6. 2^n players of equal skill enter for a tournament; they are drawn in pairs, and the winners of each round are drawn again for the next. Find the probability that two given competitors will play against each other in the course of the tournament. If $n=5$, show that the probability that a given player will either win or be beaten by the actual winner is $\frac{3}{16}$.

7. A policy register of 384 pages contains particulars of 1,920 policies, an equal number being entered on each page. 640 of the whole number are policies for £500, and 480 of the whole number have terminated. On how many pages of the register would you expect to find (a) at least one policy for £500 in force; (b) exactly one policy for £500 in force; (c) more than one policy for £500 in force.

8. A penny is tossed 6 times. Find the chance that neither heads nor tails have occurred three times in succession.

Second Paper.

1. Sum to infinity

$$(i) \quad 2 + \frac{4}{5} + \frac{4.11}{5.15} + \frac{4.11.14}{5.15.20} + \frac{4.11.14.17}{5.15.20.25} \dots$$

$$(ii) \quad \text{the series of which the } n\text{th term is } \frac{1}{n(36n^2 - 1)}.$$

2. If $\frac{f(x)}{\phi(x)}$ has an indeterminate value of the form $\frac{0}{0}$ for a particular value of x how would you determine the limiting value at that point?

Evaluate

$$\text{Lt}_{x=0} \left[\frac{\log(1+x)}{e^x - 1} \right], \quad \text{Lt}_{x=0} \left[\frac{g^{e^x} - g}{g^x - 1} \right]$$

3. The equation $512x^4 - 832x^3 - 360x^2 - 47x - 2 = 0$ has three equal roots; solve the equation completely.

4. Prove that $\frac{x}{e^x - 1} = 1 - \frac{x}{2} + \frac{x^2}{12} - \frac{x^4}{720} \dots$, and show that no odd power of x beyond the first can occur.

5. Prove geometrically or otherwise

$$\int_a^h u_t \left(\int_t^h v_t dt \right) dt = \int_a^h v_t \left(\int_a^t u_t dt \right) dt.$$

6. Show how to find the average value and the maximum value of a function between given limits.

Find the average value and the maximum value of $x(x-1)(x-2)$ between the limits 0 and 1.

7. Prove that the differential coefficient of $f(n)$ with respect to n is approximately equal to

$$\frac{2}{3} [f(n+1) - f(n-1)] - \frac{1}{12} [f(n+2) - f(n-2)].$$

8. Prove the following formulas for approximate interpolation

$$u_1 = u_3 - \cdot 3(u_3 - u_{-3}) + \cdot 2(u_{-3} - u_{-5})$$

$$u_0 = \frac{u_1 + u_{-1}}{2} - \frac{1}{8} \left\{ \frac{u_3 - u_1}{2} - \frac{u_{-1} - u_{-3}}{2} \right\}$$

and apply them to find the logs of 45, 46, 47, 48, 49, being given

$$\log 42 = 1.62325$$

$$\log 50 = 1.69897$$

$$\log 44 = 1.64345$$

$$\log 52 = 1.71600$$

Third Paper.

1. From the banker's point of view which investment is the better, and why?

(a) Three bills for 200, 300 and 100, due three, four and twelve months hence respectively, or

(b) One bill for 600, due five months hence,

commercial discount at the same rate being charged in each case.

*2. Find the present value at $2\frac{1}{2}$ per-cent per annum, convertible once every two years, of an annuity of 1 per annum for 24 years (1) if the annuity is payable biennially, and (2) if payable annually.

*3. Find the value of an annuity-certain for 20 years, the t th payment of which is $(1.01)^{t-1}$, the value to be calculated so as to yield a purchaser 5 per-cent on his whole investment throughout the term and to return him his capital at $2\frac{1}{2}$ per-cent.

*4. The whole of an issue of 4,000 bonds of £1 each, bearing interest at 5 per-cent per annum, and redeemable at par by equal annual drawings extending over 40 years, (*i.e.*, 100 bonds are drawn each year) are purchased by an insurance company at a premium of 4 - per bond. Every year on receipt of the dividend on the bonds the company takes credit in its accounts for interest at the rate of 4 per-cent per annum on the book values of the investment and employs the balance of the dividend to write down the book value of the investment.

At the end of three years the insurance company sells the remaining bonds at a price to pay a purchaser 4 per-cent per annum.

What book profit or loss does the company make?

5. An assurance fund on the 31 December 1906 was £5,000,000. During the five years ending 31 December 1911 the amount received from interest was £1,184,210 gross (income tax thereon being £59,210) and from premiums and other sources £2,500,000. The total outgo was £2,125,000. What was the gross rate of interest earned by the funds?

6. What is meant by the repayment of a loan by a cumulative sinking fund? A loan in bonds of £1 each, interest on which is payable half-yearly at the rate of 2g per-cent per annum convertible half-yearly, is being repaid by means of a cumulative sinking fund, the drawings taking place half-yearly on the same dates as the interest is payable. Show that the value of a bond just after a

drawing and n years before the expiration of the loan to yield a purchaser $2i$ per-cent per annum convertible half-yearly $= \frac{a_{2n}^{vi}}{a_{2n}^{vi}}$.

How would the formula be modified if interest were payable half-yearly, but drawings only took place annually?

*7. In constructing a table of any function $f(x)$, what are the advantages and disadvantages of expressing the working formula in the form

$$f(x+1) = f(x) + r_x \times C$$

where C is a constant and r_x a variable?

Assuming ordinary interest tables are available, explain how you would proceed to construct and check a schedule of capital and interest payments in respect of a loan of £40,000 at 4 per-cent convertible half-yearly and repayable by equal half-yearly instalments of capital and interest in a period of 30 years. Commence the schedule for the first 3 years.

How many figures in the interest tables would you require to obtain results correct in the third decimal place?

*A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART II).

First Paper.

1. In a population which had been stationary up to 31 December 1907 the rate of mortality at age 0 decreased in each of the years 1908 to 1910 at the uniform rate of t per-cent per annum. How would you estimate the effect of this change on the population at the end of 1910?

2. Find the probabilities:

- (a) that in the t th year from the present time the last survivor of three lives (x) (y) and (z) will die.
- (b) that (x) will die at least t years after (y) and at least t years before (z).

3. A company issues 100 policies through each of 60 branches and experiences 60 deaths within 2 years of issue. The circum-

stances being such that the risk of claims should be expected to be the same at all branches, estimate how many branches have probably experienced 0, 1, 2, 3, 4 deaths respectively.

Given $(1 - .01)^{160} = .37$ approximately.

4. Out of six persons at present living of the same age, two have to be selected at the end of four years to undertake certain duties. Find the probability that the number of ways in which the selection can then be made will be at least ten.

5. If $\mu_x = \frac{1}{(a_0 + a_1, c)(h_0 + h_1, c)}$ what is the form of l_x ?

6. Prove that the chance of (x) dying before (y) within one year is approximately equal to $\frac{1}{2}\mu_x + \frac{1}{2}p_{xy}\mu_{x+1}$. Give a similar form for Q_{xy}^1 pointing out any modifications that should be made if a numerical value is required.

7. A staff is recruited by the admission of 5,000 entrants every 1 January, 1,000 entering at each of the exact ages 15–19. Out of 1,000 entering at any one age 30 per annum leave from all causes, including death, before age 30, and then 15 per annum up to age 50, when all then remaining retire on pension.

(Give (1) the ultimate constitution of the staff in quinquennial age groups on 1 January of any year ;

(2) the average length of service of (a) all who enter ;
(b) all who retire on pension ; (c) all who withdraw otherwise.

Second Paper.

1. A temporary assurance for n years at an annual premium of P_{xu}^1 is effected by each of l_x persons aged x . Prove algebraically that the accumulated claims equal the accumulation of the premiums paid (a) by those who die and (b) by those who survive n years.

2. Find the value of an annuity payable for n years certain and so long thereafter as (y) may live, the first payment to be made on the death of (x) or at the end of m years if (x) be then living, the annuity to be paid yearly and to be non-apportionable to the death of (y) .

3. The annual premium for a whole life policy is .04 for five years and .02 thereafter, or .0475 for five years and .0175 thereafter. Find the uniform annual whole-life premium. Also find from the

foregoing the annual premium for an endowment assurance payable at the end of five years or previous death in terms of ${}_5P_x$ and d .

4. The value at 3 per-cent interest of an annuity, payable half-yearly and non-apportionable is 12.481. Find the value of a corresponding annuity payable quarterly with proportion to date of death.

5. A child's assurance is effected by annual premiums throughout life at age x , the sum assured being payable at death after age 21, and the premiums only being returnable at death before that age.

Find the policy values—

(a) At the end of 10 years.

(b) „ „ „ „ 25 years.

6. Find the annual premium for an educational annuity, the first yearly payment under which is to be made on (x) attaining age $x+n$ and which is to continue for t years certain, premiums to be payable during the joint lives of (x) and (y) during n years, and to be returnable should (x) die before (y) within that term.

7. Assuming that the force of mortality for male lives is represented by the formula $A + Bc^x$, and for female lives by the formula $A' + B'c^x$, show that the principle of uniform seniority is applicable to joint annuities on two lives, one being male and one female, and explain what tables are required to deal with all cases of annuities on two joint lives where each life may be either male or female.

*8. Calculate an approximate value for the single premium at H^M 3 per-cent for an increasing term survivorship assurance payable only on the death of a life aged 30 provided he die within 10 years before a life aged 60 and after a life aged 40. The sum assured to be £100 for the first year and to increase by 5 per-cent per annum.

* A Short Collection of Actuarial Tables will be supplied for use in answering this question.

Third Paper.

1. Prove the following formula for $a_x^{(m)}$ —

$$\frac{1}{j_{(m)}} - \left(\frac{1}{m} + \frac{1}{j_{(m)}} \right) A_x^{(m)}.$$

Is this approximate or exact?

$$\begin{aligned}
 2. \text{ Given} \quad a_x &= 18.370 \\
 a_{xx} &= 14.494 \\
 a_{xxx} &= 10.575 \\
 a_{xxxx} &= 9.544
 \end{aligned}$$

$$\text{Find} \quad a_{\overline{xxxxx}}^{[3]} \text{ approximately.}$$

3. For a given distribution of deaths over a term of n years it is found that the net annual premium for a double endowment assurance is not affected if the deaths at every age are increased or decreased in the same ratio. Prove that the net annual premium must be the same as that for a capital redemption policy of the same term for the amount of the double endowment receivable at the end of the term.

4. Having given the rate of interest and the values of P_x at all ages for three different tables of mortality, show clearly how you would construct tables enabling you to determine for all ages and durations the relative magnitude of the policy values.

5. A life aged x wishes to effect an assurance for £100 payable at age $(x+n)$ or previous death, with the option at age $(x+n)$ of taking, without further medical examination, a fully-paid whole-life assurance instead of the cash payment of £100. How would you find the amount of the paid-up policy which can be given and the annual premium payable for the benefit? Ignore the question of loading.

6. Give the formula you would use, and the method of calculation you would employ, for the construction of a table of annual premiums, payable for 20 years only, for a policy securing £1,000 at death, with the provision that if death occur before the age of 65 payment of the sum assured shall be deferred until the date when that age would have been attained, and that in such event an annuity of £100 a year shall be paid yearly in advance from death until payment of the sum assured.

7. Show how to form and check a table of the net annual premiums for endowment assurances at age 60, with an initial sum assured of £1,000, increasing by £15 at the end of each year.

8. How would you group joint life assurances for valuation?

Draw up a class-book showing the columns you would use.

Fourth Paper.

1. How would you treat the following items in preparing the annual accounts of an assurance company?

- (1) Purchase of a lease for 21 years in consideration of a payment down of £20,000 and a payment of £500 at the end of each year for 21 years.
- (2) Receipt of premium in respect of a policy effected some years ago at an annual premium of £20 5s., since reduced by the application of bonus additions to £10 2s. 6d.
- (3) Refund of income tax from the Inland Revenue Authorities in respect of annuities paid to persons with incomes of less than £160 per annum.

2. The following items appeared in the balance sheet of an assurance company on the 31 December 1910:

	£	£
Interest and dividends outstanding	5,300	
Less Income Tax	300	
	---	5,000
Interest accrued but not due	71,000	
Less Income Tax	4,100	
	<hr/>	66,900

The company purchased during the year 1911 Stock Exchange Securities for £500,000, the accrued interest included in the purchase price being £2,124 (gross), tax thereon £124. It sold during the same period securities for £250,000, the accrued interest included in the sale price being £955 (gross), tax thereon £55.

The amount of interest and dividends that fell due to be paid during the year 1911 was £209,600 (gross), tax thereon £12,200.

The following figures were extracted from the company's cash book for the year ending 31 December 1911: Stock Exchange Sales £250,000; interest and dividends (gross) £207,500; income tax on interest and dividends £12,100; Stock Exchange purchases £500,000.

The interest accrued but not due on the 31 December 1911 was estimated to be £74,300 (gross), tax thereon £4,300.

Make the necessary journal entries and draw up the interest and dividends and income tax accounts as they would appear in the company's general ledger.

3. A Company decides to sell Guaranteed $2\frac{3}{4}\%$ Stock (Irish Land Stock) and to invest in

(1) A new issue of Grand Trunk Pacific 4% Sterling Bonds, application list for which has just closed.

(2) Lake Shore and Michigan Southern $3\frac{1}{2}\%$ Gold Bonds.

Explain the details of each transaction.

4. Into what classes are the following securities divided in the Stock Exchange Official List?

(a) British Railway Securities.

(b) American Railway Securities.

Explain briefly the characteristics of each class.

5. Explain the various methods adopted in practice for guaranteeing that an issue of stock, shares or bonds should be fully subscribed.

6. The following particulars were extracted from the *Stock Exchange Official List*, 28 November 1911:

Interest due	When x'd. or x'n.	Per ct.	NAME	3.30 quotations, Nov. 28	Business done
30 June 31 Dec.	...	3(a)	Indian Rupee Paper, 3% (1896-97) 1916.	52-54	...
1 Jan. 1 July.	1 July	$4\frac{1}{2}$	National Rys. of Mexico, Prior Lien $4\frac{1}{2}\%$ 50 yr. Sk. Fd. Red. Gld. Bonds (\$1,000 = £205 : 15 : 2).	94½-95½%	95½
1 Jan. 1 July.	15 June	4	Kansas City Terminal Railway Co., 1st Mort. 4% Gold Bonds.	103-105	...

Explain briefly the meaning of each of the headings and what you understand from the various quotations.

7. Give a brief account of Goschen's National Debt Conversion Scheme of 1888, and explain the reasons for its success.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART III).*First Paper.*

1. Given age last birthday at entry, exact date of entry, calendar years of death and withdrawal and number existing on 31 December 1911, show how to obtain select and aggregate mortality tables. Discuss the advisability, in the particular case, of giving the rates of mortality for exact ages or for ages last birthday.

2. For a number of years an office has been granting endowment assurances with the additional benefit that a return of premium shall be made for periods of total incapacity from sickness or accident which last for more than one month. It is desired to examine the sickness and accident experience. Draft outline instructions for the staff doing the work.

3. Assuming that (*a*) the experience mentioned in the last question has been obtained from 1,000 endowment assurances, maturing not later than 60, for ages at entry 20 to 35, (*b*) the business has been running for 20 years, explain whether you would graduate the experience. Give the method you would use and show how you would construct a table of office premiums.

4. Given populations on 1 Jan. 1901, 1 Jan. 1906, 1 Jan. 1911, and the deaths for the 10 years 1901-1910 inclusive, how would you find the rate of mortality at each age? Explain how the method you suggest might be adapted to obtain aggregate rates of mortality from the books of a large life office which groups all its single life policies according to office year of birth.

5. Explain where Milne's graphic method of subdividing population and deaths, which are given for groups of ages, into corresponding single age groups differs from graduation. What are the objections, if any, to graduating populations (and exposed to risk) and deaths independently instead of graduating q_x or m_x ? Do these objections apply to Milne's method?

6. What are the tests of a good method of graduation?

Do you consider that a comparison of "expected deaths" is entirely satisfactory as a test of closeness of agreement between graduated and ungraduated rates of mortality, or would you apply some further test (*e.g.*, an "expected exposed to risk" calculated from the deaths)?

7. Given the following particulars how would you find graduated values for durations 0-26 weeks?

Number of weeks } under 2 3 4 5-13 13-26
sickness since accident)

Number of cases ... 472 1,970 1,506 324 160

(Cross ruled paper will be provided.)

8. Give any summation method of graduation and apply it to find graduated values for l_{65} , l_{66} , l_{67} , l_{68} , l_{69} , and l_{70} , from the following data:

x	l_x	x	l_x	x	l_x
55	6,776	64	5,183	73	3,064
56	6,622	65	4,986	74	2,818
57	6,472	66	4,743	75	2,571
58	6,302	67	4,522	76	2,311
59	6,141	68	4,318	77	2,054
60	5,957	69	4,060	78	1,814
61	5,780	70	3,814	79	1,590
62	5,590	71	3,572	80	1,359
63	5,383	72	3,335		

(Cross ruled paper will be provided.)

Second Paper.

1. Criticize the various methods which can be adopted for estimating the number of persons over a certain age who may be alive in a community at a date $5\frac{1}{2}$ years later than the date to which the last available census returns refer.

2. How were "duplicates" treated and dealt with in the several combined investigations which have been made into the mortality experience of assured lives and annuitants in British Life Offices?

3. An office charges for foreign residence extras which run for a term of years and then cease altogether, the policy becoming free from all restrictions. How should such cases be treated in a valuation—

(a) while the extras are current,

(b) after they have ceased?

Give any simple practical method that would in your view be sufficiently accurate.

4. The with and without-profit rates of an office earning £3. 18s. 0d. per-cent interest (net) on its funds are approximately $O^{(M)}$ $3\frac{1}{2}$ per-cent with 25 per-cent loading and 10 per-cent loading respectively; 33 per-cent of the business is without profits and the office has paid a 35s. simple reversionary bonus for 20 years and has valued by H^M and $H^{M^{(G)}}$ at 3 per-cent and carried forward 20 per-cent of its surplus.

It has been suggested that a valuation should be made either by $O^{M^{(G)}}$ at $2\frac{1}{2}$ per-cent, O^M at 3 per-cent, or $O^{M^{(G)}}$ at $2\frac{3}{4}$ per-cent with O^M net premiums. Which would you choose, and why?

5. How would you value children's deferred assurances before and after the assurance begins?

6. In making the first valuation of an ordinary life assurance company which has just completed its fifth year of existence, what matters would you specially investigate and what would be your main considerations in fixing the valuation basis?

Would you in any circumstances recommend the declaration of a bonus if a true net premium valuation did not reveal a surplus?

*7. On 31 December 1910 a life assurance society consisted of 1,000 whole-life policies for £100 each, issued on 1 January 1901 at an annual premium of £2 per-cent, the lives assured being aged 30 at entry. A net premium valuation on 31 December 1910 by the H^M 3 per-cent Table (Text-Book graduation) with the usual adjustments shows the estimated reserve was just equal to the funds—£13,142. Claims are paid two months after death. During 1911 £458 was received by way of interest; expenses amounted to 10 per-cent of the premium income and 60 per-cent of the expected rate of mortality was experienced. Four policies were surrendered on 1 July 1911, the surrender-values being 50 per-cent of the reserve values. Ascertain and analyse the profit made in 1911.

* A Short Collection of Actuarial Tables will be supplied for use in answering this question.

Third Paper.

1. An office valuing by the O^M and $O^{M^{(G)}}$ Tables at a low rate of interest takes credit for future office premiums, less a percentage. What are the advantages and disadvantages of this course and how does it, in your opinion, compare with other means of attaining the purpose aimed at?

2. An office adopts a table of long term policies carrying the right of conversion to a whole-life table at any time, except during the last n years of the term, on paying the normal premium for the age at the date of conversion. What reserve should be kept against the policy after t years,

(a) If the policy has not been converted :

(b) If the policy has just been converted ?

3. To what extent does the original American contribution method of allocation of bonuses differ fundamentally from the contribution methods employed in the United Kingdom? Draw up for whole-life policies a schedule, suitable to one of these methods, for finding the reversionary bonus to be allotted to each policy.

4. A proprietary office which has just made its quinquennial valuation finds that, after writing down securities by £100,000 and strengthening valuation reserves by £50,000, it has no divisible surplus.

In the circumstances the following alternative suggestions have been made :

(a) To declare an interim bonus on policies becoming claims during the next five years ; or

(b) To have annual valuations and distributions in future.

Discuss the relative advantages and disadvantages of these plans.

5. A company which declares its bonus on the uniform reversionary bonus system proposes to start a scheme under which bonuses will be deferred until the premiums paid, accumulated at 4 per-cent compound interest, amount to the sum assured. Explain the basis you would adopt in forming the new table of premiums and what special precautions you would take in valuing the liabilities under the scheme.

6.	Gross rate of interest last quinquennium	£4·0
	Gross rate of interest this quinquennium	4·2
	Premium income with profits	200,000
	Premium income without profits	50,000
	Expense ratio	15·5
	Funds	3,000,000
	Annual New business	800,000
	Last two bonuses	25s. compound
	Carry forward, last valuation	£60,000
	Depreciation, this valuation	150,000
	Last valuation	H^M and $H^{M(5)} 3\%$.

On this basis the surplus would be only just sufficient to provide a compound reversionary bonus at the rate of 25s. per-cent per annum.

The office has no reserves for uneven distribution of premium income and its reserves for loading anticipated, &c., are low. Would you advise an O^M valuation and if so, at what rate of interest? What bonus would you pay on the basis you suggest?

*7. Calculate Office annual premiums for—

- (a) Joint-life 3-year term assurance at ages 30, 40, and 50.
- (b) A 3-year term assurance at age 40 with the option of converting at or before the end of the term to any table at the then rates.

* A Short Collection of Actuarial Tables will be supplied for use in answering this question.

Fourth Paper.

1. Discuss the rates to be charged for and the propriety of granting last survivor deferred annuities.

2. How would you calculate an office premium for a 20-year with-profit endowment assurance at a reduced premium for the first five years?

Discuss the question of allowing the policy to participate in respect of the lower premiums.

3. How would you calculate a scale of premiums for whole-life policies allowing for a rate of lapse?

It is stated that a company, which takes account of discontinuances in calculating its premiums, requires to make a considerably higher reserve than a company charging the ordinary premium. Explain exactly what is meant by the statement.

4. A man aged 25 proposes for a whole-life assurance and is charged the premium for age 32 because his mother and one brother died of tuberculosis. He asks on what terms an endowment assurance will be granted and whether a decreasing debt will be allowed.

Give a concise outline of the points to be considered and the result you would reach.

5. What, if any, classes of risk subject, under a whole-life table, to extra premiums would in your opinion be accepted under a double endowment table at ordinary rates; and what other classes at a lower extra?

6. Mention the principal published experiences of mortality in India and state briefly their most important characteristics.

7. What are the usual conditions as to foreign residence and travel in policies which are not world-wide?

State, generally, the rates of extra premium which are charged for civilian residence beyond the usual free limits. Should any modifications be made in the case of short-term policies?

8. What are the scales of compensation in the event of death, and partial and total disablement under the Workmen's Compensation Act, 1906? To what extent can actuarial principles be used in calculating premiums and estimating liability under employer's liability insurance contracts?

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW

(PART IV).

First Paper.

1. Define, and distinguish between, the expressions Condition, Warranty and Representation, as applied to a contract. What is the effect of an innocent misrepresentation by one of the parties to a contract?

2. Describe the method of conveyance of land by means of registered transfer, as provided in the Land Transfer Acts of 1875 and 1897. State the advantages and disadvantages of that method.

3. What are the legal disabilities attaching to the following classes of persons, namely: Felons; Bankrupts; Lunatics; Infants; Aliens?

4. A, a debtor, executes a deed of assignment for the benefit of his creditors in favour of B. The deed, which includes a policy of life assurance on the life of A, is dated 28 October 1910. On 4 January 1911, B applies to the assurance company for the surrender-value of the policy. What precautions should the assurance company adopt before making the payment? State whether a different course of action should be adopted if A had died and the application was for the policy money. What special formalities must be complied with in such a deed in order to render it valid and available?

5. Describe briefly the methods by which a married woman could alienate, *inter vivos*, a policy of life assurance, (a) in 1850; (b) in 1875; (c) in 1900.

6. It is proposed to include in a marriage settlement a policy of life assurance on the life of the settlor. What special provisions should be inserted?

7. Give an account of the provisions of the Assurance Companies Act, 1909, relating to employers' liability insurance.

8. Describe the powers of investment possessed by registered friendly societies, and state what, if any, special privileges attach to such investments.

Second Paper.

1. A large company desires to institute a pension scheme for the benefit of its clerical staff numbering about 2,000. It is suggested that the future entrants shall contribute a percentage of their salary, the company contributing a similar amount. The pension it is hoped to be able to provide is at the rate of $\frac{1}{30}$ th of the average salary for each year of service. You are asked to report upon such a scheme.

State briefly how you would make your necessary investigations, referring particularly to (1) application of the scheme to members of advanced age; (2) suggested payments at death and withdrawal; (3) the nature of the contributions required in respect of existing members.

2. A friendly society, in connection with a large railway company, grants the following benefits:

- (1) Pension of 5 - per week on retirement at age 60 or earlier in the event of permanent incapacity.
- (2) Sickness allowance of 12 - per week for first 26 weeks of sickness, 6 - per week for next 26 weeks, and 4 - per week for remainder of sickness.

Between the ages of 30 and 40, members have the option of increasing their weekly contribution from 1 6 to 2 3 in order that they may be entitled to a pension of 8 - per week, and sickness allowance of 14 -, 7 - and 5 - respectively, in place of the amounts stated above. Ages at entry vary from 18 to 25. Explain how you would deal in your valuation with the question of the option to increase contributions.

3. A widows' fund charges an entrance fee of £5. 5/- and an annual subscription of £5. 5/-. The limiting age for entrants is 25. The benefits comprise an annuity to a widow during widowhood, with continuance to orphan children during their minority. Owing to legacies to the fund the benefits granted are, on the average, 25 per-cent in excess of the value of contributions payable. What points should be considered in the calculation of cash surrender-values to retiring members?

4. Your office is proposing to start what is popularly known as a "non-forfeiture" scheme, and is considering which of the following methods will be most suitable:

- (a) Revival to be allowed in all cases within two years of date of lapse, without medical examination and whether the assured be alive or dead, on payment of a fine of $7\frac{1}{2}$ per-cent on the amount of premiums due to date. Any balance of surrender-value unused, to be paid if applied for within two years.
- (b) The assurance to be continued for a certain number of years as a term policy for the original sum assured, such term depending upon the reserve-value.
- (c) Paid-up policies to be automatically granted in all cases.

Give, concisely, the arguments for and against each scheme, stating which you would advocate and any modifications which, in your opinion, would be advisable.

5. Draft suitable policy-endorsements for the following:

- (a) Reduction of sum assured in whole-life assurance.
- (b) Alteration from endowment assurance to whole-life assurance.
- (c) Licence to reside abroad.

6. Your company has under consideration a scheme for granting assurances to abstainers at reduced rates. Draft a report to the directors explaining the various methods in use and the arguments for and against the same.

Third Paper.

1. Explain briefly the meaning of "high" and "low" exchange. What are the main causes which affect the transmission of bullion from one country to another?

2. The following is an extract from the New York Exchange list of prices for September, 1908 :

United States Government Securities.			
U.S. 2% Cons. reg. Apl. 1930	...	103 $\frac{3}{4}$	104 $\frac{1}{2}$
U.S. 3% reg. Aug. 1918	101 -102
Pan. Can. 10-30 year 2% Aug. 1936	102 $\frac{5}{8}$	103	

How do you account for these quotations ?

3. Give a list of the more important rates of Exchange between London and other centres which generally appear in London daily papers. Besides giving average quotations, indicate the limits within which rates vary and state the coinage in which each rate is expressed.

4. What do you understand by : Service of a loan ; cumulative sinking fund ; New York terms ; contributories ; promissory note ; legal minimum ; bad delivery ?

5. An assurance company having a large proportion of its funds invested in British securities, has determined to broaden its field of investment. Mention three classes which you consider especially suitable at the present time, giving your reasons.

6. Discuss the following securities, pointing out the salient features and indicating any peculiarities each one may have :

Indian Rupee Paper.

East Indian Railway Deferred Annuity Capital.

Turkish 4% Guaranteed Loan.

London County 3 $\frac{1}{2}$ Consolidated Stock.

Bills of Exchange.

Mortgage Bank Bonds.

7. What are the main points to be noticed in connection with the purchase and sale of American railroad stocks and shares ?

Fourth Paper.

*1. A man, aged 30, is entitled, contingently upon his surviving his mother, aged 56, to one-fourth of the undermentioned fund. State what you consider is the utmost it would be advisable to advance upon the security and what policy should be effected and included in the mortgage. To what extent could the advance be

increased if the mother would charge her life interest as security for the interest and premiums !

£10,000 Consols.

£2,000 Bank of England Stock.

*2. A, who died on 13 November last, was entitled to the absolute reversion, expectant on the death of a lady born in February 1843, to a sum of £20,000, charged upon a fund (called the £60,000 fund) at present invested in the following stocks. The fund is charged with the payment of £40,000 in priority to the £20,000. Value A's interest for the purposes of assessment of Estate Duty.

Fund					Official price quoted by Broker
£16,000 Consols	78 $\frac{3}{8}$ - 78 $\frac{5}{8}$
£19,000 New South Wales 4	Inscribed				
Stock, 1933	105 $\frac{1}{2}$ - 106 $\frac{1}{2}$
£25,000 London & N.W.Ry. 3 %	Debenture				
Stock	82 $\frac{1}{2}$ - 83 $\frac{1}{2}$

3. A, aged x years, is entitled contingently on his surviving B, aged y years, to the reversion to a large fund expectant on the decease of the survivor of B and C, aged z years. A wishes to sell a charge of £5,000 net secured upon his contingent reversionary interest. Set out an expression for the sum that can be given for the purchase of this charge, and state what policy or policies should be set up.

4. Give and explain formulæ for the method suggested by Paterson for advances on reversions in consideration of increasing reversionary charges,

(1) if the reversion be absolute :

(2) if the reversion be contingent.

What are the points that have to be considered in applying the formulæ and what are the advantages and disadvantages of the method ?

5. State the general principles to be observed in calculating the values of policies offered for sale at public auction.

How do these principles differ from those followed in calculating surrender-values ?

*6. A loan is required by A, aged 35 next birthday, upon the security of his life interest in the following fund :

£1,000 South Australia 4 % Inscribed Stock, 1924.

£1,000 Canada 3 % Registered Stock, 1938.

£1,000 Bombay, Baroda & Central India Railway 3
Guaranteed Stock.

£1,000 London, Brighton & South Coast Railway 5
Preference Stock.

In addition to the above there is a whole-life with-profit policy for £1,000 offered as security. This policy was taken out 15 years ago (the 16th annual premium having just been paid) at an annual premium of £18, and there is £200 reversionary bonus. Estimate the amount that can be safely advanced and the rate of interest which you consider should be charged.

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions, but if the rate of interest and table of mortality the student would use in practice are not contained in these tables, he should, in giving his results, state what rate and table he would employ if they were available.

PROCEEDINGS OF THE INSTITUTE.—SESSION 1911-1912.

First Ordinary Meeting, 27 November 1911.

The President (Sir GERALD H. RYAN) in the Chair.

A paper entitled "A new method of approximating to the values of Last Survivor Annuities, and to the values of Joint Life Annuities when the advantages of Makeham's Law are not available", was read in abstract by the Author, Mr. G. J. Lidstone.

The following gentlemen took part in the discussion:—Messrs. D. C. Fraser, W. Palin Elderton, R. Todhunter, S. W. Newling, and G. F. Hardy.

Second Ordinary Meeting, 18 December 1911.

The President (Sir GERALD H. RYAN) in the Chair.

Mr. Arthur Cockburn, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "Some recent Statistical Results. A review", was read in abstract by the Author, Mr. W. Palin Elderton.

The following gentlemen took part in the discussion:—Messrs. L. C. Monkhouse, H. J. Rietschel, O. T. Falk, G. Marks, J. Pullar, S. G. Warner; Mr. Major Greenwood and Professor Karl Pearson (visitors); and the President.

Third Ordinary Meeting, 29 January 1912.

The President (Sir GERALD H. RYAN) in the Chair.

A paper entitled "The Investment of Life Assurance Funds", was read by the Author, Mr. G. E. May.

The following gentlemen took part in the discussion:—Messrs. E. W. Townley, A. T. Winter, F. Schooling, O. T. Falk, G. Marks, J. R. Hart, and the President.

Fourth Ordinary Meeting, 26 February 1912.

The President (Sir GERALD H. RYAN) in the Chair.

Mr. George Clark Hutton, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "On the principal provisions of the Law of Bankruptcy in England, with references to some decisions of interest to Life Insurance Companies", was read in abstract by the Author, Mr. N. J. Carter.

The following gentlemen took part in the discussion:—Messrs. R. C. Simmonds, J. R. Hart, S. G. Dunn, A. J. Hicks, W. Penman, Jr., R. R. Tilt, and the President.

Fifth Ordinary Meeting, 25 March 1912.

Mr. GEOFFREY MARKS (Vice-President) in the Chair.

A paper entitled "Notes on the Construction of Mortality Tables", by Messrs. W. Palin Elderton and R. C. Fippard, was read in abstract by Mr. Fippard.

The following gentlemen took part in the discussion:—Messrs. G. Green, C. W. Kenchington, E. Woods, S. G. Dunn, G. King, and the Chairman.

Sixth Ordinary Meeting, 29 April 1912.

The President (Sir GERALD H. RYAN) in the Chair.

Mr. David Spence Fraser, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "On the Superannuation and Pension Funds of certain Metropolitan Borough Councils, their Establishment, Administration, and Actuarial Investigation", by Messrs. H. W. Manly and T. G. Ackland, With Tables of Progress of Typical Funds for Officers and Workmen, and Examples, by Mr. L. E. Clinton, was read in abstract by Mr. Clinton.

The following gentlemen took part in the discussion:—Messrs. E. C. Thomas, T. Tinner, H. E. W. Lutt; Carson Roberts and A. W. J. Russell (visitors); and the President.

The Sixty-Fifth Annual General Meeting, 3 June 1912.

Mr. H. W. ANDRAS (Vice-President) in the Chair.

The proceedings at the Annual General Meeting will be found on page 438.

REPORT, 1911-1912.

The Council have the pleasure to report to the Members upon the progress of the Institute during the Session of 1911-1912, the sixty-fourth year of its existence.

There has been a *decrease* of 14 in the total number of members, as compared with the previous year. At the end of the official year in which the Institute was incorporated by Royal Charter the number of Members was 434, while twenty years later, at 31 March 1905, it was 881. Since that time the numbers have been as follows:

On 31 March	Fellows	Associates	Students	Honorary and Corresponding Members	Total
1906	232	301	367	22	922
1907	248	303	383	22	956
1908	253	313	421	22	1,009
1909	254	325	400	19	998
1910	259	335	348	21	963
1911	267	339	308	20	934
1912	278	354	268	20	920

The following schedule shows the additions to, and the changes and losses in the membership which have occurred during the year ending 31 March last:

Schedule of Membership, 31 March 1912.

	Fellows	Associates	Students	Corres- ponding Members	Total
i. Number of Members in each class on 31 March 1911 .	267	339	308	20	934
ii. Withdrawals by					
(1) Death	1	3	1	...	39
(2) Resignation or otherwise	6	28	...	
	266	330	279	20	895
iii. Additions to Membership					
(1) By Election	2	25
(2) By Order of Council	13	...	
(3) By Re-instatement	1	4	5	...	
	267	336	297	20	920
iv. Transfers					
(1) By Examination:					
<i>from Associates</i>	7
<i>to Fellows</i>	7
	274	329	297	20	920
(2) By Examination:					
<i>from Students</i>	4
<i>to Fellows</i>	4
	278	329	293	20	920
(3) By Examination:					
<i>from Students</i>	25
<i>to Associates</i>	25
v. Number of Members in each class on 31 March 1912 .	278	354	268	20	920

[Continued on page 434.]

Dr.			Revenue Account for the					
1911.			1912.					
£	s.	d.	Amount of Funds at the beginning of the year—			£	s.	d.
0,736	15	3	General Fund (including Stock of Publications, other than <i>Journal</i>)			9,861	9	7
412	3	10	Messenger Legacy Fund			424	11	2
303	2	7	Brown Prize Fund			312	4	5
0,452	1	8				10,598		
			Subscriptions—					
793	16	0	Fellows			819	0	0
680	8	0	Associates			708	15	0
334	19	0	Students			286	13	0
87	13	6	Probationers			97	2	6
1,896	16	6				1,911	10	6
25	4	0	One Annual Subscription compounded for			10	10	0
...			Fines on Reinstatement			3	3	0
1,922	0	6				1,925		
			Application Fees—					
4	4	0	Associates			4	4	0
25	4	0	Students			14	3	6
30	9	0	Probationers			34	13	0
59	17	0				53		
122	17	9	Profit on Sales of Publications			139		
			Dividends and Interest—					
289	16	4	General Fund			303	2	10
12	7	4	Messenger Legacy Fund			12	14	9
9	1	10	Brown Prize Fund			9	7	4
311	5	6				325		
2,868	2	5				£13,041		

Publications Account for the					
£	s.	d.			
673	15	1	Stock (excluding <i>Journal</i>) at the beginning of the year		
...			Expenditure on Account of A. W. Watson's Lectures on Friendly Societies		
...			Cost of New Edition of "Short Collection of Actuarial Tables"		
17	13	9	Binding and Advertising		
122	17	9	Profit on Sales		
£814	6	7			

Balance Sheet,					
£	s.	d.	LIABILITIES.		
9,861	9	7	General Fund		
233	9	2	Messenger Legacy Fund		
191	2	0	Accumulated Dividends		
424	11	2			
200	0	0	Brown Prize Fund		
112	4	5	Accumulated Dividends		
312	4	5			
85	1	0	Examination Fees for year 1912		
51	10	11	Sundry unpaid Accounts		
0,734	17	1			

<i>year ending 31 March 1912.</i>				<i>Cr.</i>					
1911.				1912.					
£	s.	d.	Journal—	£	s.	d.	£	s.	d.
645	2	0	Printing of Nos. 244, 245, 246, 247, 248	771	13	9			
86	5	0	Clerical assistance	86	5	0			
731	7	0		857	18	9			
233	1	8	<i>Less</i> Sales during the year	280	3	1			
498	5	4					577	15	8
74	13	4	Library—Binding, Purchases, &c.				76	8	0
68	13	6	Meetings				64	10	4
352	16	3	Examination charges	303	15	6			
219	9	0	<i>Less</i> Fees received from Candidates (1911)	225	15	0			
133	7	3					78	0	6
265	13	0	Tutors for classes in Parts I and II	296	2	0			
181	13	0	<i>Less</i> Fees received from Students	212	2	0			
84	0	0					84	0	0
4	2	8	Legal Charges				66	16	0
600	0	0	Office Expenditure—Rent	600	0	0			
306	8	0	Salaries	352	6	6			
107	14	7	House expenses	73	13	5			
14	13	8	Corporation Duty	15	5	11			
30	18	4	Fire and other Insurance	29	8	11			
131	19	7	Stationery and Printing	118	12	9			
36	5	0	Furniture and Fittings	15	18	3			
46	1	3	Postage and Telegrams	43	18	6			
27	14	9	Sundries	8	2	10			
1,301	15	2					1,257	7	1
105	0	0	Donation to King Edward Memorial Fund		
...			Donation to Sir Francis Galton Laboratory Fund				52	10	0
...			Honorarium for Lectures on Friendly Societies, by A. W. Watson (charged to Messenger Legacy Fund)				63	0	0
10,598	5	2	Amount of Funds at the end of the year as per Balance Sheet				10,720	18	0
				<i>Examined and found correct, 1 May 1912.</i>					

<i>year ending 31 March 1912.</i>						
£	s.	d.		£	s.	d.
341	13	10	Sales			302 13 11
...			Received in Settlement of Claim on Fire Insurance Policy for Depreciation			20 0 0
472	12	9	Stock (excluding <i>Journal</i>) at the end of the year			371 19 8
			<i>Examined and found correct, 1 May 1912.</i>			
£814	6	7	J. CHAS. WARDROP,	} <i>Auditors.</i>	£694 13 7	
			H. J. PEARCE,			
			H. LUCEY,			

31 March 1912.

£	s.	d.	ASSETS.	£	s.	d.
2,460	0	0	£3,000 Natal 3 per-cent Inscribed Stock	2,460	0	0
1,128	0	0	£1,200 Metropolitan Railway 3½ per-cent Debenture Stock	1,128	0	0
2,279	11	3	£2,000 Great Eastern Railway 4 per-cent Debenture Stock	2,279	11	3
965	0	0	£1,000 Great Northern Railway Preferred Ordinary Stock	965	0	0
1,660	10	0	£1,350 Great Western Railway 4½ per-cent Debenture Stock	1,660	10	0
498	3	6	£500 Dominion of Canada 3½ per-cent Registered 1930-50 Stock	498	3	6
491	18	6	£500 New South Wales 3½ per-cent Inscribed 1930-50 Stock	491	18	6
472	12	9	Stock of Publications (excluding <i>Journal</i>) in hand	371	19	8
...			Cash on Deposit Account	300	0	0
779	1	1	Cash on Current Account and in hand	733	17	1

			<i>Examined and found correct, 1 May 1912.</i>			
			J. CHAS. WARDROP,	} <i>Auditors.</i>	£10,889 0 0	
			H. J. PEARCE,			
			H. LUCEY,			

There are also 181 candidates admitted as Probationers, and 59 as Students conditionally on their passing Part I of the Examination. These are not included in the above Schedule of Membership.

The Council have, with great regret, to report the loss by death, since the last Annual Meeting, of two Fellows, Sir Henry Harben and Mr. W. Hughes; three Associates, Messrs. T. J. W. Buckley, A. Powell, and G. Wilson; and one Student, Mr. G. M. Van Homrigh.

The Annual Subscriptions, together with admission and other fees, amounted to £2,416. 1s. 0d., as compared with £2,382. 19s. 6d., received in the previous year. The net Income and Expenditure for the year were £2,443. 0s. 5d., and £2,320. 7s. 7d. respectively.

The stock in hand of the Institute publications on 31 March was as follows:

No. of Copies	Description of Work
20,957	Parts of <i>Journal</i> .
761	Index to Vols. 1 to 40.
25	<i>Text-Book</i> , Part I (New Edition).
489	„ Part II (Second Edition).
647	Government Joint-Life Annuity Tables.
746	Select Life Tables.
726	A Short Collection of Actuarial Tables (New Edition).
1,366	Frequency-Curves and Correlation (W. P. Elderton).
61	Messenger Prize Essay (Friendly Societies).
47 <i>in cloth</i>)	Lectures on Finance and Law (Clare and Wood Hill).
2,415 <i>in paper</i>)	
1,545	Lectures on the Companies Acts (A. C. Clauson).
1,294	Lectures on the Law of Mortgage (W. G. Hayter).
742	Lectures on the Measurement of Groups and Series (A. L. Bowley).
1,550	Lectures on the Construction of Tables of Mortality, &c. (G. F. Hardy).
1,135	Lectures on Stock Exchange Investments (J. Burn).
332	South African War Mortality (F. Schooling and E. A. Rusher).
316	Barrand's Paper on Life Assurance Law.
1,724	British Offices' Valuation Tables.
653	Transactions of the Second International Congress of Actuaries.
1,300	Examination Questions, 1908-11.

The following papers were submitted at the sessional meetings of the Institute, namely:

27 November 1911.—“A new method of approximating to the values of Last Survivor Annuities, and to the values of Joint Life Annuities when the advantages of Makeham's Law are not available.”—Mr. G. J. Lidstone.

18 December 1911.—“Some recent Statistical Results. A review.”—Mr. W. Palin Elderton.

29 *January* 1912.—“The Investment of Life Assurance Funds.”—Mr. G. E. May.

26 *February* 1912.—“On the principal provisions of the Law of Bankruptcy in England, with references to some decisions of interest to Life Insurance Companies.”—Mr. N. J. Carter.

25 *March* 1912.—“Notes on the Construction of Mortality Tables.”—Messrs. W. Palin Elderton and R. C. Fippard.

29 *April* 1912.—“On the Superannuation and Pension Funds of certain Metropolitan Borough Councils, their Establishment, Administration, and Actuarial Investigation.”—Messrs. H. W. Manly and T. G. Ackland. With Tables of Progress of Typical Funds for Officers and Workmen, and Examples.—Mr. L. E. Clinton.

For the Examinations held in the United Kingdom and the Colonies on 22, 23, 24, 25, 26 and 27 April 1912, 234 entries were received, namely :

87	for Part	I.
27	I, (§) 3.
57	II.
44	III.
19	IV.

The results will be duly announced.* The Council warmly acknowledge the valuable services of the Board of Examiners, and also those of the Honorary Supervisors at centres other than London.

In December last the Council accepted, with much regret, Mr. T. G. Ackland's resignation of the Editorship of the Journal. The Council take this opportunity of recording their thanks and great appreciation of the valuable services rendered to the Institute by Mr. Ackland in this connection since his appointment in February, 1905. Mr. R. Todhunter and Mr. J. Spencer have kindly carried on the editorial duties since Mr. Ackland's resignation, and the Council are pleased to be able to report that these gentlemen are willing to continue to act as Joint Editors for the present.

During the year an enlarged edition of the Short Collection of Actuarial Tables has been issued. The publication will no doubt be of practical value to the Members generally, although prepared mainly for the use of Candidates in the Examinations.

A series of Lectures on Friendly Societies was delivered during the Session by Mr. A. W. Watson. The Lectures will be published in book form at an early date, and it is believed that the volume will prove of considerable value to Students, and be equally useful to the profession at large.

The passing of the National Insurance Act has greatly increased the need for the services of Actuaries, and the Council have to record in this connection their satisfaction at the appointment of a leading member of their body, Mr. A. W. Watson, to the important positions of Chief Actuary to the National Health Insurance Joint Committee and Actuarial Adviser to the Chief Registrar of Friendly Societies. It is also gratifying to the Council to report that the National Health Insurance Joint Committee have appointed an Actuarial Advisory Committee, under the chairmanship of Mr. G. F. Hardy, Past-President of the Institute, to advise them on actuarial matters arising out of the administration of the

* See p. 436.

Act; the other members of the Advisory Committee are Messrs. Gordon Douglas, D. C. Fraser, G. J. Lidstone, and A. W. Watson.

During the year the President received the honour of Knighthood. The Council greatly appreciated the compliment thus paid to Sir Gerald Ryan personally, and through him to the Profession, and they have by Resolution conveyed their congratulations to Sir Gerald. The terms of the Resolution appeared in the number of the Journal for July, 1911.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS, 1912.

Examinations were held on the 22nd, 23rd, 24th, 25th, 26th and 27th of April 1912, in the United Kingdom, the Colonies, and India, at London, Liverpool, Edinburgh, Melbourne, Sydney, Montreal, Toronto, Ottawa, Winnipeg, Calcutta, with the following results.

The successful candidates are placed in two classes only, the names being printed in alphabetical order in each class.

PART I.

Eighty-seven candidates sent in their names, of whom seventy-eight presented themselves (sixty-six in the United Kingdom, and twelve in the Colonies), and twenty-two passed, namely:—

Class I:

Barnsley, J. C.
Smith, V. R.

Tambe, M. R.
Wallis, H.

Class II:

Bazell, R.
Clegg, C.
Cole, G. H.
Cooper, W. A.
Corble, E.
Dean, F. M.
Frame, J.
Gostelow, C.
Hurd, H. G.

Iyer, N. Rama.
Jennings, A.
Kearney, R. A. M.
Rider, W. W. H.
Rotwand, T.
Thurston, O. Fa.
Thomas, E. M.
Varney, F.
Woffindin, R. H.

PART II.

Fifty-seven candidates sent in their names, of whom fifty presented themselves (forty-two in the United Kingdom, and eight in the Colonies), and seventeen passed, namely:—

Class I:

Alison, S. H.

Class II :

Brenton, W. P.	Manly, G. W.
Edwards, A. J. C.	Olifiers, E.
Evans, A. W.	Pickworth, E. B.
Gopp, J. I.	Rotwand, T.
Gravatt, H. C. A.	Shaw, D. W.
Johns, A. H.	Stockman, G. D.
Ledger, R. J.	Trachtenberg, H. L.
Linton, M. A.	Warwick, R. W.

PART III.

Forty-four candidates sent in their names, of whom forty-three presented themselves (thirty-eight in the United Kingdom, and five in the Colonies), and eighteen passed, namely :—

Class I :

Cockburn, A.

Class II :

Barrett, W. G.	Nathan, E. B.
Edwards, H. A.	Perry, S. J.
Fraser, D. S.	Phillips, E. W.
Harvey, P. N.	Robertson, B.
Hawes, E. E.	Rowland, S. J.
Hutton, G. C.	Sharp, H. G.
Laing, J. M.	Smither, H. B.
Lever, E. H.	Stocks, J.
Lewty, F. A.	

PART IV.

Nineteen candidates sent in their names, of whom eighteen presented themselves (all in the United Kingdom), and twelve passed, namely :—

Class I :

† Henry, A.

Class II :

† Ball, S. R.	† Leigh, S. G.
† Clemens, F. B.	† Reeve, G. M.
† Deck, J. G.	† Sturt, H. R.
† Doucet, G. D.	† Thompson, J. H. R.
† Jones, L. A. M.	† Wenyon, H. J.
† King, A. E.	

Those marked (†) have now completed the Examination for the Class of Fellow.

PART I, § 3.

(COMPOUND INTEREST AND ANNUITIES.)

Twenty-seven candidates, who had already passed, or been exempted from, Part I of a Syllabus prior to 1908, entered for this section alone, of whom twenty-two presented themselves (eighteen in the United Kingdom, and four in the Colonies), and nine passed, namely :—

PART I.—THIRD PAPER ONLY.

Gopp, J. I.	Muckle, C. P.
Gravatt, H. C. A.	Taylor, F. G.
Harrison, A. L.	Taylor, F. R. S.
Lithgow, J. H. F.	Welch, L. G.
Martin, W. A.	

By Order of the Council,

A. LEVINE.

Chairman of Board of Examiners.

L. F. HOVIL,

R. R. TILT.

Joint Honorary Secretaries.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Sixty-fifth Annual General Meeting of the Institute was held on Monday evening, 3 June 1912, at Staple Inn Hall, Holborn. Mr. H. W. Andras (Vice-President) in the Chair.

Before beginning the business of the meeting, the Chairman referred to the absence of the President. The members would have heard with great regret of his serious illness. Sir Gerald Ryan had been about to go to the United States and would not, therefore, have been able in any case to occupy the Chair that evening. He had had a letter from him, dated before his illness, in which Sir Gerald said :—" My dear Vice-President, I understand " you have been kind enough to say you would take my place in the chair " at the annual meeting of the Institute on Monday, 3rd June, and you " would add to my obligations to you if you would express to the members " my feelings of regret that special business across the water may prevent " me from being present on so important an occasion. I regret this very " deeply, because it would have afforded me the opportunity of expressing " to all the members of the Institute my sincere appreciation of the courtesy " and support which I have invariably received during my two years' " occupancy of the Chair. I shall always look back with extreme pleasure " on this eventful and happy period of my life.—Yours truly, G. H. RYAN." He was very glad to be able to state that the President was making good progress and hoped to be in his usual robust health at no distant date.

The Report of the Council (given on p. 430) having been taken as read,

The CHAIRMAN, in moving the adoption of the Annual Report and Accounts, copies of which had been circulated among the Members, said that it would be noticed that there had been a decrease of 14 in their numbers. Happily that was not so large a decrease as in the two previous

years. The membership of the Institute had declined since 1908 from 1,009 to 920. It was not altogether possible to account for that. At the same time the Council did not consider it a very unfavourable state of affairs, inasmuch as the number of Fellows, out of a total of 920 members, was 278 as against 267 last year, and the number of Associates 354 against 339. So although there had been a slight decrease in the number of members the quality had been kept up; the proportion of qualified actuaries to the whole number had increased. There were 181 candidates admitted as Probationers, who would doubtless come on and increase the number of members in the next schedule of membership. The Council regretted to report the loss by death of two well-known Fellows—Sir Henry Harben and Mr. W. Hughes; three Associates—their good friend Mr. T. J. W. Buckley, whom they all knew so well, Mr. A. Powell and Mr. G. Wilson; and one Student—Mr. G. M. Van Homrigh. The annual expenditure was within the income, the net income and the expenditure for the year being £2,413 and £2,320. The list of publications was of great interest to all members, and of very much greater worth than their value in £. s. d. They were of great utility to the students and to all members of the Institute; and the Council hoped the list would be added to before the next annual meeting by the very valuable Lectures which had been given by Mr. A. W. Watson on "Friendly Societies." He did not know whether the present was the proper time to make such a statement, but he had always looked forward to the occasion when the Institute would have a Text-Book for the actuarial subjects of Parts III and IV. That was a matter perhaps which was rather for the new President and Council to deal with. It had also been suggested that the Institute should have a Treatise written by a competent man on the Differential and Integral Calculus in its bearing on actuarial subjects. He thought that would be a very useful book, especially to the students for Parts I and II.

Referring to the Examinations, which were full of interest to the Examinees—and he took the opportunity of congratulating those present who had passed their examination,—he had in his hand some very interesting statistics. In Part I, in 1912, 66 sat and 16 passed—a percentage of 24·2. In 1911 62 sat, and the percentage of passes was 14·5. In 1910 the percentage of passes was 19·2, namely, 10 out of 52 who sat. In the present year, therefore, the percentage showed some improvement. A larger number of candidates presented themselves and 16 passed. At the same time he did not consider the figures altogether satisfactory, and it had surprised him that the percentage of passes was not larger. With regard to Part II, 42 sat in 1912, and the percentage of passes was 28·6, as against 43·4 in 1911 and 33·3 in 1910. That was lower than the previous two years. It seemed rather a low percentage having regard to the fact that there was a Text-Book for Part II. However, it was fairly satisfactory. With regard to Part III, there had been a great improvement in the percentage of passes; 38 sat and 44·7 per-cent—17 out of the number who sat—passed the examination, as against 36·8 and 28·8 in 1911 and 1910 respectively. With regard to Part IV, 66·6 per-cent—that was to say two-thirds out of the number of 18 who sat—had passed, as against 71·4 in 1911 and 62·5 in 1910. He had given these percentages at length because he thought they might specially interest the members. There was another point with reference to the examinations which was also of very great interest, and that was the proportion of candidates attending the Institute classes who were successful in the examinations of Parts I and II. In 1912 the percentage was 62·5 as against 44·4 in 1911 and 55·5 in 1910. That was in Part I. With regard to Part II, in 1912 the percentage was 58·3; in 1911, 30·4, and, in 1910, 44·4. So that the percentage this year of passes from the classes for Parts I and II had been very satisfactory and very much better than in the two previous years. That reflected great credit not only on the pupils but also on those who conducted the classes.

Since the last annual meeting the National Insurance Bill had become an Act and had been discussed *ad nauseam*. It was shortly coming into force, and perhaps it would be more discussed then. It was open to a great deal of criticism. At the last annual meeting Sir Gerald Ryan had expressed the hope that the word "Actuary" would be substituted for the word "Valuer" in the clause relating to the valuation of the assets and liabilities of Friendly Societies. The Institute had not been successful in accomplishing that, but they hoped that they might bring influence to bear on the officials of the Treasury to be very careful in their interpretation of the word "Valuer" when appointing actuaries to value under the Act. He would conclude by formally moving the adoption of the Annual Report and Accounts.

1900 32

Mr. G. J. LIDSTONE said it was quite unnecessary to add anything to the full remarks which the Chairman had made on the subject of the report, and he would therefore content himself with formally seconding the resolution which had been moved from the Chair.

The motion was then put and carried unanimously.

ELECTION OF OFFICERS.

A ballot was then taken for the election of the President, Vice-Presidents, Council and Officers for the ensuing year. The Scrutineers reported that the following Fellows, recommended by the Council, had been duly elected:

President.

FREDERICK SCHOOLING.

Vice-Presidents.

HENRY WALSINGHAM ANDRAS.
GEOFFREY MARKS.

GEORGE JAMES LIDSTONE.
WILLIAM PEYTON PHELPS, M.A.

Council.

THOMAS GANS ACKLAND.
HENRY WALSINGHAM ANDRAS.
ARTHUR RHYS BARRAND.
*JOSEPH BURN.
ROBERT CROSS.
WILLIAM PALIN ELDERTON.
DUNCAN CUMMING FRASER, M.A.
GEORGE FRANCIS HARDY.
JAMES ROBERT HART.
LEWIS FREDERICK HOVIL.
ABRAHAM LEVINE, M.A.
GEORGE JAMES LIDSTONE.
GEOFFREY MARKS.
*VYVYAN MARE.
ALFRED MOORHOUSE.

WILLIAM PEYTON PHELPS, M.A.
SIR GERALD HEMMINGTON RYAN.
FREDERICK SCHOOLING.
JOHN SPENCER.
WILLIAM RICHARD STRONG.
HERBERT CECIL THISSELTON.
ROBERT RUTHVEN TILT.
GEORGE TODD, M.A.
*RALPH TODD HUNTER, M.A.
*HAROLD MOLTKE TROUNCER, M.A.
*SAMUEL GEORGE WARNER.
ALFRED WILLIAM WATSON.
JAMES DOUGLAS WATSON.
ERNEST WOODS.
FRANK BERTRAND WYATT.

Treasurer.

SAMUEL GEORGE WARNER.

Honorary Secretaries.

LEWIS FREDERICK HOVIL.

| ROBERT RUTHVEN TILT.

MR. GEO. R. JELlicoe then proposed the re-election, as Auditors, of Messrs. H. J. Pearce and H. Lucey, who, he said, together with the retiring Auditor, Mr. J. C. Wardrop, merited the thanks of the Institute for their services during the past year. He had very great pleasure in moving the election of Mr. W. G. Titmuss as a new Auditor for the current year. He was sure Mr. Titmuss would worthily maintain the traditions of the office.

The motion was duly seconded, and carried unanimously.

MR. S. G. WARNER, in proposing a vote of thanks to the President, Vice-Presidents, Council, Officers, Examiners and Honorary Supervisors to the examinations at centres other than London, said that in reading the terms of the resolution he had probably done all that was required to commend it to the meeting. But it would be ungrateful on the part of the members who had profited so much by the services of those gentlemen during the past year to be satisfied with a mere formal resolution, and it certainly would be so in the case of the President. He was sure he need do no more than mention the name of Sir Gerald Ryan as an occupant of the Chair of the Institute to remind its members that in Sir Gerald they had had one who had adorned the office and had filled it as worthily as anyone could have done. They all recalled the grace, dignity, tact and wisdom with which Sir Gerald discharged the duties of his responsible post. Most of them had heard him in his contributions to the various discussions, and had listened to words of wit and wisdom which were exceedingly appropriate to the subject in hand, and which would remain in their memories. In the other responsibilities which had fallen upon the President during the past two years, responsibilities which had been wider and weightier than usual on account of the public interest which had been directed to the actuarial profession in connection with important legislation, they knew there again how wonderfully Sir Gerald Ryan had represented them. He need not say how deeply they regretted his recent sudden serious illness, nor how happy they all felt that he was well on the way to complete recovery.

The Vice-Presidents, Council and Officers, whom, perhaps, he might excusably take together as representing the corporate interests of the Institute, undertook a responsibility which was an ever-increasing one. The functions of the actuary in regard to the State were widening, and the Institute held the premier position as the guardian of the interests of the profession. The members believed that in the hands of the Council and Officers those interests were safe. The Institute had now been in existence for 64 years. He believed that 65 years was, in the case of individuals, the orthodox age at which retirement might take place, and a little well-earned leisure be entered upon. But that did not apply to the Institute, because as it approached that rather venerable age duties seemed to be accumulating upon them far beyond anything they had contemplated in the past. They hoped and believed that their profession would rise to the occasion, and discharge those responsibilities in a manner worthy of all its traditions; and they thanked the Council, Officers and Vice-Presidents for being, during the past year, the worthy custodians of those interests, and worthily discharging the duties committed to them. The Examiners deserved the members' special sympathy and consideration, because their work was of an exceedingly engrossing character and sometimes rather a thankless duty. Theirs, again, was a task which was increasing in magnitude, complexity and importance as the years went by. To all of those, then, the members would like to render the tribute of their thanks for having discharged well and worthily the responsibilities which they undertook.

He wanted just in a word to add another resolution which required no comment. It was to the effect "That the members of the Institute of Actuaries, assembled at its general meeting, desire to express their sympathy with Sir Gerald H. Ryan, its retiring President, in his present illness and their earnest hope that he may speedily and completely recover."

Mr. W. R. DAY remarked that if the occasion were an ordinary one he supposed he would be saying enough if he formally seconded the resolution, but the occasion, as far as he was concerned, was a unique one, for he had not had the pleasure of attending a meeting of the Institute for something like twenty years. He had been an exile in a far corner of the Empire—in Australia. It was not only his own personal desire which brought him to the meeting that evening, but he had been specially asked to attend by the actuaries in Sydney, as represented by the Actuarial Society, of which he had the honour to be President, and to convey to the Members and the President and the Council of the Institute their very fraternal good wishes. He could assure them that the actuaries in Sydney had the highest feelings of loyalty to the Institute. They might not always agree with everything that went on in the Institute and that was done there, but at the same time they were exceedingly loyal to the Institute, and had no wish whatever to sever their connection or set up on their own in any way. If those present wanted to test what he had just said about the loyalty of Australian actuaries, he would invite them to Australia, and he could assure them of a most cordial and hearty welcome, a welcome which had already been proved by two very distinguished actuaries—Mr. R. P. Hardy and Mr. George King—who had within the last few years visited Australia.

The other part of the resolution was one he was very sorry to have to second, because he had hoped he would have the pleasure of seeing Sir Gerald Ryan in the chair. He would especially like to convey the sympathy of Australian actuaries to Sir Gerald Ryan in his illness.

The motion was then put and carried unanimously.

Mr. F. SCHOOLING, who was very heartily received, said on his own behalf, and on behalf of the Vice-Presidents, Council and Officers for the ensuing year, he had to return their most heartfelt thanks for the honour the Members had done them in electing them to their respective offices. He felt that indeed it was an honour, but, at the same time, he felt the responsibility of the office for which he had been chosen, a responsibility, perhaps, made the greater on account of the very eminent men who had preceded him in the chair, one of the greatest being Sir Gerald Ryan, to whom they had just passed a vote of sympathy, in which he hoped Lady Ryan would also be included. It would be his endeavour during his term of office, with the aid of the Council, to maintain the traditions of the Institute and to wisely examine with their help the many problems which in these modern days were bound to come up for consideration.

Mr. R. CROSS proposed a vote of thanks to the Auditors for their services during the past year. A quarter of a century ago he had himself acted as one of their Auditors, and at that time the income of the Institute was £1,500. Now it was about £2,500. Then the total funds were about £3,000; now they were £11,000. Consequently the Auditors' duties had increased in weight and importance, and were performed, no doubt, much better now than in years gone by. He thought the Institute was to be especially congratulated on having as its Auditors during the past year the chief representatives in London of two famous Scottish offices, and he thought the Institute had been wise in selecting them, especially as it was proverbial that gentlemen from the other side of the Tweed knew well how to take care of funds.

Mr. H. COCKBURN seconded the motion, which was unanimously carried.

The Meeting was then adjourned to Monday, the 25th November, 1912.

Additions to the Library.

The following works have been added to the Library since the publication of the *Journal* for October 1911:

*By whom presented
(when not purchased).*

Accountants and Auditors, Society of Incorporated

List of Members, &c., 1911-12.

The Society.

Accountants, Institute of Chartered, in England and Wales.

List of Members, 1912.

The Institute.

Actuarial Society of America.

Transactions, 1911-12.

The Society.

Containing *inter alia*—

"Liberality of Modern Policies", by H. Moir.

"A Pension Fund Method", by C. C. Ferguson.

"A determination of the Constants in Makelam's Formula by the method of Least Squares. Illustrated by graduations of the American Experience Table", by J. S. Thompson.

"Net Premiums and Reserves for policies giving disability benefits", by E. B. Fackler.

"Concerning the American Experience Table of Mortality", by S. A. Joffe.

"Report of Mortality Experience among annuitants resident in the United States and Canada", by A. Hunter.

"Extended Insurance", by A. A. Welch.

"Survivorship and Deferred Survivorship Annuities. Some points raised by recent rulings and legislation," by H. N. Sheppard.

"On the methods used in the construction of the l_x^{ua} column with a new method of calculating values. ('Waiver of Premium' benefit)", by S. H. Pipe.

"On the determination of the 'expected mortality on net amount of risk' and 'interest required to maintain reserve.' (Note on Gain and Loss exhibit)", by M. Davis.

"Mortality Experience of the Aetna Life Insurance Company under its ten year renewable term policies", by M. H. Peiler.

"Select rates of mortality amongst impaired lives and the probabilities of lives becoming impaired", by P. C. H. Papps.

"Workmen's Compensation Benefits", by W. A. Watts.

"Gill's Mortality Table", by S. A. Joffe.

Actuaries, Faculty of

Transactions, 1911-12.

The Faculty.

Containing *inter alia*—

"On a method of calculating the present values of prospective Pensions based on Salary, and the expected amounts of the future Pension payments", by J. J. McLauchlan.

"Research in Life Assurance", by L. P. Orr.

"On Rates of Premium for discounted bonus assurances", by G. W. Richmond.

"The Agents' Ledger—can it be dispensed with", by R. Murrie.

*By whom presented
(when not purchased).***American Mathematical Society.**

Transactions, 1911-12.

*The Society.***American Statistical Association.**

Transactions, 1911-12.

*The Association.***Anderson, (L. A.).**Valuation of assessment and stipulated premium policies.)
Madison, Wis. 1911.)*The Author.***Assecuranz Jahrbuch.**Herausgegeben von A. Ehrensweig. Vols. 32 and 33.)
Svo. Wien. 1911-1912.)*Purchased.***Australian Mutual Provident Society.**Mortality Experience, 1849-1903. 4to. Sydney. 1911.)
Sixty-third annual report, 1912.)*The Society.***Austria-Hungary.**Bericht der Arbeiter-Unfall-Versicherungs-Anstalt für
das Königreich Böhmen, 1910.)Die Privaten Versicherungsunternehmungen in den im
Reichsrathe vertretenen Königreichen und Ländern
im Jahre, 1908, 1909.)*The Austrian
Government.*Versicherungswissenschaftliche Mitteilungen der
Mathematisch - Statistische Vereinigung des
Österreichisch-ungarischen Verbandes der Privat-
Versicherungs-Anstalten, 1911-12.)*The Society.*Zeitschrift für öffentliche und private Versicherung.)
1911-12.)*The Editor.***Bachelier (L.).**

Calcul des Probabilités. Vol. I. 4to. Paris, 1912.

*The Author.***Bankers.**

Journal of the Institute of Bankers.)

List of Members, 1912.)

*The Institute.***Barrand (A. R.).**On some legal points arising in Life Assurance Practice,)
1896. Further notes on some legal aspects of)
Life Assurance Practice, 1907. In one vol. Svo.)*The late
Wm. Hughes.***Belgium.**

Bulletin de l'Association des Actuaire Belges.

The Association.

Bulletin du Comité central du travail industriel, 1911-12.

*Le Comité.*Bulletin du Comité permanent des Congrès inter-
nationaux d'Actuaires, 1912.)*Le Comité.*Bulletin du Syndicat des Compagnies d'Assurances-vie
populaires opérant en Belgique, 1911-12.)*Le Syndicat.*Compte Rendu des Opérations et de la Situation de la
Caisse Générale d'Épargne et de Retraite. 1911.)*The Belgian
Government.***Besso (M.).**La Previdenza Sociale nel Risorgimento. 4to. Rome.)
1911.)*The Author.*

*(By whom presented
when not purchased).*

"Biometrika."

Volume VIII, Parts III and IV.

Purchased.

Containing *inter alia*—

"On the relation of stature and weight to pigmentation", by Ethel M. Elderton.

"Supplementary tables for finding the correlation coefficient from tetrachoric groupings", by P. F. Everitt.

Birmingham, Insurance Institute of
Transactions, 1910-11.

*The Insurance
Institute.*

Brecher (Dr. B.).

Versicherung auf fremden tod. 8vo. Wien. 1912.

The Author.

Browne (E.) and H. Kingsley Wood.

The law of National Insurance. With introduction
and notes. 8vo. 1912.

Purchased.

Cameron (J. C.).

Treatment of sub-standard risks. Texas. 1912.

The Author.

Canada.

Canadian Annual Financial Review, compiled by W. R.
Houston. With Appendix. Ob. 8vo. Toronto.
1911, 1912.

Purchased.

Carr (A. S. Comyns), W. H. S. Garnett and J. H. Taylor.

National Insurance. To which is appended the Act of
1911, with notes. 8vo. 1912.

Purchased.

Clarke (Orme).

The National Insurance Act, 1911. Being a treatise
on the scheme of National Health Insurance and
Insurance against Unemployment created by
the Act, with the incorporated enactments, full
explanatory notes, tables, and examples. 8vo. 1912.

Purchased.

Clifford (W. K.).

The common sense of the Exact Sciences. 5th Edit.
8vo. 1907.

Purchased.

Clifton (E. C.) and A. Grimaux.

English-French and French-English Dictionary, com-
piled on a new plan. 2 vols. La. 8vo. 1910.

Purchased.

Dawbarn (C. Y. C.).

Employers' Liability and Workmen's Compensation.
4th Edit. 8vo. 1911.

Purchased.

Dawson (W. H.).

Social Insurance in Germany, 1883-1911. Its history,
operation, results, and a comparison with the
National Insurance Act, 1911. 8vo. 1912.

Purchased.

Denmark.

Beretning fra Forsikringsraadet for aaret, 1910.
Another Copy.

*Danish
Government.
H. W. Andras.*

*By whom presented
(when not purchased).*

Deutscher Verein für Versicherungs-Wissenschaft.

- Sammlung von Versicherungsbedingungen Deutscher
Versicherungsanstalten. Fünfter Teil. Die in
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OBITUARY.

Mr. William Hughes.

AT the Meeting of the Institute of Actuaries on April 29 the President said that before passing on to the regular business of the evening he had to refer with deep regret to the death of the late Mr. William Hughes, a distinguished predecessor of his in the Chair of the Institute. At the time of his death Mr. Hughes had been a Fellow of the Institute for forty years, and during that long period he invariably took a deep interest in the work of the Institute. His professional career might be said to have culminated in 1902, when he was elected to the high position of President. Many members of the Institute had a vivid recollection of the ability and dignity with which he performed the onerous duties attaching to the position, and it would be many a day before they forgot the urbanity and personal charm which endeared him to a wide circle of friends and were especially conspicuous at the Actuarial Congress held in America, where he represented the Institute with so much distinction and success. He was sure every member would mourn the loss of one who left behind him such a long record of faithful and unostentatious service to the profession. The Council would in due course convey its condolences to the family of their deceased friend.

Mr. A. H. Bailey.

THE actuarial profession will have learned with deep regret of the death of Mr. Arthur Hutcheson Bailey, which took place at Bournemouth, whither he had recently removed for the benefit of his health. With him has passed away the last link with the earliest days of the Institute of Actuaries. He was one of the small band of candidates who presented themselves for the first examination of the Institute, and at the sessional meeting on the 24 June 1850, he was awarded the Certificate of competency and immediately elected a Fellow.

Mr. Bailey was born on the 12 October 1823, and died on the 25 August last. After leaving school he entered an architect's office, but he commenced his insurance career shortly afterwards, when, in 1841, he was appointed to a clerkship in the

Protector Life Association, of which the late Mr. Charles Jellicoe was the Actuary and Secretary. When, in 1847, that Company was amalgamated with the Eagle, he followed Mr. Jellicoe, who was appointed Actuary and Secretary of the amalgamated Company. On the retirement of Professor Sylvester from the Actuary and Secretary-ship of the Equity and Law Life Assurance Society in 1855, Mr. Bailey was appointed to succeed him, and in 1861 he accepted the appointment of Actuary to the London Assurance Corporation, where he remained until his retirement in 1895.

Trained in the best school under the ablest of masters, and possessing skill, courtesy, urbanity, sound judgment and strict integrity, he was eminently fitted to deal with that large consulting practice which came to him during a long and active professional career.

Mr. Bailey was essentially a practical actuary, taking a broad and sound common sense view on all questions relating to management and finance. Although a staunch advocate of the net premium method of valuation for ascertaining profits, he nevertheless agreed that under certain circumstances and conditions it was capable of modification, and he himself departed from it when his foresight clearly showed him that it was to the advantage of all concerned that he should do so.

He had a great affection for the Institute, and seldom missed a meeting. He served in all the honorary offices of that body, and was elected President in 1880. To his professional brethren he was always ready to give assistance and advice, and particularly to the younger members, many of whom will hold him in grateful remembrance. His familiar figure, with the kindly, genial face, will be greatly missed, but his memory will be honoured and revered by all who knew him.

H. W. M.

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